Fluid and Particle Mechanics

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Lecture - 50
Laminar and Turbulent Flows - 1

So, the last few classes, you have actually been seeing you know various applications right where interaction between fluid and particles come into the picture. So, such situations many of times you might have used certain relations, certain empirical correlations and so on. So, most of them you might have actually dealt with complex geometries. What we are going to do today is to look at flow situations which you know the geometry can be simple, but the flow can still be complex.

So, when I say the flow being complex, I actually meant turbulent flow ok. So, earlier part of the course, we have really looked at what is laminar flow in various geometries you have been able to really solve the equations in an exact form and you have been able to calculate what is the velocity profile what is the pressure drop and so on. So, we are going to look at slightly different situation now, where the flow itself become complex even if the geometry is very simple. So, that is where the turbulence comes in and we are going to see you know how turbulence comes what can we do, when you have fluid flow is turbulent what is the best that we can do so, that you can go ahead with you know designs. So, that is the idea now hm.

So, let us say you let us take any simple case let us say you are you have flow through flow between two parallel plates. So, that is where we actually started our class.

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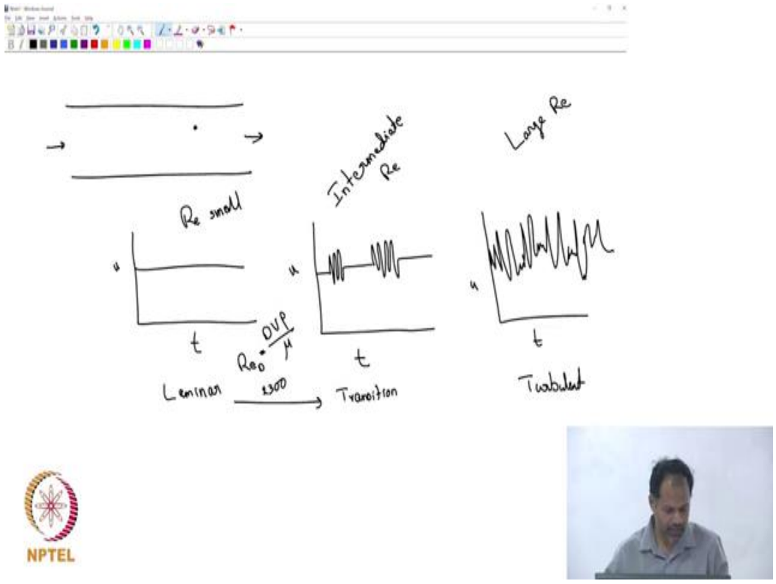

We have a let us say two plates and there is a fluid flow in it. And let us say you are interested in finding out what is the velocity and to find out velocity you actually you are going to do an experiment and you are going to put a probe a velocity probe to measure velocity at a particular point let us say.

So, there is a fluid that is flowing and let us say you want to measure at some location, what is the velocity ok. So, you have put a probe into that system and that the probe actually gives you what is the value of velocity and you are plotting that velocity as a function of time. So, you are continuously measuring the velocity and plotting it ok. So, we will assume that the flow is steady ok. So, there is a constant flow rate through the system. So, if you plot velocity at that point let me just denote it using u at some random location as a function of time.

If the flow rate is very low ok so, when I say flow rate is very low that is really characterize by low Reynolds number ok. So, if you define a Reynolds number and if Reynolds number is sufficiently small, we will see how small it is a is a small. But let us say if Reynolds number is very small and you have put the probe and you measure the velocity and plot it as a function of time the flow is steady you are going to see that the curve is going to look like that velocity is not changing as a function of time. So, you will get just a straight line.

Now, let us say you increase the flow rate when you increase the flow rate what happens to Reynolds number? It is going to increase. So, let us say you keep increasing your

Reynolds number, but you continue to measure it. So, this is what happens if Reynolds number is small ok. Now beyond some certain Reynolds number what you are going to see is that if you measure the velocity, it will more or less be constant, but in between you will see some fluctuation and then it will again come back to a constant and then again there will be a burst and then again it is going to come back to its constant value. There will be sudden bursts in the flow at various points.

