

Fluid and Particle Mechanics
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Lecture - 55
Effect of rough Walls

So, we are looking at flow turbulent flow in the pipe that is what we just did in fact, right. We calculated V average divided by or other V max. So, what else then you may write V average; V average by V max is equal to 1 divided by 1 plus 1.3 square root of f ok.

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The image shows a digital whiteboard with handwritten notes. On the left, the equation $\frac{V_{avg}}{V_{max}} = \frac{1}{1 + 1.3\sqrt{f}}$ is written. To the right, two cases are shown: for $Re = 10^4$, $\frac{V}{V_{max}} = 0.8$; and for $Re = 10^8$, $\frac{V}{V_{max}} = 0.9$. Below these calculations is a diagram of a pipe with a velocity profile represented by five horizontal arrows of increasing length from the wall to the center, indicating a parabolic-like turbulent profile. The whiteboard interface includes a toolbar at the top and an NPTEL logo at the bottom left.

So, if you have not gotten it, just go back and try and do it ok. I just wanted to know, whether you have done your homework or not. So, what is that? Yeah, so V average by V max is this for the turbulent flow. So, the difference is that so, we talked about this yesterday, V average by V max is half for laminar flow and this is the factor that comes out for turbulent flow. So, let us see what this means ok. Typically, let us say if I take a Reynolds number of 10 raised to 4, then V by V max comes out to be 0.8.

If I take Reynolds number as 10 raise to 8, then V by V max comes out to be 0.9 ok, just some number. So, given Reynolds number you know, how to calculate friction factor, we derived the expressions for that. So, if you use that friction factor and substitute, this is what you get ok.

So, what does it mean; as Reynolds number get larger and larger V by V , this average velocity divided by the maximum velocity becomes closer and closer to 1 ok. When can average velocity become very close to the maximum velocity? When the profile is very flat ok, when the profile is parabolic, you had that factors 2. Now, this is that the average velocity is very close to maximum velocity; that means, if you look at the velocity profile, the velocity profile is going to be almost flat, something like that. So, that is what the velocity profile going to look like.

So, that is the information you have really gotten. And in some sense now, this should start making sense to you, because turbulence flow or turbulent flow is full of fluctuating velocities. There are extra mechanisms for momentum transfer. So, the gradients are not maintained that well ok. The gradients get actually smoothened out very fast ok, that is why you are getting this flat profile for the turbulent flow, is that clear, doubts ok. So, what we are going to look at maybe next 10 minutes is some effect of rough walls.

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Effect of rough walls $\rightarrow$ does not affect laminar flow
affects turbulent flow

Wall layer

$$y_m^+ = 5$$

$$\frac{y_m u^+}{\nu} = 5$$

$$\frac{y_m}{D} = \frac{5 \nu}{u^+ D} = \frac{5 \nu u_{avg}}{u^+ u_{avg} D} = 5 \left(\frac{\nu}{D u_{avg}} \right) \left(\frac{u_{avg}}{u^+} \right)$$

$$\frac{y_m}{D} = 5 \left(\frac{1}{Re_+} \right) \sqrt{\frac{8}{f}}$$

So, when I say roughness so, you let us say you take any you know any material let us say even if it is P V C pipe or iron pipe the finishing that typically, looks very smooth, but when do we call it smooth? When do we call it rough?

Here, when you look at it you may not see that it has got any roughness at all, it may be like very small, but if you actually put it under a microscope you will at see that it is not really flat it has got some you know troughs and some you know uneven area right that

you would definitely see. So, what happens is that even though it is such a small scale, it starts affecting the flow field. It does not effect a laminar flow, but it can effect turbulent flow. We will see why it effects turbulent flow.

So, rough walls does not affect laminar flow, affects turbulent flow ok; so, to see that we need to go back and check our viscous sub layer or the wall layer. Remember, we said there are if you look at the turbulent flow near a wall we divided it for a into 3 parts right. There was a layer very close to the wall, where the viscous stresses are much larger than the turbulent stresses right. So, that is what we call the wall layer and we do a velocity profile and we said up to what extent the viscous sub layer or the wall layer is right, does anybody remember the number up to what distance would you see the wall layer?

Check your notes; 5 what is 5 y^+ ok, when y^+ plus if 5 up to 5. So, from 0 to 5 you are expected to see the viscous wall layer. So, let us say y^+ plus is equal to 5. So, from 0 to 5 is the viscous wall layer. So, let us say y^+ plus maximum is 5 that you are supposed to get. So, let us write it as y^+ , how is y^+ plus defined? What was it non-dimensionalised with respect to? Is y^+ max, is 5 or y^+ max divided by let us say D .

Where, D is now the diameter of that pipe is simply $5 u^*$ divided by μ right is equal to 5 sorry, I made a mistake no this is y^+ divided by there is a D here. So, I am just dividing D on both sides, because I am trying to what I am going to calculate now is the thickness of the wall layer compared to the diameter of the pipe. So, I am looking for a fraction which is y^+ max divided by the diameter of the pipe yeah. No. So, y^+ so u^+ plus is equal to y^+ plus.

