Fluid and Particle Mechanics
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Lecture - 57
Pipes of non-circular cross section

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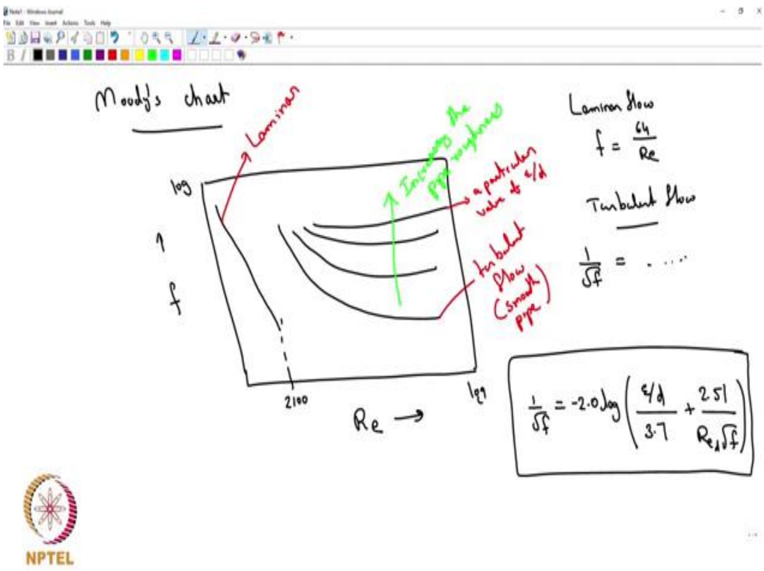

And that leads to what we call the Moody's chart. So, let us see what we have gotten what we are going to do plot a graph of friction factor on y-axis and Reynolds number on x-axis and we will do within a log coordinate system for laminar flow f is equal to 64 by R e. So, if it plot it in a log coordinate system, how would it look like? it will be a straight line of slope minus 1.

So, that is what is going to be laminar flow, some you know transition Reynolds number somewhere around 2000, 2100 let us say, you would start seeing turbulent flow. We calculated an expression for turbulent flow right. It was that some 1 by root f is equal to some expression we have calculated put log that and you will see a curve like that ok. So, this is for laminar flow that is for turbulent flow, but in a smooth pipe.

What happens is that if you start in incorporating the roughness factor, you will going to get a curves like that ok. So, that is basically to go up in this direction this is increase in the roughness. If this is you know each of the squares would be correspondent to particular value of epsilon by d. So, such a diagram becomes very useful because then you do not need to remember any correlation just use that given Reynolds number looked at your friction factor and use them for your design. They all the calculation that we have done in the last few classes is compiled into such a diagram.

If they have tried to come up you know relations we need everything together one such useful expression, if you do not want to use the graphs 1 by root f is equal to minus 2.0 log of epsilon by d by 3.7 plus 2.51 divide by Reynolds number d by root f, this expression is also sufficient to be centre calculate friction factor.

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon}{3.7d} + \frac{2.51}{Re_d\sqrt{f}}\right)$$

Yeah, yes when I have written log it is base 10, if I have written ln that would be natural logarithm. It is a general people have tried to fit that equation and reasonably reproduces the flow. So, you would use either of them actually you know you could use any of these things ok, but whenever you are going to do a design problem, you do not want to stick to exact numbers.

You would want to you know say that you know whatever number you get, you want to give some cushion to it, you will say you know I will be take 15 percentage or 20 percentage extra whatever is required. So, there things get adjusted. So, you could either you will equation or the graph, the graph is definitely going to be better so that is more discharge. Next, so, let us look at the flow you know that we had so far, we started with laminar flow then we did turbulent flow in a pipe then we talked about roughness ok. So, we are ready to do the next complication, what would be that in your opinion?

What is you know just imaginary life scenario where you want to design a pump oh yeah or a piping system rather. The next thing that, yeah you said something right now, know? Power yeah. So, power we can calculate because once you know pressure drop then you can you may just multiply it with the flow rate and you get it.

So, you know we could encounter more scenarios for example, so far we assume the pipe to be circular, need not be circular. You may have a square pipe or a triangular pipe especially you know for example, heat exchanges and stuff like that you might have different cross sections. What do we do then? Do we do the all the calculations that we have done so far for a different cross section, it becomes extremely hard.

