Fluid and Particle Mechanics
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Lecture - 53
Turbulent Stress and Turbulent Shear Layer

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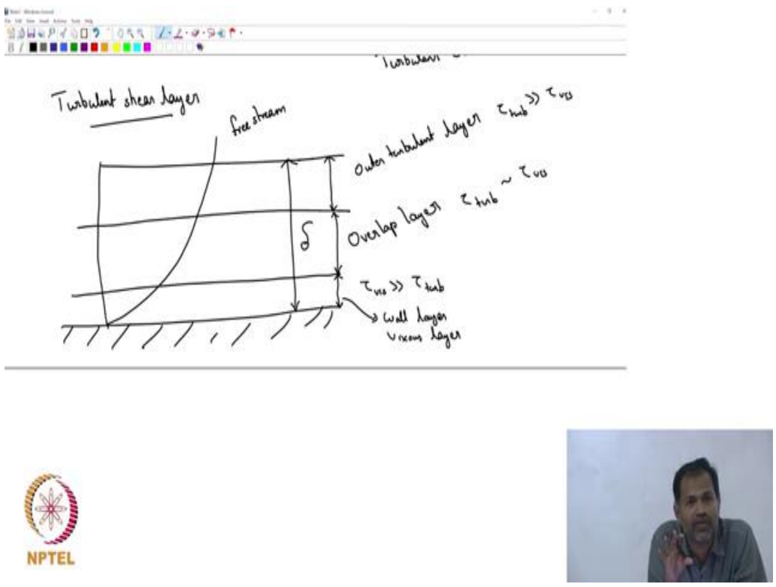

So, the first thing I want to talk about is Turbulent Shear Layer. So, we are going to give up our governing equations completely and we are just going to look at how does actually turbulent flow look like ok; if it is near a wall and whether that information give us some lead towards solving flow in a pipe ; that is a stuff.

So, what I have is a wall; a plain wall and there is a fluid that is flowing on top of it. I am not saying what is on the other side; I am assuming that the other side actually goes to infinity. If you look at the velocity profile; it is going to look something like that and it will go to wherever it wants to you know where whatever the profile that said far away or from at infinity. We are interested in a region where the velocity profile is affected by the plate.

So, you can imagine that the fluid when it is near the region very close to the plate; should experience that friction you know by the plate a lot. But when you if you go infinitely far from that plate then you know the fluid should not see whatever effect that you see should be independent of the plate. So, we are going to restrict to that region where the fluid is seeing the plate and I am going to give let us say some length which we do not know what it is as some delta ok.

So, delta is the length in which the fluid flow is really affected by the plate ok. And I am going to now split this into three parts and that is been found useful; the first layer I will just divide it and then talk about. The first layer a layer very close to the wall; what do you think the turbulent stresses would be in that region? It would be almost negligible because the viscous action will be very strong because the solid is introducing it is giving a lot of friction to the fluid flow. So, that is the region where the viscous stresses will be much larger than the turbulent stresses ok.

So, this region is where tau let us say viscous is much larger than the tau turbulent stresses and we call that layer wall layer or viscous layer ok. Now, this is really the outside fluid; so, that is let us call it a free stream; a free stream is basically that fluid which is not affected by the solid wall. The region between the free stream and a little bit inside that is called outer turbulent layer and in between, you call something as overlapped layer.

So, why I am writing down it has three different layers is because the laws that you know govern the fluid; so, let me talk about this first. So, in the wall layer we found that tau viscous is much larger than tau turbulent ok. How about outer turbulent layer? It will be exactly the opposite tau turbulent will be much larger than tau viscous.

So, in the outer turbulent layer you will have tau turbulent will be much larger than tau viscous; in overlap layer tau turbulent will be similar to tau viscous. So, that is actually the difference between these three layers ok. We are saying that if you are looking at turbulent flow, we need to worry about viscous stresses and turbulent stresses very close to the wall viscous action will be so strong; turbulent stresses will be small compared to viscous stresses.

So, you have a wall layer, but if you go sufficiently far from the wall then you will have turbulent stresses being large. So, turbulent stresses will be larger than the viscous layer viscous stresses and in between there may be a region where the viscous stresses and turbulent stresses will be of a similar you know nature.

