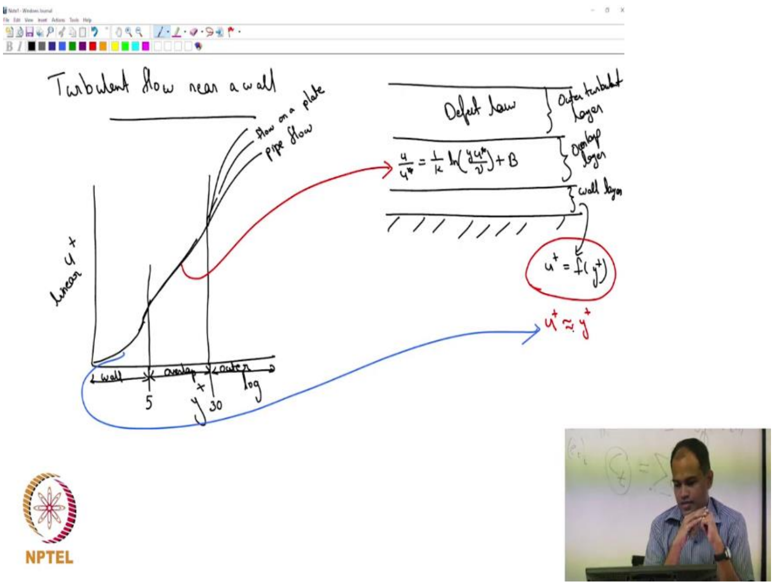
Fluid and Particle Mechanics
Prof. Sumesh P. Thampi
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 54
Turbulent Flow near a wall and in a pipe

In the last class we started analysing Turbulent Flow, we looked at that turbulent velocity profile near a wall right and we had talked about three different layers of flow field.

(Refer Slide Time: 00:43)



So, we are looking at. So, that is the wall we have and then for convenience or rather we found that the structure of the turbulent flow near the wall can be divided into three parts the first layer is what we called as the wall layer and in the wall layer the viscous stresses are much larger finished, the layer which is very close to the wall where we called as the wall layer where the viscous stresses are much larger than the turbulent stresses. And we had from dimensional analysis we said u plus is a function of y plus where we had defined u plus and y plus as the two non dimensional relevant variables as a velocity variable and a distance variable.

And then we had the overlap layer where we had a logarithmic law that we wrote down which was u by u star is equal to 1 over k ln y u star by nu plus B and then we had an outer turbulent layer where we wrote down a defect law. So, that is what essentially we did in the last class. So, what I am going to do now. So, I am just going to draw a diagram showing the velocity profile that is there in these three layers and I am going to do that using u plus and y plus. So, u plus and y plus are the velocity and the distance variables that I am going to use now throughout and I will keep my x axis as log and y y axis as a linear scale ok.

So, the first part that I am going to draw is the velocity profile in the wall layer and that is where I have u plus equal to a function of y plus we really do not know at the moment what is f, but it is been now seen experimentally that u plus is approximately equal to y plus is a good enough approximation ok. In other words in the wall layer the velocity profile is simply u plus is equal to y plus ok. So, if you use that; so you are basically going to get something like that its a log linear plot, so it would not be straight line ok. So, that is what the wall layer would look like.

Then we have the overlap layer in the overlap layer the law is already logarithmic. So, in this plot its going to look like a straight line. So, I will have; so it will be a smooth connection I will have a logarithmic layer and then you will have the outer layer which we do not know something else. So, that is my wall layer, this is my overlap layer and that is my outer turbulent layer and here we already know the law. So, this the law that is governed by is that here in the outer layer that is what we have plotted ok.

Now, the point is that this outside layer the outside layer depends upon what is really your; so this could be multiples. Meaning that one could be a pipe flow, one could be a flow on a plate and so on. So, whatever be the main flow that you have that is seem to be affecting only the outer layer the overlap layer and the wall layer are independent of the flow configuration that you really working on ok.

