Fluid and Particle Mechanics
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Lecture - 52
Laminar and Turbulent Flows - 3

So, we were looking at the governing equations and we realized that there is no point in really looking for an exact solution and then we figured out that you know we would actually look start looking at a mean quantity like mean pressure, mean velocity and so on. But then we also found that the mean velocity the mean quantities actually do not satisfy the governing equations that is where we ended up ok. So, that was the point and therefore the fact that we realized was that we need to go to another framework, we cannot just proceed the way we have been doing it for laminar flow ok.

We can do that, but before we do that I need you to introduce you to this term called Turbulent Stress and to do that we need to look at the equation where we stopped last time. We will see what is turbulent stress in that equation and then we will see how to attack turbulent flow problems is that clear.

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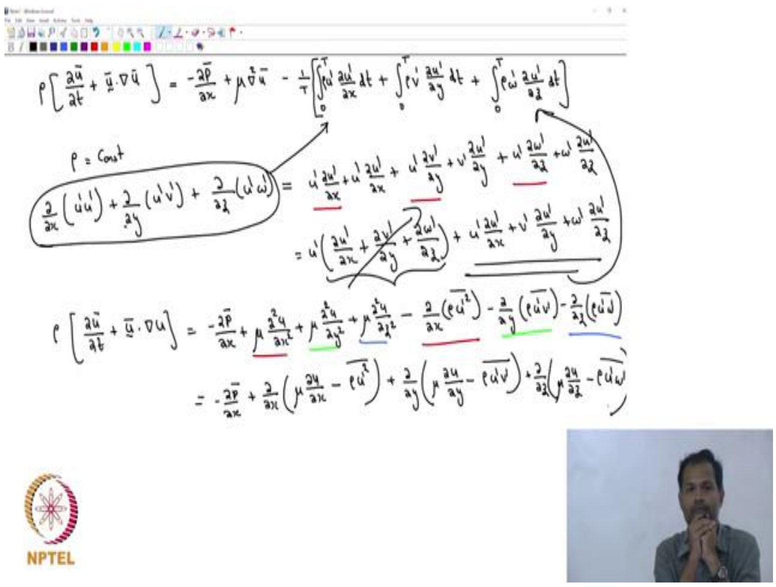

So, can you all look at the equation that we left last time for the mean flow in let us say x direction. So, we use just u for the x component of velocity right is this right this is what we had yeah and we realized that those terms that I have written on the right hand side which actually contained the primed variable. So, the primed variables are the fluctuating parts of the velocity field u prime v prime w prime are respectively the fluctuating velocities in x y and z direction and they gave rise to certain quantities right. Those quantities are extra terms that have come up in the mean flow governing equation and we said that is actually a complication.

And therefore, we cannot really solve for the mean quantity we need to know what these fluctuations are. So, there is a nicer way to write these fluctuations and that is what we are going to do first, let us say we want to say we are looking at incompressible fluid we are going to look at rho being constant. So, rho is a constant let us see what is del by del x of u prime u prime plus u prime v prime plus u prime w prime sorry, we do not want that plus del by del y of u prime v prime plus del by del z of u prime w prime is ok. I am just trying to look at a quantity which I have looked at as products of the fluctuating components and I am taking some derivatives I will I want to connect that expression to the first expression that I have written.

So, this is and I am going to simply apply the product rule. So, that will give me u prime dou u prime by dou x plus again u prime dou u prime by dou x plus u prime dou v prime by dou y plus v prime dou u prime by dou y plus u prime dou w prime by dou z plus w prime dou u prime by dou z just expanded the products. And, if you look at terms this and that and that; that is nothing but u prime into dou u prime by dou x plus dou v prime by dou y plus dou w prime by dou z plus, the other quantity that we have is u prime dou u prime by dou x plus v prime dou u prime by dou y plus w prime dou u prime by dou z.

$$\frac{\partial}{\partial x}(u'u') + \frac{\partial}{\partial y}(u'v') + \frac{\partial}{\partial z}(u'w')$$
$$= u'\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}\right) + u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial y} + w'\frac{\partial u'}{\partial z}$$

So, I have written all these things wrong this is correct now, so that is what we must have derived in the last class ok. So, the why I want to do this is because what is this that 0 and how do we know that 0 we are talking about the fluctuating components. But we know that the mean quantities and fluctuating quantities separately satisfy the continuity equation we proved it in the last class ok. So therefore, this quantity is 0 and therefore what I have on the left hand side is same as what I have on the right hand side.

But, if you look at these quantities on the right hand side this is what I actually have inside the integral in all these three terms is that right or in other words what my intention is to write down this quantity in the square bracket with this this particular one this particular quantity I want to substitute and use there that is the idea, should I say anything should I explain again yes no should I say no ok.

So, then I am going to substitute the quantity that I have circled dou u bar by dou t plus u bar dot grad u equal to minus dou p by dou x plus mu the laplacian ok. The laplacian of u I am going to write it in terms of components right, different components del square u by del x square plus mu del square u by del y square plus mu del square u by del z square minus ok. The quantity that is inside the integral del by del x of rho u prime square bar minus del by del y of rho u prime v prime bar minus del by del z of rho u prime w prime bar what have I done.