In fact, it you cannot even do that for many of the things ok. So, therefore, you define something called a hydraulic diameter, have you heard hydraulic diameter? So that is where it comes.

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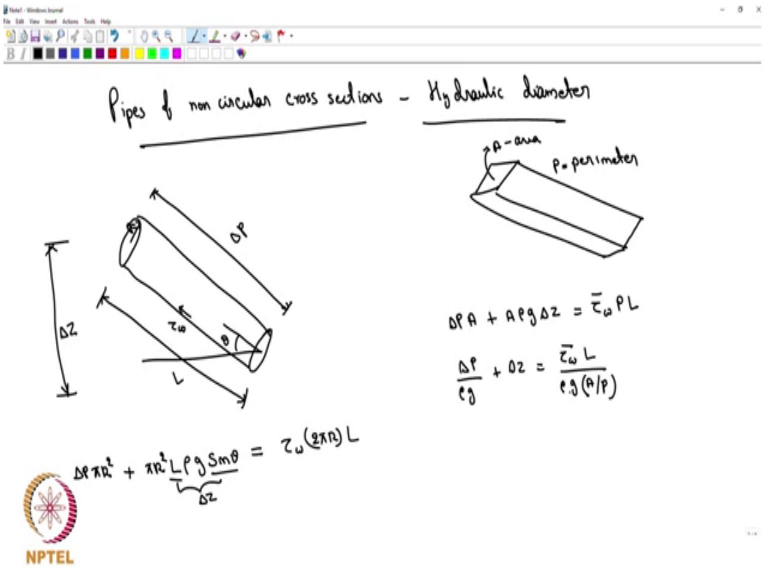

So, we want to look at pipes of non circular. So, if you define it, we need to go back and look at how did we originally define our friction factor um. We analyzed laminar flow in a pipe, we at some pipe which we must probably you would have drawn something like that a circular pipe of radius R and let us say some length L with the pressure drop let us say delta P then angle theta and some delta Z and if tau w is the wall shear stress. You would have written down some forces balance then you note a check your notes and tell me, what is the force value that we wrote for this configuration?

So, there is a pipe and then you have applied a pressure difference which makes it flow, it is inclined and therefore, weight is playing a role and of course, the viscous friction is acting against it. So, you need to apply you can write down on a pressure balance which would mean that if delta P into pi R square is the force acting along the pipe plus the weight which would be pi R square times L times row times g let us say sin theta should be balance by the viscous friction that is acting on the walls which would be tau w times 2 pi R into L, this is the equation that you might see in your notes.

Now, what we want to do is we want to look at the write down similar expression for a non circular pipe let us say some non circular pipe with some side a everything else remaining the same you can write. So, let us call this as an area of a and so, A is the area and P is the perimeter ok, you can write down the same force balances delta P into A plus this we must have simplified no L times sign theta would have given you a delta is z.

So, A times rho g times delta z is equal to tau w into p into L just generalizing it in terms of A and P, one thing that you need to be careful is that when we talked about the circular pipe ok, the wall shear stress was a constant quantity, it does not matter where you are actually looking at or were you are calculating tau w were because nothing was changing along the theta direction or z direction, but that is no more true when you talk about non circular cross section depending up on where you are looking at tau w could change.

So, we can say tau w is some average tau w that we have calculated, yeah. So, that would mean that the P plus that we let us say divide by rho g plus delta z is equal to tau w bar and L divided by rho g A by P.

$$\Delta P \pi R^2 + \pi R^2 L \rho g \sin\theta = \tau_w (2\pi R) L$$

$$\frac{\Delta P}{\rho g} + \Delta Z = \frac{\overline{\tau_w} L}{\rho g (A/P)}$$