So, the wall layer which is dominated by your viscous stresses and the overlap layer is really independent the profiles are independent of what is the outer flow field the outer flow field is the only when that is really dependent upon the exact configuration that you are looking at. So, that is why I have plotted a wall layer I have plotted this one line for the wall layer and for the overlap layer because they are like universal profiles and then things basically start changing all right.

And this wall layer in this coordinate y plus is approximately up to 5 and the overlap layer is found to be approximately up to 30 ok. So, that is these are all experimental measurements this is now way to really calculate that if you write it down in terms of y plus up to y plus of 5 you will see that you know u plus is equal to y plus is a good approximation. And from y plus being 5 to 30 you would see that u plus goes as approximately logarithmic of 5 plus and beyond that these neither of these two laws are there will you will start seeing different different profiles.

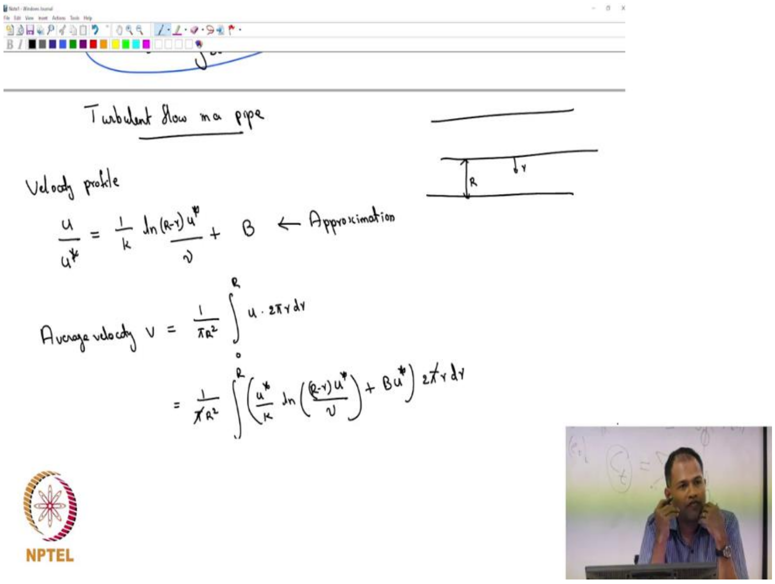
Why u plus is equal to y plus is between y plus being 0 to 5. So, in that region approximately u plus is y plus. So, see only thing that we derived was u plus is a function of y plus we did not know now I am saying that experiment show that u plus is equal to y plus. We did not show u plus is equal to y plus, we calculated u plus as a function of y plus. Any other questions?

I do not think it is really necessary it could be anyway, but I think these profiles are probably stable. So, typically I think you would see like this you can see that its. So, for example, it was laminar it would have been parabolic right. So, its some signatures of like that the curvature should be like that the other curvature I think the flow the flow is never stable that is why you do not see it.

So, that is actually the important point even though we introduced overlap layer as something which is to connect these two regimes, it turns out that overlap layer is very big and it is a straight line its a log its a straight line than log linear scale ok. So, the logarithmic law is a very robust law it turns out and it spans quite a length as you see its not a small region. In fact, what we are going to do now onwards is to going to skip the wall layer and the outer layer I am going to stick or use the result of just logarithmic layer ok.

So, now we are ready to look at turbulent flow in a pipe. What is our ultimate aim? Our aim is to get the pressure drop which we would write it as using friction factor and we want to relate friction factor with Reynolds number that is our aim. So, f was is equal to 64 by Re was the relation that we found for the laminar flow we want to now get a similar relation for turbulent flow. So, that given Reynolds number we can calculate the pressure drop.

(Refer Slide Time: 08:42)



So, what we are going to assume now is that; so again. So, the first approach that we had right starting from the governing equation is going to be impractical. So, what we are going to do is having seen that this is the particular structure of the you know turbulent flow and wall layer being very small we are going to assume that overlap layer is majorly the you know the velocity profile inside the pipe. So, in the pipe if you have the wall layer will be very near to the wall right and then we have a large region of the overlap layer.