And why I am talking about these things is because now you know that in the governing equation which actually contains the viscous stresses and turbulent stresses; depending upon which one is important or which one is more you will get different solutions. So, in the wall layer you know you are going to get some profile, in the outer turbulent layer you are going to get some other profile, in between you are going to get something else and so on; is that clear? Yeah.

No, it will not be viscous free the; it will definitely have viscous stresses as well less turbulent stresses.

Student: They are almost same right?

They are almost same. So, they are equally important; both of them will be there. So, in other way of saying it is that; so, that was the governing equation we entered away with right.

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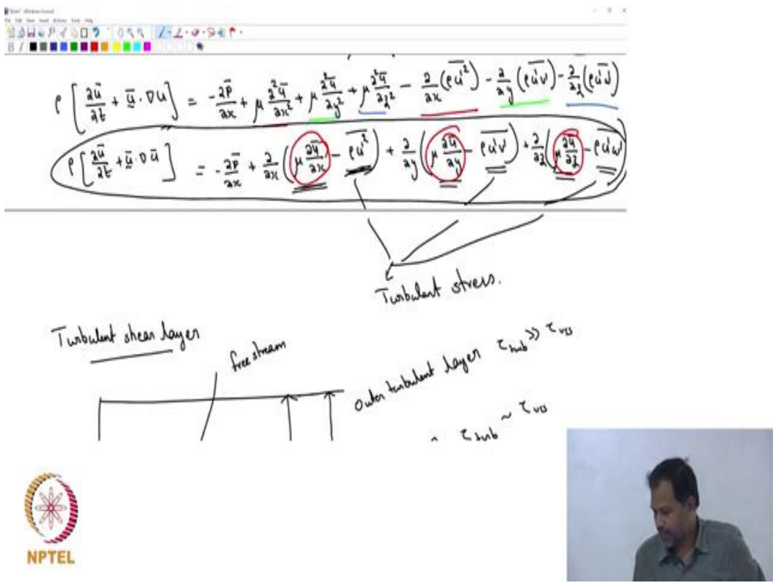

So, let me write the left hand side also right that was the governing equation. If I am solving for the viscous wall layer or viscous layer what this in this equation what all terms we can I put to 0? If I am doing that I can actually put this to 0 that to 0 and that to 0 and then tried solving it because I know that turbulent stresses will be small and therefore, you are going to get some profile some velocity profile right.

Now, this must be undo it; undo it ok. Now if I am solving for the outer turbulent layer that is very close to the free stream, then I know that the turbulent stresses are much bigger. So, then I will put this and that to 0 and that then I am going to get a different solution ok;

in the overlap layer I cannot do either of them I need to solve for the full equation. So you now you realize that you are going to get three different solutions ok. So, that is why we are actually dividing it into three different parts and that is convenient because we anyway do not know how to solve the full thing that is why hm.

Doubts?

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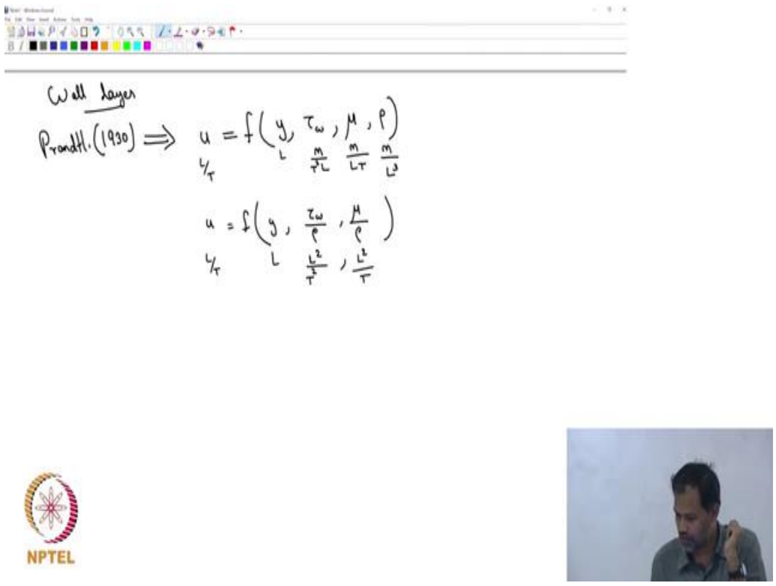

So, we are primarily going to depend upon experimental results and dimensional analysis. So, the first thing that we are going to analyze is the wall layer and this is suggested by Prandtl in 1930 ok.