We do not know a priori when it is going to happen, how it is going to happen, but that is what you see you do not care you will further increase the Reynolds number let us say ok. So, this is some intermediate Reynolds number. Now you go to really large Reynolds number and large Reynolds number would really mean that your flow rate probably is very high ok. If you do that where you will measure velocity as a function of time you would see that it is just a random signal something of that sort. There is nothing like you are going to you are not going to see anything constant about it ok.

Even though you can see that there is a mean which is constant, but it is really go in to be a noise because the velocity is continuously changing in time. So, the left hand picture when Reynolds number is very small that is when we call the fluid flow is laminar and the right hand side when the Reynolds number is very large. When you see this noisy signal, you will call the fluid flow is turbulent and in between when you see this it is actually the transition from laminar to turbulent. So, this is what we have as a laminar flow, this is what we have as turbulent flow and this is when we have transition between the two.

And remember this is not with respect to time this is with respect to Reynolds number that I am talking about. Is that ok? So, let us just see what it looks like. You might have already seen several instances. See you have heard about Reynolds experiment right.

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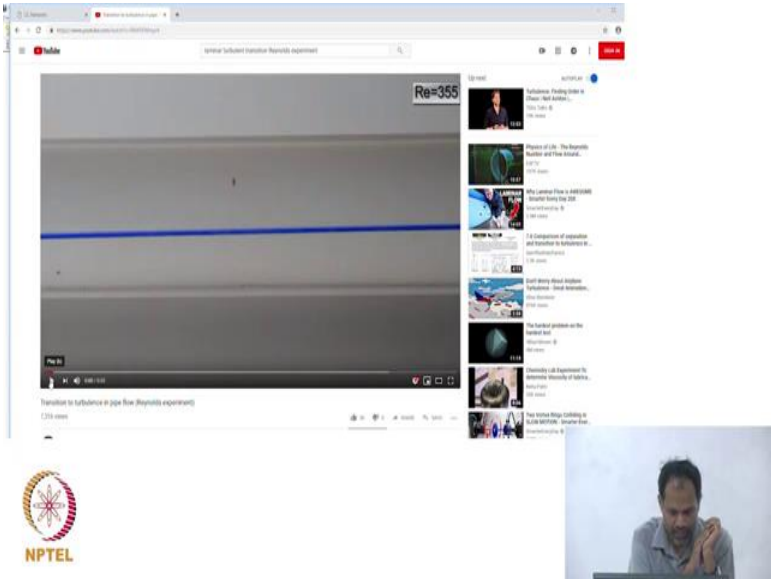

So, what you have is I do not know I think this is actually a pipe where the fluid is flowing. The fluid I think I do not know why one of one way or the other and the blue line that you see is basically some ink that you have inserted ok. So, what would you expect? If the fluid flow is laminar that ink is just going to be carried along with the fluid flow ok.

But if the fluid flow is very turbulent, then the fluid will see this fluctuating velocity sorry the ink will see this fluctuating velocity so, the ink will start actually spreading to different regions. So, that is what this experiment is; let us just watch it. So, I think initially. So, on the right hand side corner you can see the Reynolds number. So, Reynolds number is 355, now they are hiking of the Reynolds number.

So, the flow becomes sort of unstable, but it still smooth now this is basically the transition where you start seeing some bursts, the flow rate is increasing you can clearly see that. Now the burst become completely dispersed. So, that is basically the turbulence.

Display that again. So, it does not look like a pipe because the Reynolds number seems to be very small.

Yeah.

What is that?