So, let us say that we will just use the overlap layer as a velocity profile and try to see what is it going to look like and how does it match with experiments and what we are going to see is that its going to be a robust approximation or the approximation that the entire turbulent velocity profile is given by that logarithmic law is a good approximation, but we have to do it and see that its a good approximation ok. So, I am assuming that the entire layer is now just going to be given by the overlap layer law or the logarithmic law or in other words what will I write the velocity profile will be now given by u by u star is equal to 1 over k ln y u star by nu right. So, y u star by nu plus b.

So, this is the logarithmic law that we wrote down. So, now, I am going to talk about a pipe whose radius is R and let us say we are using going to use a cylindrical coordinate system in which r is the radial coordinate. So, y I will replace with R minus small r because y is the distance from the plate. So, in this coordinate system R minus r will be the distance from the plate. So, this is the approximation that we are doing. R minus no we had defined y to be the distance from the plate. So, now, the distance from the plate would be just capital R minus small r.

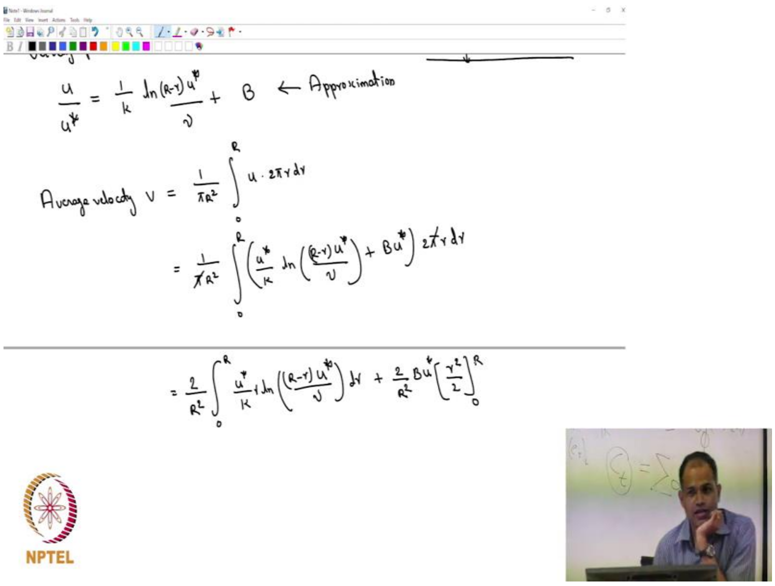
$$\frac{u}{u^*} = \frac{1}{K} \ln \frac{(R-r)u^*}{\nu} + B$$

U star; look at your notes and see whether this is consistent whatever we wrote in the last class should be consistent with this expression. So, you feel a little more comfortable with that expression. Where u star was defined as? What is the name of what is the what is it called?

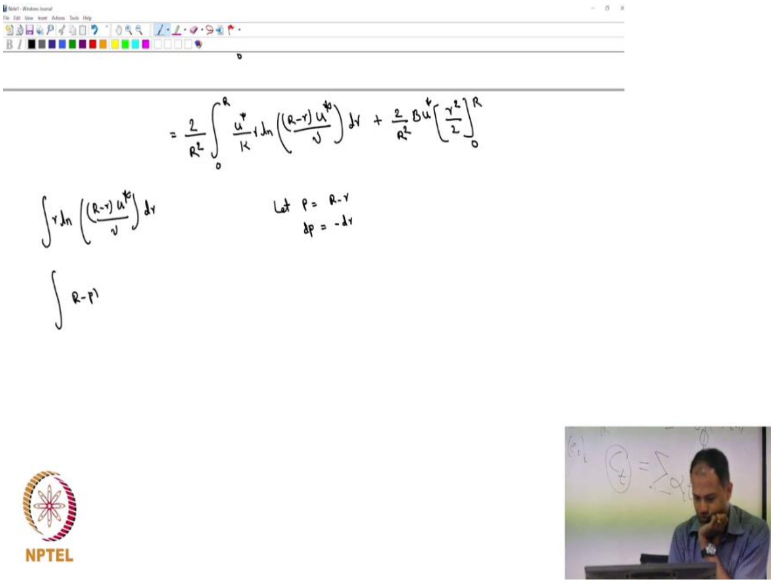
Friction velocity which is defined as square root of ok. So, u star is the friction velocity ok. So, if this is the case let us actually calculate the average velocity in the pipe ok. So, right now we have just taken it as an approximation average velocity. How do we calculate average velocity? Let us call it let us say v is equal to 1 by. So, its a circular pipe pi R square integral 0 to R u into 2 pi r d r is that right. So, this is the velocity integrated over the entire surface divided by the surface area. So, we know the expression for u. So, substitute and integrate and see what the velocity average velocity is.