He said let us try to do write down some expression for the velocity profile, but we will just do a non dimensional analysis. And he said the velocity profile is just going to be a function of how far it is from the wall that makes sense because it has to depend upon where it is ok. And instead of talking about how you are driving the flow, we will talk about a wall shear stress. So, you remember we talked about wall shear stress, we connected the wall shear stress to pressure drop and so on.

So, instead of talking about what is driving the flow; we will really talk about what is the force that is acting on the wall ok. We will use that as the quantity because that is a useful quantity to talk about and that is what is really responsible for the flow. So, we will talk about that driving force in terms of tau w and fluid properties like viscosity and density; I cannot neglect viscosity because as we are talking about the viscous layer now or the wall layer ok.

So, this is what actually he experimentally found also. So, he says that they should therefore, exist some relation connecting these quantities. So, if you write down some expression like this; now you know how to proceed right, what is the next thing to do? We need to do the non dimensional analysis. Can you do the non dimensional analysis and find out what happens, what is the x what is; what are the two non how many non dimensional numbers would you expect?

How many variables are there?

Student: 5.

5, how many fundamental dimensions are there?

So, how many non dimensional numbers? Yeah, let us get them.

Student: Sir.

Yeah.

Yes.

Viscous stresses will be; so you are talking about.

Free stream right? Yeah, yeah.

That is right, it will be very close to what is happening in the outer turbulent layer. So, that; so that we are we are keeping it free stream because that is a region where which is completely unaffected by walls. So, for example, if it was a uniform flow that is coming ok, we would say that free stream is basically uniform it will not see the wall, but in the outer turbulent layer you will still see the wall because that is the region where things are changing from the overlap to the completely uniform region. So, there will be some region where we could expect that the turbulent stresses are much larger, but still it is affected by the wall yeah.

It is basically going to be the sum of both tau w ok. So, it is basically the stress this is basically the force that you need to apply to maintain the flow though in this. So, by this analysis already tells you that when you talk about just tau w, we will be able to talk about only a viscous contribution to that because tau turbulent will be much smaller there. So, dimensions of u; L by T, y is length, tau w is Newton's per meter square. So, m L T square L square; so m viscosity is gram per centimeter second m by L cubed.

$$u = f(y, \tau_w, \mu, \rho)$$

$$\frac{u}{\sqrt{\tau_w/\rho}} = f\left(\frac{y\rho}{\mu\sqrt{\frac{\rho}{\tau_w}}}\right) = f\left(\frac{y\sqrt{\rho}}{\mu\sqrt{\tau_w}}\right)$$

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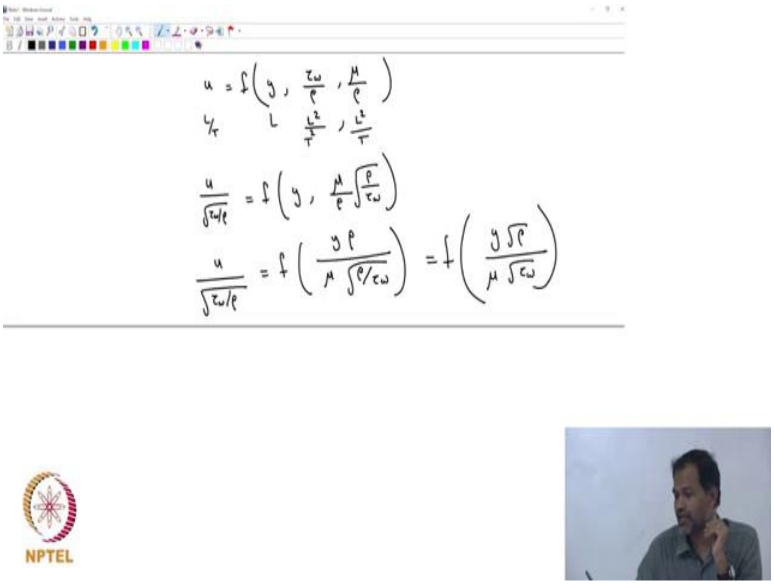

I just used epsilons method you could do whichever you like, you could end up with some other combinations of these two quantities that I have written based on what method you have done. Yeah. Yes.