$$\frac{\Delta P}{\rho g} + \Delta Z = \frac{2\overline{\tau_w} L}{\rho g R}$$

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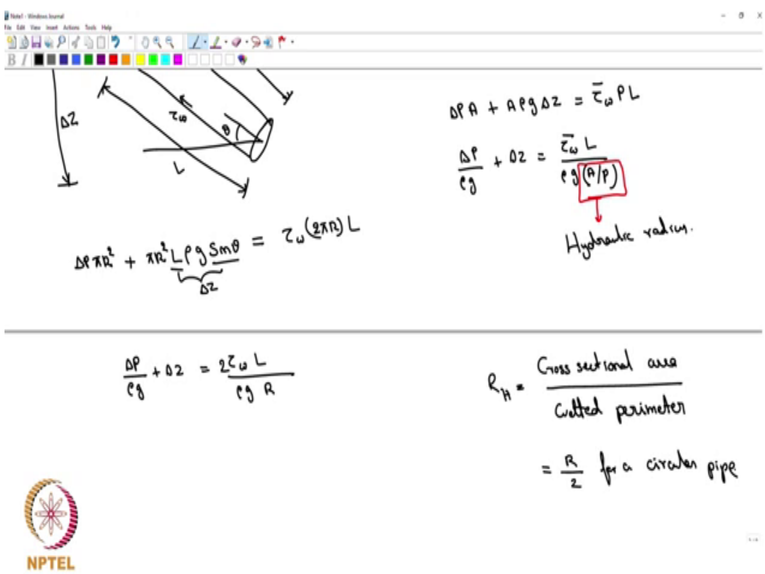

So, what you see on the left hand side is what you originally had, now you end up with an expression which contains tau w area and perimeter and so on and this quantity is what is called hydraulic radius. May in other words R H is nothing, but cross sectional area divided by wetted perimeter and that is actually for a circular pipe that will be equal to R by z right.

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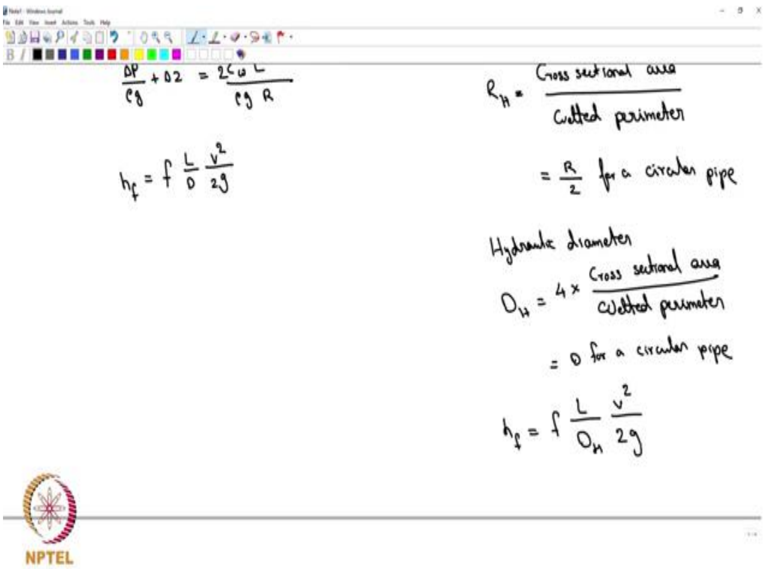

So, then define a hydraulic diameter D H is equal to 4 times cross sectional area divided by wetted perimeter. So, that mean D H is equal to D for a circular pipe um. So, in case of a circular pipe, we wrote h f the head loss is equal to friction factor times L by D v square by 2 g. In case of a noncircular pipe, we will write similar relation h f is equal to f times L by D hydraulic v average square will be different v square divide by 2 g, everything we did for the circular pipe we just borrow it, but replace all the diameters with the hydraulic diameter.

$$Hydraulic\ diameter, D_H = \frac{4\ X\ Cross - sectional\ area}{Wetted\ periemeter}$$

$$h_f = f\frac{L}{D_H}\frac{V^2}{2g}$$

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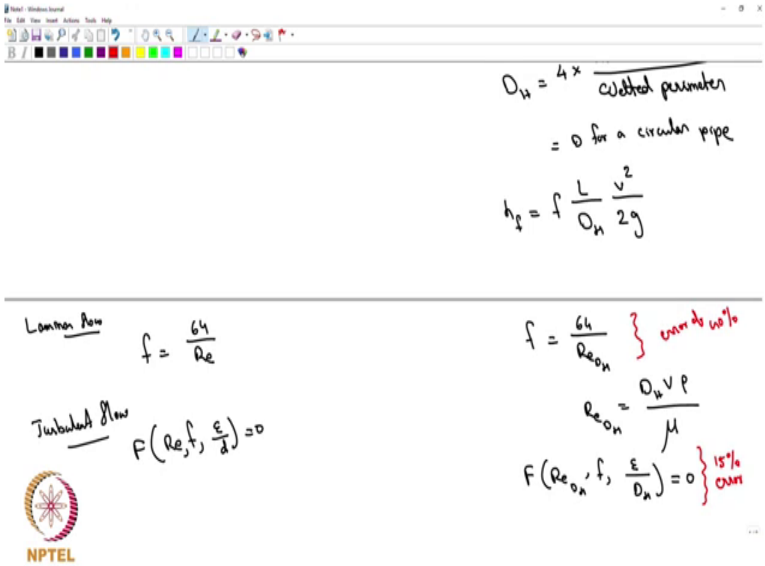