$$v = \frac{1}{\pi R^2} \int_0^R u.2\pi r\, dr = \frac{1}{\pi R^2} \int_0^R \left(\frac{u^*}{K} \ln\left(\frac{(R-r)u^*}{\nu} \right) + Bu^* \right) 2\pi r\, dr$$

(Refer Slide Time: 13:35)

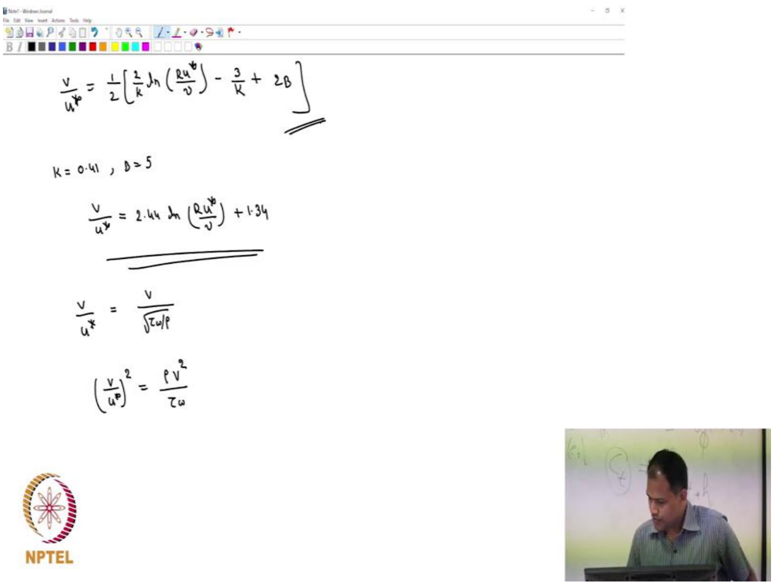


(Refer Slide Time: 15:20)



You can just go ahead and integrate it will take a couple of steps. So, that is what you need to integrate now right, there are two different terms one containing log and then there was a constant multiplied by R.

(Refer Slide Time: 16:06)



$$\frac{v}{u^*} = \frac{1}{2}\left[\frac{2}{K}\ln\left(\frac{Ru^*}{\nu}\right) - \frac{3}{K} + 2B\right]$$

Where you get something like that that is what I. So, and the typical value of K is 0.41 and B is 5 and if you substitute you get V by u star is equal to 2.44 ln Ru star by nu plus 1.34

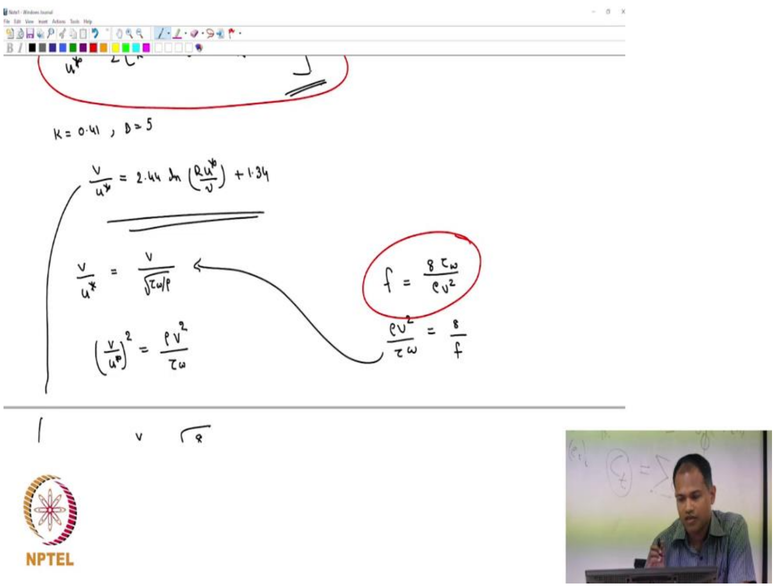
ok. So, that is the average velocity that you get for a turbulent flow in a pipe if you assume that its given by the overlap layer completely. Now, we are directly going to now evaluate the friction factor.