Diffusion will happen, but diffusion will happen over a very small time scale sorry very large time scale because diffusion is a very slow process ok. So, if there was no fluid flow, you would have seen that of course, the ink is basically you know getting diffused. But you are not letting that to happen your fluid flow is sufficiently bigger than that ok. So, laminar let us transition and there goes the turbulence. Now we can actually see it in a quite a lot of situations.

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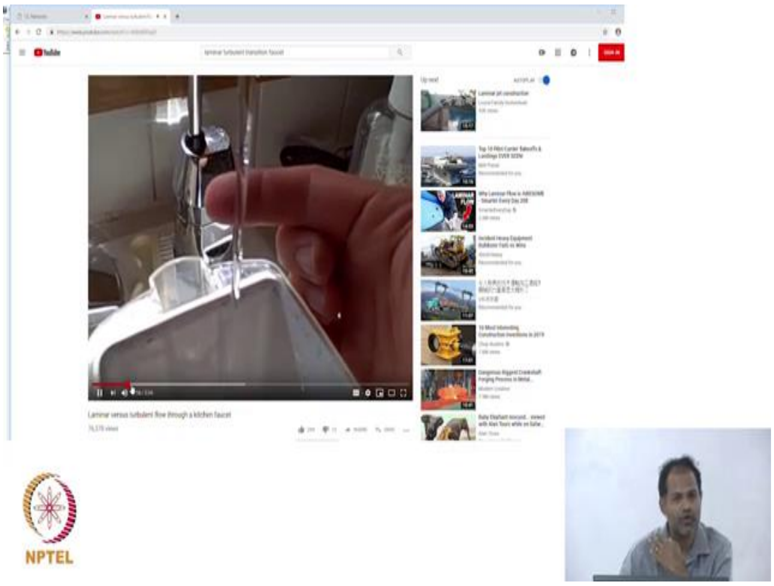

Let us just check this out. So, so, this is a fluid flow that is coming from a pipe and then you must have seen this right when you open it very lightly you basically have a very thin you know thin stream that is coming and that will look very clean and neat if the water is actually clean ok. But if you start increasing, then you will see that it basically becomes hazy ok. So, that is nothing, but laminar to turbulent flow transition. So, he is just going to put the finger I think, you can see that clean stream and therefore, you will be able to see the finger if you look through that that is because there is no velocity fluctuation everything is like a laminar; it is a basically laminar that is for flowing down. But if you increase the flow rate further. I hope he shows that; maybe I chose the wrong video. So, anyway you could try that I think this might show no this is also same.

There it is, this a nice one there you see ok.

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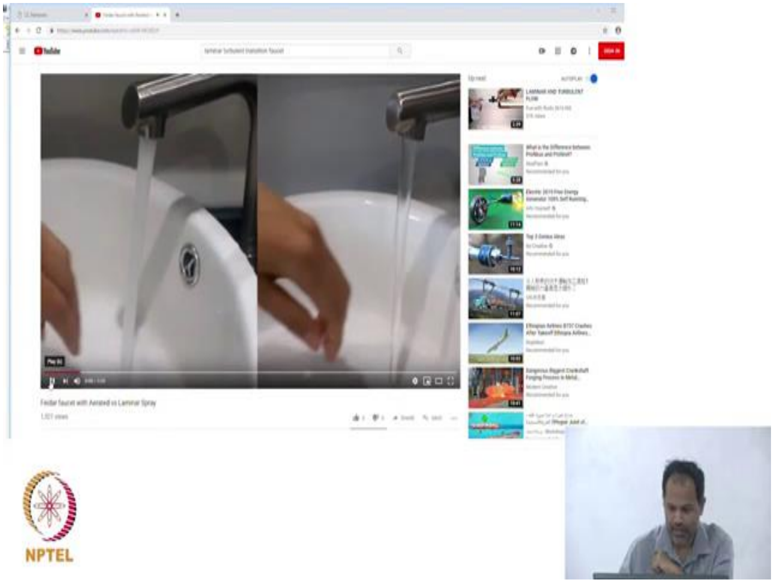

So, on the right hand side you see the laminar flow and on the left hand side you see the turbulent flow hm. So, even in a pipe in an industry this is exactly what you are going to see ok. So, what we have done so, far let us say we calculated the pressure drop in a pipe right we derived nice relations, but all those are useful when the flow is laminar, but not when it is turbulent. So, there is a clear distinction, you can see that visually when now mathematically also that is going to be a distinction and therefore, we need to know what is the velocity profile in a pipe when the fluid flow is turbulent ok.

