Fluid and Particle Mechanics
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Lecture - 56
Roughness in Turbulent Pipe Flow

So let us start. So, what did we learn yesterday? We were looking at Turbulent Flow in a pipe right and we realized that something that you do not see, ok. So, visually when you see it you do not perceive it really that the small amount of roughness the small unevenness that be there on a pipe can affect your turbulent flow, or in other words the pipe roughness affects turbulent flow that is peculiar to turbulent flow. And we saw why it is the case right, we saw that if you look at the wall layer which is very close to the wall; the thickness of the wall layer is so small at that scale it becomes comparable to the roughness itself.

So, in real numbers it could be less than a millimeter and the wall layer is as that small and therefore, if you really look at the wall layer, wall layer is not on a flat surface, wall layer is on a surface which is very uneven. And such an uneven such a complicated you know geometry could break your viscous sub layer or the wall layer. So, that is what we were restored, ok. So, let us just make it quantitative so, that you can do calculations.

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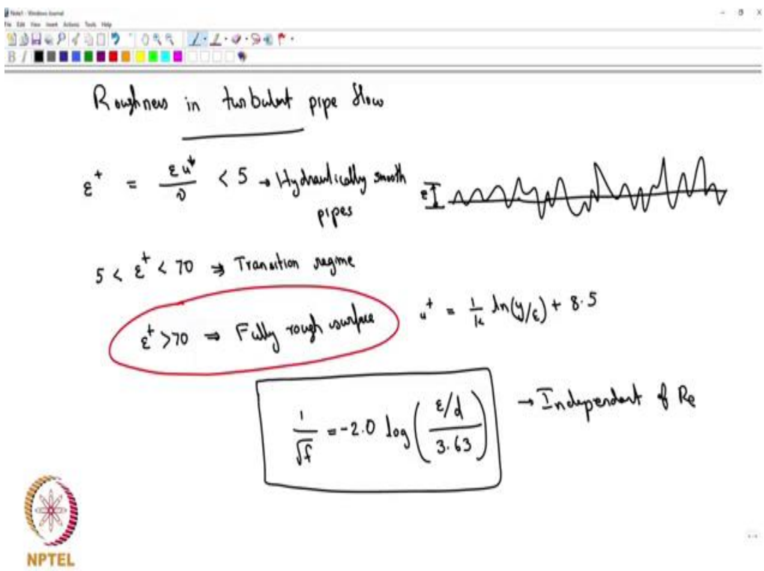

So, roughness in turbulent pipe flow. So, what we need first is some way of characterizing them. So, one way we need is we need some way of characterizing the roughness ok. So, roughness how would you imagine the roughness of the surface to be, very uneven like for example, if you know let us say that is a plate that you are looking at plate or surface.

If you start looking at it under a microscope you would see something you know very uneven like that. So, you can imagine that thick line that I have drawn is the main thing that you typically see, but if you start zooming in then you will see that it is very rough some arbitrary of arbitrary nature.

So, you need some way of characterizing this roughness, and one thing that you could use that is typically used is some length scale for this roughness, ok. So, let us say epsilon we will define a length scale epsilon which is a measure of this roughness scale, roughness length or roughness height rather. You can imagine epsilon is let us say some average height or some average or root means square height some height, some typical height of the roughness, ok.

So, that is a scale in which you have these variations let us call that epsilon. We can argue that it is not one; its one quantity is not sufficient maybe you should talk about multiple quantities to really characterize, but we of course, do not want to bring in so many things because it makes the analysis complicated. So, we want to you know stick to what we can use practically. So, epsilon let us say is the thickness that characterizes the roughness and the quantity that we want to look at is epsilon plus.

So, epsilon plus would be what? Epsilon divided by what is the scale that we would use to define epsilon plus how did we define y plus? Y. So, v divide we define y plus as y u star by nu. So, we will define it as epsilon u star by nu where nu by u star is some length scale that we have. Why we do that? Is because we said the thickness the highest or the thickness of the wall layer in terms of this plus quantity is 5 approximately right, we said y plus is approximately 5 and up to 5 you see the wall layer. So, we can say that if epsilon plus is let us say less than 5 what is that mean? That means, that if the height of the roughness is going to be smaller than the height of the wall layer or another words height of the wall layer is going to be bigger than the roughness.

So, therefore, the wall layer will not see the roughness and therefore, if epsilon plus is smaller than 5, then you can say that my pipe is actually smooth and I do not have to really worry about it about the roughness, ok. So, we say that epsilon plus is less than 5 we call it hydraulically smooth pipes, ok. In other words the thickness of the roughness is smaller than the height of the wall layer. Is that, ok?