$$\frac{v}{u^*} = 2.44 \ln\left(\frac{Ru^*}{\nu}\right) + 1.34$$

So, there are two ways of doing it one is what we followed in our laminar flow calculation where you know calculate the pressure drop connect it with the turbulent sorry the wall shear and then do it or you can do it in a simpler fashion here. If you look at this quantity v by u star is v divided by u star was defined as square root of tau w by rho, so; that means, v by u star square is rho v square divided by tau w correct. And what is the connection between tau w and the definition of friction factor? You can just check your notes and tell.

$$\frac{v}{u^*} = \frac{v}{\sqrt{\frac{\tau_w}{\rho}}} \rightarrow \left(\frac{v}{u^*}\right)^2 = \frac{\rho V^2}{\tau_w}$$

(Refer Slide Time: 18:23)



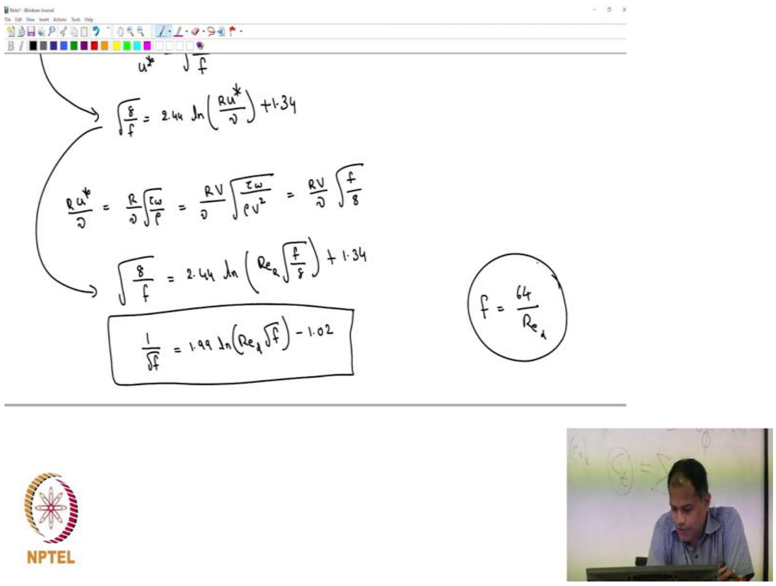
In terms of wall shear stress, it will be there where we introduced friction factor for the first time. Somebody says 8 tau w by rho v square check whether this is there now in the notes. So, that means, tau w by rho v square is what or rather rho v square divided by tau w is equal to 8 by f right. So, if I can actually directly therefore, write down a relation v by u star is equal to square root of 8 by f agreed. And why am I writing down because I already know the expression for average velocity, but the average velocity the relation between average velocity in friction factor we now know. So, therefore, I can immediately say that the expression that we really have this expression is nothing, but root of 8 by f is equal to 2.44 lon not y r; R u star by nu plus 1.34.

$$f = \frac{8\tau_w}{\rho v^2}$$

$$\frac{v}{u^*} = \sqrt{\frac{8}{f}}$$

$$\sqrt{\frac{8}{f}} = 2.44 \ln\left(\frac{Ru^*}{\nu}\right) + 1.34$$

(Refer Slide Time: 21:39)



We really want to get it in terms of Reynolds number, so, we want to make that logarithm in terms of Reynolds number, can you make that logarithm argument of that logarithm in terms of a Reynolds number? The argument R u star by nu, so u star if I replace with square root of tau w by rho and again write it as tau w rho v square where v is the average velocity I can write that square root term in terms of f again. So, basically I get Reynolds number times square root of f by 8 that argument. Check whether its right.