So, our objective now is going to derive an expression for that. So, that we can calculate what is the pressure drop and therefore, come up with the right pump. So, that you can do you know pump fluids yeah. So, for a pipe typically this Reynolds number where the laminar to transition happens is approximately when Reynolds number is 2300, when Reynolds number is defined based on the diameter which is D V some average velocity rho divided by mu ok. So, you could define Reynolds number based on radius diameter anything. So, when it is defined based on diameter and when it is approximately slightly greater than 2000, you would actually start seeing the transition. So, just to give you what are the typical regimes.

$$Re_D = \frac{\rho V D}{\mu}$$

It can it can start forming those bursts. So, before that you can almost certainly say that the flow will be laminar.

So, it must be either because the fluid the this may not be a cylindrical pipe now the other thing is that its very sensitive to external disturbances any small vibration is sufficient to basically make it up. So, if you want to exactly fine 2100 or 2300 in a lab you need to basically make sure that everything is vibration free and so, on otherwise it will pick up. So, it must be one of those reasons. And it should be sufficiently far from your inlet and so, on and what is also there at the outlet yeah.

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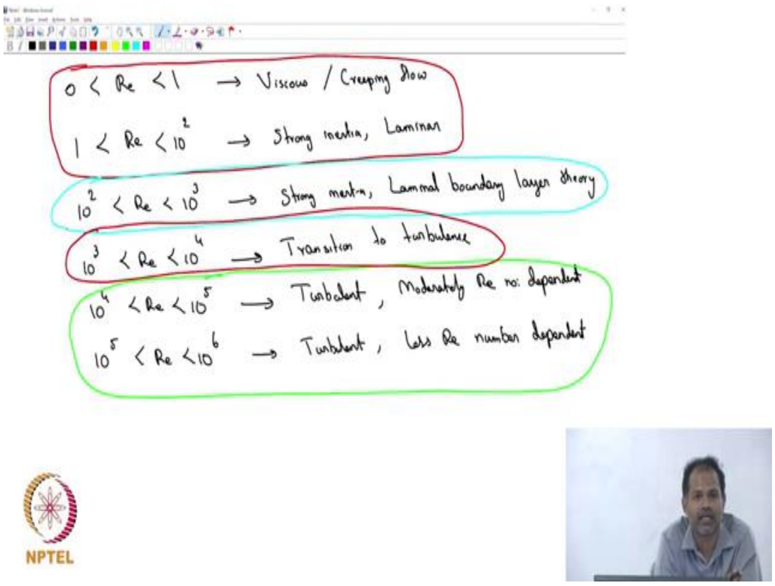

So, just to characterize the regimes. So, 0 less than Re less than 1.

So, Reynolds number is what is going to characterize it in that regime if Reynolds number is very small the flow is very viscous and one another name that you use is called creeping flow. So, if Reynolds number is saw between let us say 1 and 100 the inertia of the fluid is going to become a important, but still you are going to get a laminar flow. So, this is basically strong inertia, but the flow will still be laminar; between 10 raise to 2 and 10 raise to 4 is based you have the transition to turbulence. In fact, you could have one more in between let us say 10 raise to 3 because 10 raised to 3.

So, this is where you have again strong inertia, but something called laminar boundary theory is useful and let us say if it is between 10 raise to 3 to 10 raise to 4. This is when you have transition to turbulence and when it is 10 raise to 4 to 10 raise to 5 this is turbulent and this is one could say I will explain moderately Reynolds number dependent ok.