Yeah, but you do not know what is then other quantity right; let us say the u prime v prime. So, for example, what is the correlation between u prime and v prime ok. So, there is no real way to see that though there might be this convention issue again what is this stress ok? So, do you want to write your stresses minus mu dou u by dou y which case you would see that they are adding up. So, there must be much something more to that.

So, it is going to take away; so, in terms of momentum transfer this is going to add up to your momentum transfer mechanism. So, if you think about how we interpreted viscosity; we said it is the fluctuating motion of the molecules that was resulting in viscosity.

Now, we are saying that the fluid itself is fluctuating ok. So, therefore in addition to viscous mechanism there is this other mechanism that also will transfer momentum across ok; so, that is really the meaning of turbulent stresses. So, if you have wall and you are applying a force that forces; not only transferred because of the molecular motion, but also you could think about it as the fluid itself is basically fluctuating and therefore, things get transferred much faster ok. And you will start seeing more and more uniform things than you know things having a gradient and we will see that as an example.

You got some form like that; I want to write this because that is going to be useful ok.

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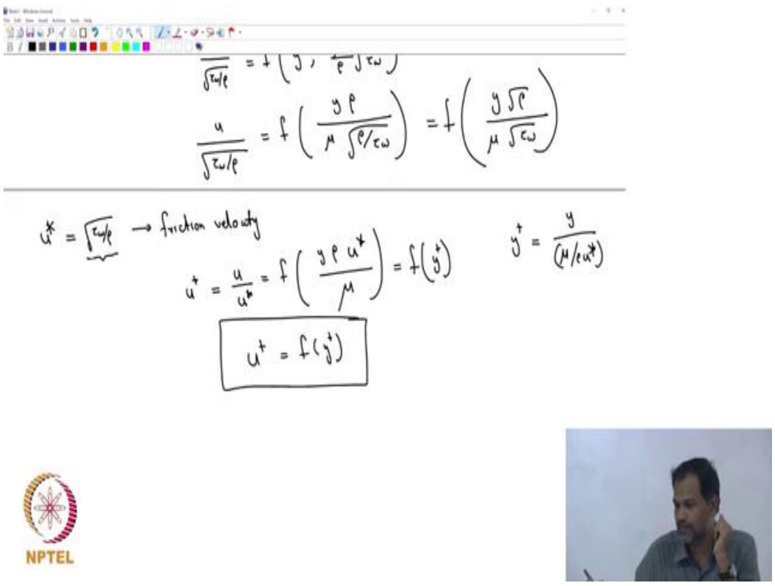

Few more terms or definitions; root of tau w by rho which is really a measure of shear stress on the wall is known as friction velocity and you typically represented it as u star what a bit the star alright.

And therefore, my relation is u by u star is equal to f of y; rho, u star by mu. So, if I replace u root of tau w by rho it is just a u star. And u by u star is typically represented as a u plus and this is represented as a function y plus, where y plus is y divided by what will be mu by rho u star ok. So, this is just and therefore, u plus is equal to a function of y plus; it turns out that it is a basically the right or the; so u plus and y plus are really non dimensional numbers ok; that you would want to use to represent the velocity profile in the wall layer.

$$u^* = \sqrt{\frac{\tau_w}{\rho}} \rightarrow Friction\ velocity$$

$$u^+ = \frac{u}{u^*} = f\left(\frac{y\rho u^*}{\mu}\right) = f(y^+)$$

Just you could have written it in terms of the non dimensional numbers that you have gotten I am just following some textbook. I have not done anything I have just written down u is proportional to some quantities and non dimensionalized. And when I got a quantity called the root of tau w by rho, I called it friction velocity; is this actually does it have the units of velocity? You could verify that put stress divided by density and square root of it ok. Should I explain anything again? Should I go through again? Just going to say a few things ok.