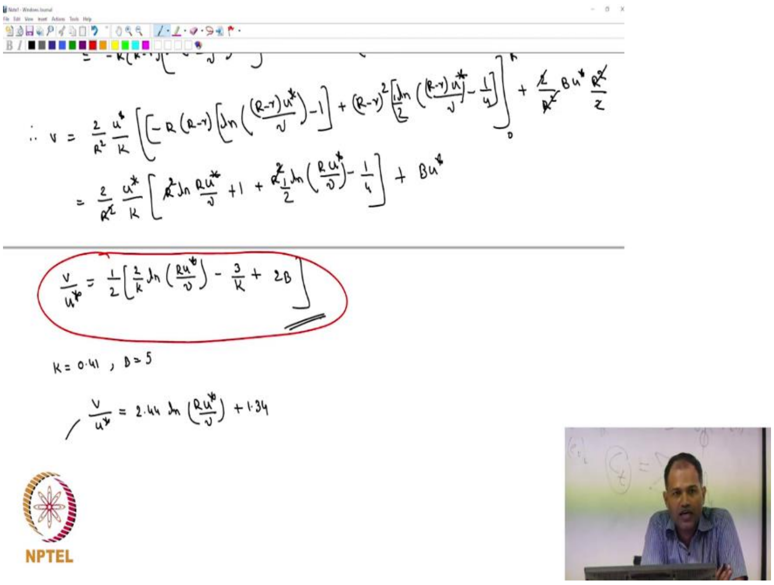
$$\frac{Ru^*}{\nu} = \frac{R}{\nu}\sqrt{\frac{\tau_w}{\rho}} = \frac{Rv}{\nu}\sqrt{\frac{\tau_w}{\rho v^2}} = \frac{Rv}{\nu}\sqrt{\frac{f}{8}}$$

In other words what we are saying is simply square root of 8 by f is equal to 2.44 logarithm of Reynolds number based on the radius times square root of f by 8 plus 1.34. If you rewrite 1 by square root of f is equal to 1.99 log Reynolds number based on the diameter times root f plus sorry minus 1.02, that is the relation that we wanted that is the relation between Reynolds number and the friction factor.

$$\frac{1}{\sqrt{f}} = 1.99\ln\left(Re_d\sqrt{f}\right) - 1.02$$

So, let us just go through what we have done and then may be. So, we started looking at the flow through the turbulent flow through the pipe and we said the velocity profile is simply given by the overlap layer log which is the logarithmic law. We assume that that is the case and then we went ahead and calculate an expression for the average velocity.

(Refer Slide Time: 24:47)



And the expression for the average velocity came out to be that average and expression for the average velocity came out to be that ok. Our intension is to calculate friction factor, so we wanted to calculate friction factor which we could have done by calculating the pressure drop and calculating the wall shear and so on. Instead of that what we have done is we knew that average velocity wall shear stress friction factor are all related.

So, we are essentially exploiting that relation which is nothing, but that and rewriting the velocity profile in terms of friction factor and Reynolds number directly that is all we have done. And therefore, we ended up with this expression which is 1 by root f is equal to one approximately 2 ln Re root f minus 1.02. As suppose to what was in laminar flow? That I am putting the suffix d to just show that the Reynolds number is calculated based on the diameter and not on radius ok. So, here you can actually see given Reynolds number if you want to calculate friction factor in laminar flow it was much easier, but how is it in turbulent flow? There. So, I have changed it. So, here we had R v by nu which you can write it as Reynolds number based on the diameter divide by 2.