So, the equations that we had used earlier the ones which he derived will are strictly for a laminar flow and when the flow is laminar. So, so this is the regime that we really have talked about the equations that we used hm.

But typically when you go to large scales you are going to deal with large pipes large flow rates and the flow invariably turns turbulent and therefore, we ideally would like to go to larger Reynolds numbers, but and therefore, we would really like to go somewhere there ok. So, we need a now equations for that, just to mention that this let us say the one it is in between ok. This is still laminar flow, but you need you know there is something called a laminar boundary layer theory which we probably will just touch upon towards the end of the class. So, just theories are very useful in that Reynolds number regime and this one this the transition to turbulence, there are its very difficult to do either theory or experiments in that regime and typically what you do is you just avoid design doing any design in that regime ok.

So, when you do it you avoid you either stick to laminar flow or you stick to turbulent flow where you have some knowledge of it in between if you come rather you basically change your parameters since that you can do a better design with either of the flow ratios. So, that is what it is. So, we want to go to that green highlighted one now any questions?

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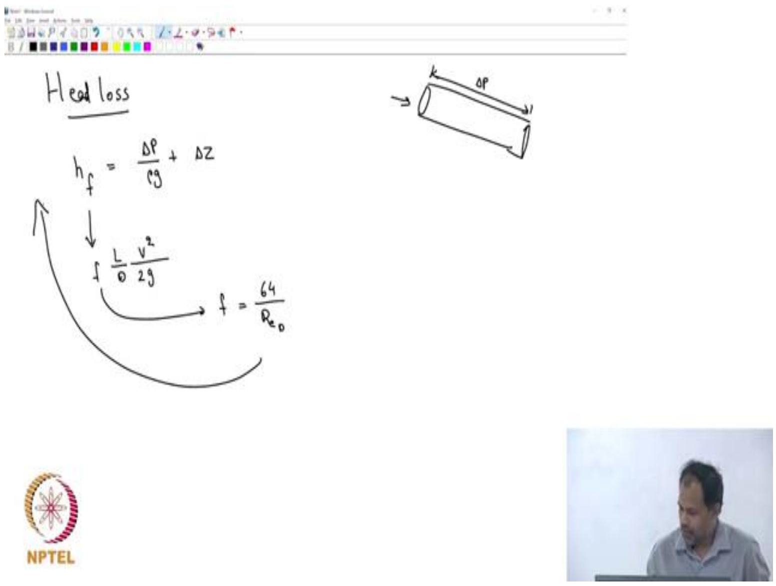

So, we talked about head loss you remember what is head loss?

Yeah, go ahead something where you actually had a pipe you were let us say you know fluid is flowing from one side to another and you wanted to characterize what is the pressure drop that is you know through a pipe for example. So, we probably would have taken some pipe we said you have fluid that is flowing and then there is a pressure drop of delta P and we would have calculated h f the head loss as delta P divided by rho g plus some delta is z if you want you can turn your pages and check that the relation is correct ok.

So, if delta is that; that means, if there is no difference in the elevation between two ends, it is just related to the pressure drop itself otherwise is the fact that the you know the effect of gravity will help ok. So, that is where the delta is that is coming in. So, hf is nothing, but really a measure of the frictional losses ok. So, the moment you know hf that is what you can prescribe and that is what you can actually use for your design. So, the idea is typically to calculate what is hf the head loss.

Now, the way we did it is we basically defined something called a friction factor and we read wrote down some relation between friction factor and head loss how did it relate hf into f does this friction factor.

Something of that sort way and then we would have finally, related our friction factor to Reynolds number that ok. So, the idea of doing this thing was that you specify the Reynolds number that will tell you what is friction factor that will tell you what is the head loss ok. So, this is really this way that our calculation should go ok, but though we derived it the other way ok. Now this calculation that we have done is for laminar flow and therefore, we would want to repeat this calculation for a turbulent flow. Now there is a challenge. So, the velocity profile that we saw right it was very noisy it was highly fluctuating.