So, what have we been doing this let us revise; we started with looking at the governing equations; we wrote down a expression for the mean flow. We realized that the mean flow governing equations now not only contains viscous stresses, but it also contains turbulent stresses. And therefore, we need some sort of a constitutive relation for turbulent stresses which we do not know ok.

And therefore, we are leaving it at that point and then trying to do something simpler and to do that we started by looking at turbulent flow very near to the wall ok. And then we are saying that look I can take that region where the wall is affecting the turbulent flow and I may be I will be dividing it into three different regions, depending upon what is the magnitude of viscous stresses versus the magnitude of turbulent stresses; that helps me to identify three regions.

One region in which tau viscous is much larger, in the other region tau turbulent is much larger and in between they are of comparable magnitude and depending upon which one is more or less, you are going to get different solutions ok. So, now that is; so the first region we call wall layer, then there is an overlap layer and then there is an outer turbulent layer ok.

And then what I did is that now I will go with what Prandtl said in 1930 and he says that look the velocity profile in the wall layer can be written down as a function of few variables, which is distance from the wall and the thing that is driving which is tau w and the fluid properties such as viscosity and density. And therefore, we wrote down that equation and when we write down something like that the first thing that we should do is to non dimensionalize and reduce the number of variables; we found out what is the non dimensional form of that function.

And that came out to be just u plus is equal to f of y plus where we do not know what is f at the moment ok. And u plus and y plus are just two non dimensional variables and during this calculation, we also define something called a friction velocity which came out to be root of tau w by rho; which is happened to have the dimensions of velocity; it is no velocity at all, it is just a magnitude of tau w that is all that is all we have done.

So, having done the wall layer; the next thing that we want to do is the outer turbulent layer.

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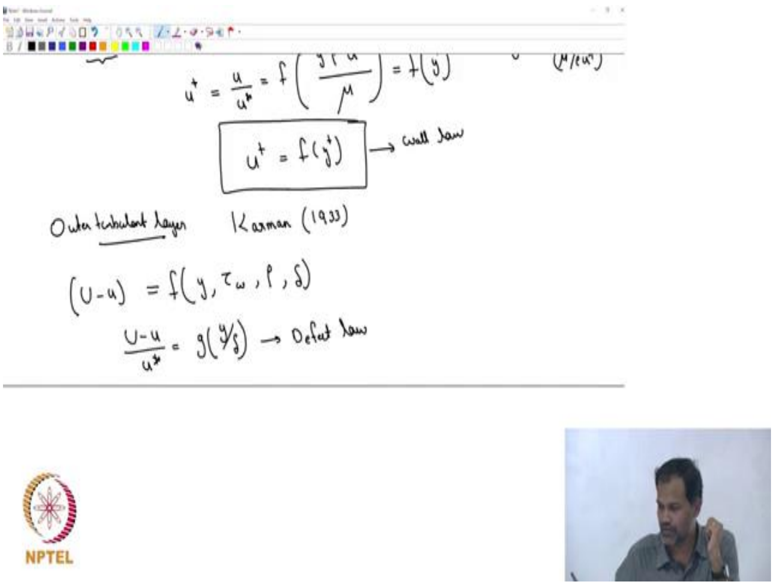

And for that ; so these; all these calculations are essentially inspired by experiments ok, or rather led by experiments when you change things and see what is happening and therefore, you try to write down the expressions.

So, what Karman suggested is that let us say in your free stream ok; the outside the turbulent layer the velocity is u; some u which could be a function of space. He said let us write down U minus u; so that is the free stream velocity minus velocity in the turbulent layer as a function of; of course, distance from the wall ; the wall shear stress which is driving it. But this time the viscosity is not going to come into the picture; you will still have density ok. And he says this is also going to be a function of delta which we defined originally where delta refer to the total height of that entire layer ok.

So, this is now if you ask me why he wrote down that, one can only say that it has been seen in experiments and therefore, you are writing down; I do not have any more explanation to that. So, he suggested that these are the quantities that seem to be dependent on the quant U minus u ok; the free stream velocity minus turbulent velocity is a function of few other things. And therefore, you could again non dimensionalize and try to write down a non dimensional expression for the outer turbulent layer; can you do that?

$$(U - u) = f(y, \tau_w, \rho, \delta)$$

So, U is the velocity of the fluid that is outside this entire layer that we are considering and small u is the velocity in that layer. So, we are really finding out how; what is the difference of that layer compared to the outside layer, so it is just that difference. And I do not think it would make any difference if you were to write it in terms of u, but it is conveniently or this away he suggested; so, and therefore, this is called a defect low. In fact, I mean you know basically it is how much is that difference we will see that yeah.