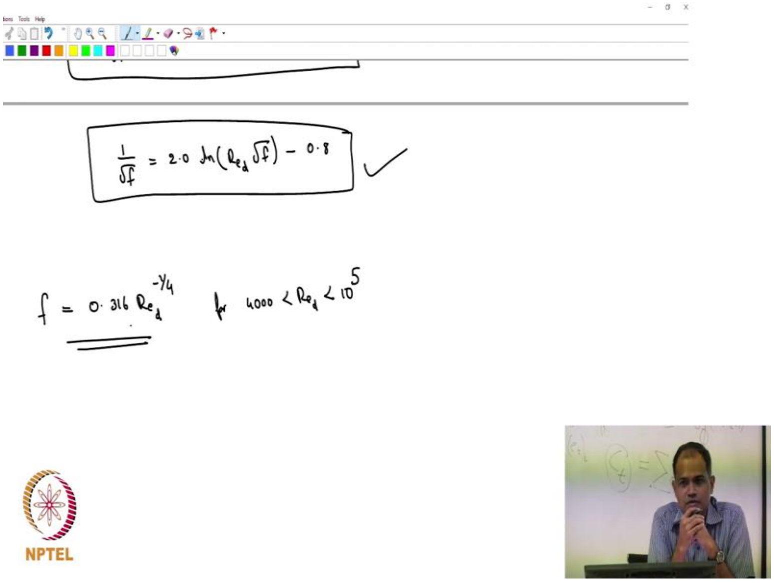
This is Re d. So, there will be log of root 2 that will come which I think is adjusted in that pre factor you can check.

So, I what I wanted to tell was given Reynolds number how would you calculate f using this expression as a straight forward?, Its a non-linear expression which contains a square root and a logarithm ok. So, what can you do, how do you solve how do you solve non-linear algebraic equations? Go ahead you want to tell something. Graphically is that the only method that you have learnt in your third semester, tell? Numerically what methods have you learnt?

I do not know the probably the best is Newton Raphson ok. So, this is a nice expression where you can apply all those methods and calculate its a very complicated equation, but its an extremely useful equation therefore, typically those problems are illustrated using this equation. So, that is where you will see the use, anyway so, this equation.

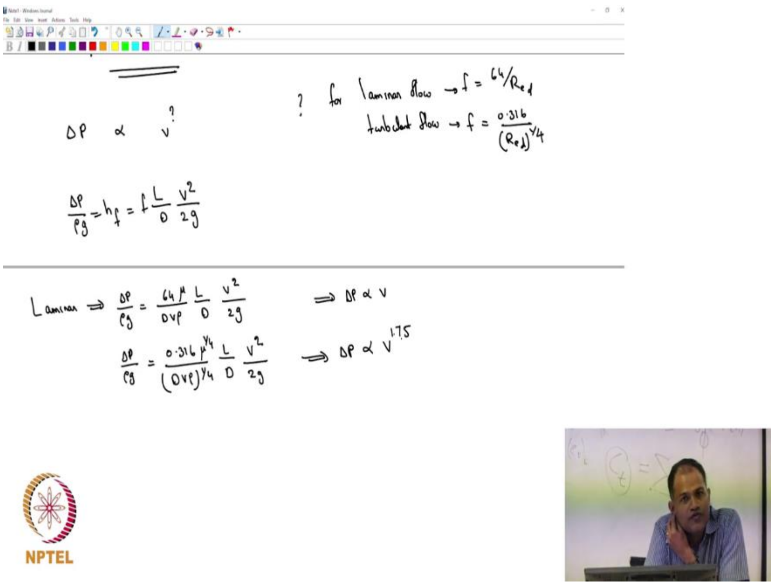
So, remember we assumed overlap the logarithmic law and calculated it, but instead of that let us say you wanted to calculate sorry the it is compared with experiments and the actual coefficients have been replaced a little bit 1 by root f is equal to 2.0 ln Re d root f minus 0.8 ok. So, this seems to be fitting very well with the experimental data which means that our original approximation of replacing the velocity profile just with the logarithmic law was a good approximation because we have ended up with equation very similar to what is found experimentally ok.

(Refer Slide Time: 28:17)



So, that is the only justification for replacing the or using the logarithmic law. So, you should be really using this expression if you want to calculate a friction factor which is a straight forward relation ok. So, if Reynolds number is not very large which is let us say 10 raise to 4, 10 raise to 5 then you could use this direct relation which is a empirical fate.