We are just talking everything in terms of these plus quantities rather than talking about real dimensions, hydraulically smooth pipes sorry. If 5 less than epsilon plus less than approximately 70, then we say it is a transition regime, the transition is in terms of roughness, ok. Because if epsilon plus is greater than seventy then we call it a full rough surface, ok.

$$\epsilon^+ = \frac{\epsilon u^*}{\nu} < 5 \rightarrow Hydraulically\ smooth\ pipes$$

$$5 < \epsilon^+ < 70 \rightarrow Transition\ regime$$

$$\epsilon^+ > 70 \rightarrow Fully\ rough$$

So, epsilon plus is greater than 70 means the wall layer thickness is much smaller than the roughness itself, ok. So, then it is so high, so you basically have a; some classification so, you can say that on one limit you basically have very smooth pipes and on the other hand you have very rough pipes and in between you know anything that could vary you basically have a transition regime. And as usual just like in the flow, here also if things are in between smooth and fully rough calculations are harder, but otherwise so, that is why this classification, is that clear.

So, what is that we can do in the fully rough region; in the fully rough region so, let us say in this region the overlap low basically gets modified I will just take it for granted I will just write down y u plus is equal to 1 over k ln y by epsilon plus 8.5.

So, one thing that you can notice is that see when we talked about the wall layer we had defined a y plus and we defined a y plus as y u star by nu there was no really length scale that was characterizing it. So, we just defined it as y u star by nu, but now when you talk about the roughness you have brought a new length scale into the system that new length scale is epsilon.

So, it turns out that you can define u plus as in terms of a non dimensional quantity which is y by epsilon, everything else the numbers like you know k 8.5 all determined from experiments you cannot do much about it, but this is. So, the point is that it is still logarithmic, it is a logarithmic law the numbers are going to be different and it is also dependent upon the roughness height, is that clear. Yeah that epsilon is the height.

This is for a fully rough surface. More than 30 would be the what was that, . so y to 30 we had the overlap layer. So, what is the question? How do you know that?

Ok yeah, that might be true. So in fact, what we are saying is that. So, epsilon your saying is 17. Yeah. So, you basically see that the wall layer is so thin that parts of your overlap layer is occupied by the trusts itself of the roughness in some sense or maybe all the way up to their. So, in other words you are going to see essentially the overlap layer with. Yeah I think that is all it is; I am not sure whether you can really characterize that structure because that graph that we drew was for strictly smooth pipes.

So, now, we are you know essentially borrowing some of those ideas to talk about roughness. So, in principle some of those numbers might change that must be what is happening in reality, but if you just stick the borrowing then we are really into the overlapping, yeah. So, this is the expression for u plus when you have a, what a rough pipe. So, if you know u plus you know out to go ahead and calculate the friction factor and relationship between friction factor and Reynolds number the way you did it yesterday should we try.

See finally, we do not care what exactly u plus is we need a relation between friction factor and Reynolds number. So, it is I never have given you what is u plus let us go ahead and calculate what is the relationship between friction factor and Reynolds number. What would you calculate first?

Yeah. So, you can calculate the average velocity and then hopefully you will be able to manipulate that expression to give your desired relation. You can go back and finish your calculation not expected to get this relation friction factor. And what you find is that you wanted to get the expression connecting Reynolds number and friction factor, but you do not see that ok, Reynolds number will go away and only factor that would remain would be epsilon by d, where epsilon is the thickness of the roughness and d is the diameter.

So, that is definitely a non dimensional number and it looks like at fully you know rough surface is what we called the friction factor is independent of the Reynolds number. It basically depends only on the thickness thick or basically depends only on the roughness, yeah.

More the roughness; so, there is a limit to which it is affecting your you know it is disturbing your wall flow, ok. So, see these things are not independent for example, that Reynolds number would depend upon your average velocity, its coming from your flow pattern itself. So, you do not really need both Reynolds number and epsilon by d cube characterize your flow it looks like epsilon by d is the only quantity that will play a role within these approach. Epsilon will be for the pipe, yeah. Yeah.

So, that is why it’s independent of the fluid flow now. Except, wait delta p will still be dependent upon the fluid flow f is a non dimensional quantity and therefore, its independent of fluid properties. So, you will get an f and when you connect your f to let us say h f and then q a delta p all the properties density viscosity everything. So, that is a another beauty of non-dimensional number there is right that you are getting everything independently, yeah.

Yeah. So, I am saying that the logarithmic low is still so, it is experimental observation and the fact that what we have seen for a smooth pipe can be borrowed. So, there are some calculations can be done where you know you can take different layers and try to match and so on, but for us it is an experimental observation.

Alright so, you try to win it. Yeah so, more importantly it is independent of form. So, we have written down so many relations, it is sort of hard to you know keep everything in mind. So, what people have done is they have plotted all of them, ok.