So, look at the quantities that are inside the square bracket ok. So, the quantities that are inside the square bracket I am replacing it with this general expression that I have circled ok. So, this as that is because we found that the quantities are same the one which I have circled and the quantity that is inside the square bracket ok. But each of those quantities are integrated from 0 to t and divided by t, so by definition they are all average quantities. So, I have put basically average of each of them.

So, the first is del by del x of u prime square so this is del by del x of rho u prime square bar it is basically the mean quantity. So, not mean quantity it is the mean of the fluctuations or mean of the square of the fluctuation. So of course, you know that that is not zero we talked about it last time, so that’s why you get all this nonzero quantities on the right hand side ok.

Is equal to I am going to combine now, let us say that and that is minus dou p bar by dou x plus del by del x of mu dou u by dou x minus rho u prime square averaged, I am going to combine that and that to get plus del by del y of mu dou u by dou y minus rho u prime v prime averaged I am going to combine that and that to get plus del by del z of mu dou u by dou z minus rho u prime w prime averaged that is ok. So, yeah so I was just going to ask you what is that quantity mu dou u by dou x.

$$\rho\left[\frac{\partial\bar{u}}{\partial t}+\bar{\bar{u}}.\nabla u\right]$$

$$=\frac{-\partial\bar{p}}{\partial x}+\mu\frac{\partial^2\bar{u}}{\partial x^2}+\mu\frac{\partial^2\bar{u}}{\partial y^2}+\mu\frac{\partial^2\bar{u}}{\partial z^2}-\frac{\partial}{\partial x}\left(\overline{\rho u'^2}\right)-\frac{\partial}{\partial y}\left(\overline{\rho u'v'}\right)$$

$$-\frac{\partial}{\partial z}\left(\overline{\rho u'w'}\right)$$

$$=\frac{-\partial\bar{p}}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial\bar{u}}{\partial x}-\overline{\rho u'^2}\right)+\frac{\partial}{\partial y}\left(\frac{\partial\bar{u}}{\partial y}-\overline{\rho u'v'}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial\bar{u}}{\partial z}-\overline{\rho u'w'}\right)$$

That is tau right so that is basically we defined a constitutive relation. So, originally we had the tau itself in the governing equation then we base r we used Newton's law of viscosity and replace that tau with mu dou u by dou y or mu dou u by dou x whichever ok, so that is how these terms that you have gotten ok. So, those are stresses that is really associated with the viscous mechanism right, those are the viscous stresses ok. Now what we are saying is that effectively if you look at the equation for the mean flow it is not viscous stresses alone, but there is something else also that is appearing along with those stresses right because each of them contain.

So, the first stress actually contain something extra here, the second one also contain something extra, third one also contain something extra, so that extra stress is known as turbulent stress ok. So, it is the stress that is coming because the flow field itself is fluctuating.

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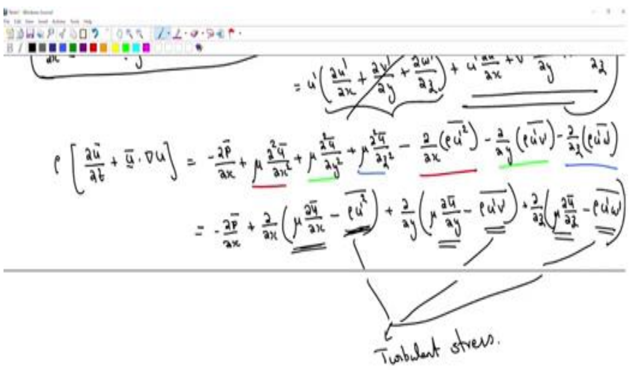

So, that is what gives rise to what you call as the turbulent stress ok. So, every place where you actually had this stress you could say that ok, there are two stresses one is coming from viscosity another is coming from the fact that the fluid field is fluctuating is that clear ok. So, remember now we originally when you derived the Cauchy's equation right, you actually had this stress you did not know what to do then. So, you said we will make use of Newton's law of viscosity and therefore you converted that stress into velocity and proceeded. So, you get everything in terms of velocity and therefore you were able to solve the equations.

Now, you have actually ended up with a similar situation, where there are quantities which are these primed variables which you do not know and therefore what we need is something like Newton's law of viscosity ok. In other words we need an expression for the turbulent stress ok, so that is the way people have been thinking about these terms and people try to derive various terms, none of them are as good as what Newton's law does ok.

So, we still do not know: what is the best way to do it there are various approaches. So for example when you look at people talking about turbulent flow and how they solve it they will also tell what kind of a turbulent model that they have used ok. So, that is what generally or really the field of turbulence modeling. So, it is really an unsolved problem we do not know what is the best equation that we want to do ok, we want to replace these fluctuating quantities with some mean values. But we really do not know what is the best thing to do but there for you there are certain models that you would use when you really want to solve a flow problem.

So, if you think you know you can contribute maybe this is one of the area that you should think about, you know where what is the best way to replace the those fluctuating quantities or is this split type of mean versus fluctuation itself is the right way to do it. Anyway so that is that is what I wanted to say about turbulence stresses. Now, realizing that we cannot take this route we are basically going to use a bit of non-dimensional analysis some insights from the experiments and then try to come up with you know expressions and quantities that we can use in our usual design routine.