(Refer Slide Time: 29:39)

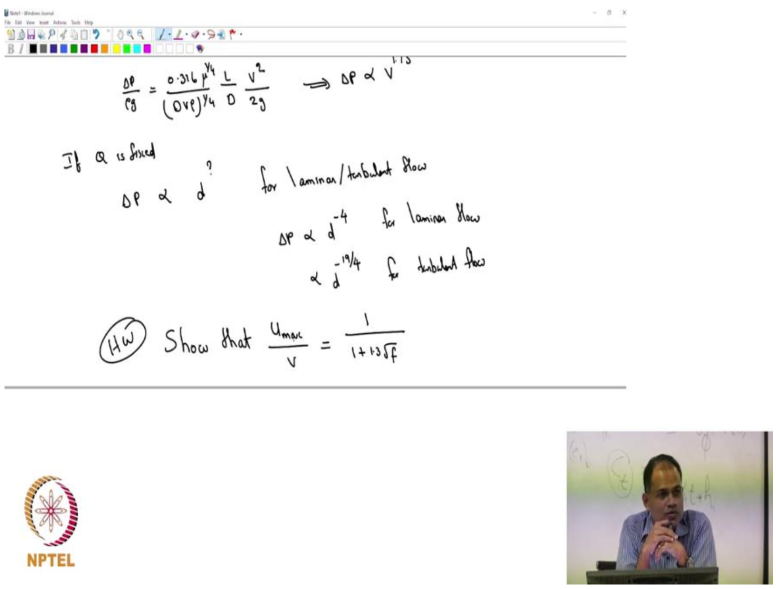


So, this is going to be useful because I want you to calculate what is delta P proportional to the let us see which function we let us do velocity first velocity to the power of question mark, what is that question mark for laminar flow and turbulent flow can you find out?

So, what I am saying is that there is a another equation which is valid over a small region ok. So, that is a useful approximation. So, use this useful approximation and tell me delta P is proportional to what power of V? For laminar flow and turbulent flow. So, for laminar flow of course, you should not you should use your older expressions that we already knew and turbulent flow you can use this nicer expression 0.316 times Re to the power of this. You know question like for example, you double the flow rate you double the velocity what happens you pressure drop you want to calculate what would you do that is what I am asking.

So, the first thing is to write delta P in terms of friction factor that is our basic definition substitute for f and see what is the power of V that you get. So, for laminar flow f is that turbulent flow f is. So, its almost square dependent ok. So, if you change the if you double the average velocity laminar flow the pressure drop will increase it will double, but for turbulent flow it will go by 4 times almost.

(Refer Slide Time: 31:58)



If the flow rate is fixed what is delta P proportional to d to the power of question mark where d is d for a for a laminar and for the turbulent flow ok. Flow rate is fixed through the pipe what is going to be the dependence of pressure on diameter or radius. Are you following what I am saying which is just trying to explore with look at those relations and see what the areas dependencies are ok. So, you will see this that it goes d to the power of minus 4 for laminar flow and close to minus 5 for turbulent flow ok, so; that means, that pressure drop decreases as you increase the you know the size of the.

Choose the volumetric flow rate. So, see you want to term something fixed and now you have the choice of selecting the pump as well as the size of the pipe what is that you would want to do would you take a smaller pipe or would you smaller pipe or would you take a bigger pipe? If you take a smaller pipe then it says that the pressure drop is going to be larger and larger ok. So, then what you would want to do is you want to reduce your electricity consumption you would want to go for a less pressure drop when you would say that you will take a larger pipe, but what is the issue with taking larger pipe? The initial cost becomes extremely high maintenance will become large ok.

So, therefore, there is a competition now, when you are asked to design something whether you want to reduce your electricity usage or whether you want to reduce other cost ok. So, that is the relevance of this relation and of course, laminar and turbulent whether that flow rate also makes a difference ok. This I have written down a relation at the end U max that is the maximum velocity divided by the average velocity. What is maximum velocity by average velocity for a laminar flow?

So we did that or you did it in one of these quizzes. So, show that its actually this factor for your turbulent flow just use the expression that you already have and try to manipulate and you should try to get this relation. Try it we will talk more about this tomorrow.