Why we are taking? So, y is going to be the exact location ok; delta is going to be the total length; the total length that is affected by the wall ok. Now, so as I said that I mean I have no answer; why it should be a function? But this is what is see and therefore, we are putting it into the expression.

No; no delta is the total height. So, delta is the total height of the fluid layer that is affected by the wall. In fact, later you will learn that that is called a boundary layer thickness, while y is going to be the exact position. So, you can expect that you should get something proportional to y by delta really like you know radius of the pipe or whatever.

See, we will get something like this just u by u star; U minus u by u star is a function of y by delta y by delta is the only other parameter that is going to come. So, this expression u plus is equal to is a function of y plus is called as Wall law and this is called a Defect law or ; yeah Defect law. So, so this is the region where Wall law is applicable, that is the region where we have the Defect law applicable and therefore, this region which I am shading is blue should be a place, where things change from this blue to the green ok.

$$\frac{U-u}{u^*} = g\left(\frac{y}{\delta}\right)$$

So, the expression that should be applicable in the overlap layer should be things, it should be an expression which smoothly changes from the Wall law to the Defect law ok. So, that is the way it was interpreted and therefore, people were actually able to derive an expression for overlap layer. Some expression that will match with these two regimes ok.

So, I am talking about something in between where if I say that if my y is very small; then it should be like my wall layer. If my y is very large, it should be my thing you know defect layer; so what could be something in between ok. We do not derive it we are just going to take the result and proceed I will write down.

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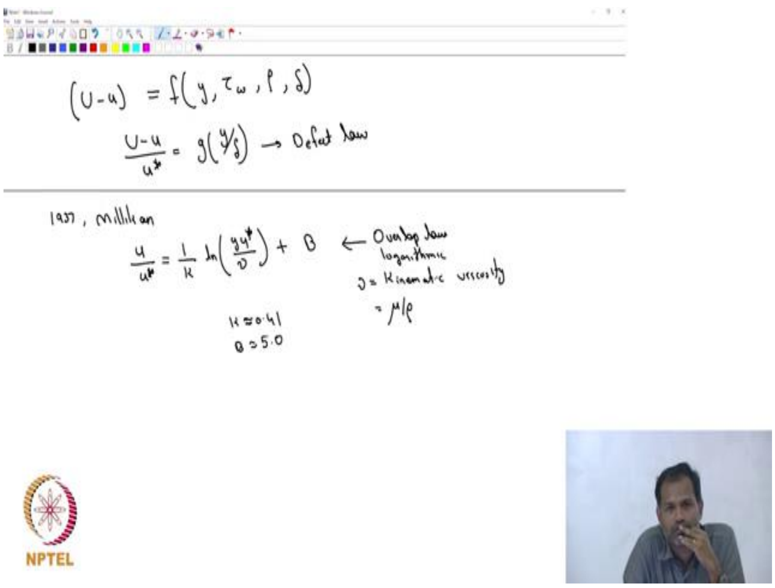

So, this was actually done in 1937 by Millikan and he said; if you write u by u star is equal to 1 over K lon y u star divided by nu plus B; this is the so called Overlap law. This Overlap law will smoothly merge into the Wall law and the Defect law ok.

$$\frac{u}{u^*} = \frac{1}{K} \ln\left(\frac{yu^*}{\nu}\right) + B$$

Now, nu what is nu?.

Mu by rho; this is also called a logarithmic law and therefore, sometimes this is referred as logarithmic overlap layer, there is a log function connecting the velocity with the distance. It is found out that experimentally K is approximately equal to 0.41; B is approximately equal to 5.0 hm. So, in some sense what we have done is; we just looked at a turbulent shear layer and we wrote down three different expressions, each applicable in a layer ok. And we will try to see how these expressions can be utilized further to analyze turbulent flow in a pipe and we will see that in the next class.