So, for a laminar flow, we had f is equal to 64 by R e. For the circular pipe here, we will define f is equal to 64 divided by R e H or rather D H; D H where the Reynolds number is now defined based on the hydraulic diameter. So, R e D H is D H times v times rho divided by mu.

For turbulent flow, we will get some relation containing Reynolds number, friction factor epsilon by d equal to 0. Here we will have you we will use the same relation, but we will use Reynolds number based on D H, f, epsilon divided by D H is equal to 0.

$$f = \frac{64}{Re_{D_H}} \; [Laminar\ flow]$$

$$f\left(Re_{D_H}, f, \frac{\epsilon}{D_H}\right) = 0 \; [Turbulent\ flow]$$

So, what I am saying is that everything they we have done so far, we will just borrow the same frame work just replace our all diameter with hydraulic diameter, assume that that is going to work. If you are wondered how good it is, this relation is going to give you things with an error of 40 percentage cubes this relations seems to be a approximately 15 percentage . So, this depends on what kind of geometry and so on, this are definitely not good relations, but what we definitely we do not want to do you no elaborate calculation for any other geometry just use the same framework ok.

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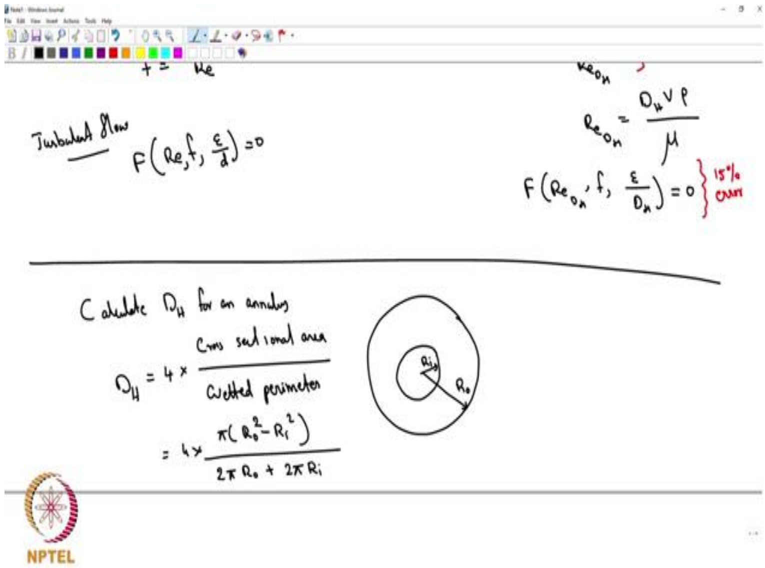

Let say you have a fluid that flowing between two pipes, have you already been you did heat transfer shall until you heat transfer heat exchanger and so on. So, imagine you have a fluid that is flowing between two pipes and you are interested in calculating the pressure drop for the fluid which is between in the gap in the annulus, what would be the hydraulic diameter.

It is going to be 4 times cross sectional area by wetted perimeter. So, now, you see right where is cross sectional area coming from and where is wetted perimeter coming from. Cross sectional area is coming because that is the one which is going to multiply with your pressure drop; wetted perimeter is coming because that is where your shear stress is acting ok. So, this really ratio what would this be, 4 into cross sectional area is pie into R o square minus R i square divide by wetted perimeter would be 2 pie R o plus two pie R i right both inside and outside both the cylinder both the surface is then generate you no it is not a shear stress it simplifies and see what you will get. So, that is how you should do the calculations.

$$D_H = \frac{4\pi(R_o^2 - R_i^2)}{2\pi R_o + 2\pi R_i}$$