So now, y^+ plus is up to 5 is when you have the viscous layer it is just that we do it on a logarithmic axis. So, the value is still 5 yeah, it is not log y^+ plus. What is that?

Is it the other way? Let us look at $y^+ u^*$ by μ . Now, is it correct? $y^+ u^*$ by yeah. So therefore, I should change this ν by u^* is equal to 5ν divided by no, in the u^* there ok. I want to bring in an average velocity, because I want to write it down in terms of Reynolds number star, let us say we define a u average D and I will put a u average upstairs also.

So, that I can write it as $5 \text{ times } \nu \text{ divided by } D u_{\text{average}}$, which will become Reynolds number times u_{average} divided by u^* is equal to $5 \text{ times } 1 \text{ by Reynolds number times}$

u average by u star, we wrote down an expression for u average by u star yesterday. What was that? Not u max ok, not the one that you calculated. Now, use u average by u star.

In terms of friction factor you would have it in your notes, root of f by 8 by f root of 8 by f y max by D ok. So, that so, this is what the thickness of the wall layer is going to be compared to let us say the diameter of the pipe, let us say you take a pipe of 10 centimetre diameter.

$$y_m^+ = 5$$

$$\frac{y_m u^*}{\nu} = 5 \rightarrow \frac{y_m}{D} = \frac{5\nu}{u^* D} = \frac{5\nu u_{avg}}{u^* u_{avg} D} = 5 \left(\frac{\nu}{D u_{avg}} \right) \left(\frac{u_{avg}}{u^*} \right) = 5 \left(\frac{1}{Re_D} \right) \sqrt{\frac{8}{f}}$$

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$$\begin{aligned} \frac{y_m u^*}{\nu} &= 5 \\ \frac{y_m}{D} &= \frac{5\nu}{u^* D} = \frac{5\nu u_{avg}}{u^* u_{avg} D} = 5 \left(\frac{\nu}{D u_{avg}} \right) \left(\frac{u_{avg}}{u^*} \right) \\ \frac{y_m}{D} &= 5 \left(\frac{1}{Re_D} \right) \sqrt{\frac{8}{f}} \end{aligned}$$

$$D = 10 \text{ cm}, Re_D = 10^5 \Rightarrow f = 0.018$$

$$\frac{y_m}{D} = 0.001 \Rightarrow D = 10 \text{ cm}$$

$$y_m = 0.01 \text{ cm} = 0.1 \text{ mm}$$

And, let us assume that Reynolds number that I have is 10 raise to 5 ok. So, a factor Reynolds number of 10 raise to 5 can give you a friction factor 0.018 and therefore, you must be able to substitute and calculate what is y m ok. So, y m by D comes out to be 0.001. So, what am I trying to do? I am just trying to estimate how big the wall layer is going to be ok, in terms of some real number. So, I am just giving some numbers and trying to see how big it is going to be for a Reynolds number of 10 raise to 5 and I find that that max the high height maximum compared to the diameter is going to be .001. So, that would mean that if I have a 10 centimetre pipe, if D is 10 centimetre y m is going to be 0.01 centimetre ok, what is that 0.1 millimetre ok.

So, if I take a pipe let us say 10 centimetre diameter ok, the thickness of the wall layer is going to be very tiny ok, less than a millimetre, very close to the surface of the pipe ok. So, that us the number that comes out to be so, it is really thin, one thing is that remember we yesterday, when we did the calculation we threw away all that thing that was coming from the wall layer and the overlap layer did the logarithmic layer did very well is 1 is 1 reason is this that the contribution that is going to come from is extremely small.

Second is that the thickness is so small that would so, if you go to that scale ok, if you start zooming in into the thickness of that wall layer would you see the roughness or not you would definitely see it right. In other words, if you look at the wall layer the wall layer is not really on a flat plate. It is actually seeing all the crusts and troughs and it is actually moving on a very uneven surface agreed ok, because the thickness itself is very small.

So, that and then if some there is a flow that is happening on a very rough surface that flow is going to get affected by the roughness, that flow is not going to be smooth. Actually another way of saying it is that that wall layer itself gets destructed, because of the roughness, the roughness that you might not have imagined would play a role actually that destroy your wall layer and will affect the entire flow field ok. Therefore, pipe roughness effects turbulent flow that is the take away point, is that clear. Yeah. So, because it affects the wall layer, the other layer is on this layer.

Because of the continuity yeah so, for example, you need to maintain some stress balance that will get destructed. So, the entire thing will get destructed. In fact, the log layer will remain as a log, but you know the arguments and the constant everything will get changed. Therefore, the friction factor expression that you have gotten should be where re looked at in talking the context of the rough pipe yeah.

So, for the laminar flow, it is basically going all the way there right. So, the layer is there is nothing like a wall layer that is confined to the wall. There may be a small region where the roughness could affect it, but basically the viscous action will immediately smoothen it out, because it is you know spanning all the way up to the radius. So, that is why it does not get affected.