So, what do we do with that? So, let us say we again have you know flow some flow in a pipe for example, and let us say the flow is turbulent we; that means, the velocity the flow is no more unidirectional you can really say that.

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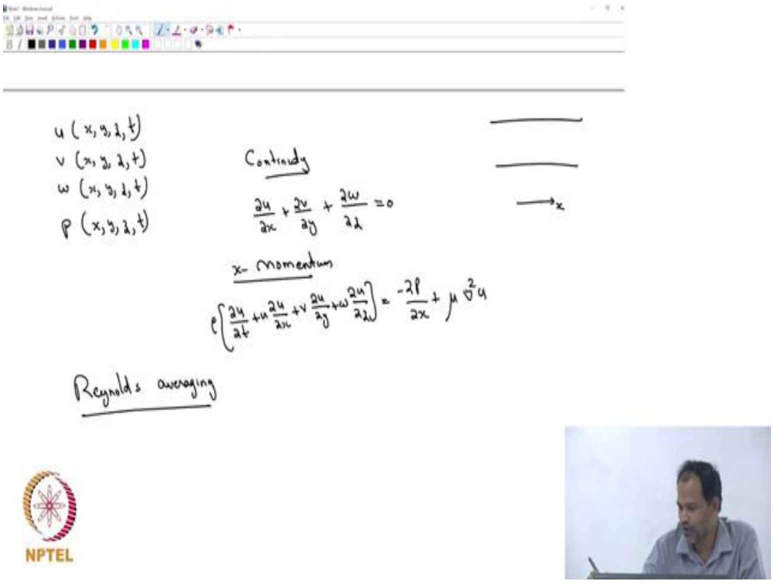

So, if you have it and the velocity is highly fluctuating; that means, and if I call let us say this direction as x hm. So, typically we will write down the three components of velocity u v and w ok. So, we have u v and w and in this case what is going to happen is that u will be a function of x y z and t ok. Similarly v is also going to be a function of x y z and t w is also going to be a function of x y z and t and even the pressure is going to be a function of x y z and t you measure any of them you are going to see that fluctuation ok. So, that may see tells that you the governing equations that we use right we cannot do all those simplifications that we had used earlier.

So, we need to invent something better. So, can we invent something better is the question. So, let us just write down let us say the continuity equation and the moment the conservation equation, these are the two equations that we had and we just kept our manipulating that. So, continuity equation in Cartesian coordinates was just dou u by dou x plus dou v by dou y plus dou w by dou z is equal to 0 and we had the momentum conservation let us just write down the x momentum equation that is the x momentum equation right when do we say something is a solution to this equation? When that expression satisfies this equation for example, we had the parabolic profile if you substitute that profile into this equation with the right boundary conditions you will find that it basically satisfies ok.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\nabla^2 u$$

Now, what is the kind of solution that we are looking for in a turbulent flow? We are looking for a equation which basically is going to do like this we are going to look at a fluctuating kind of a expression an expression which is fluctuating. And as of now we have no idea or we do not know any equations that can actually predict like that or that can be represented with a completely fluctuating field ok. So, therefore, it is basically useless to look for a solution, which is going to predict exactly those fluctuations because at the moment we even do not know, but we cannot give up right we need to do design. So, what is the best that we can do? So, the this person Reynolds right who did this Reynolds number experiment that you are familiar with he is a something interesting about that statement no ok.

So, see he came up with this idea called Reynolds averaging, ok. What he said is this let us say you have a flow which was that right this one on the right hand side where the velocity field was continuously fluctuating. We are saying that we do not know any function that can exactly prescribe those fluctuations, but why should we care about let us only worry about a mean value because we are only interested in the mean value. So, let us write down everything in terms of a mean velocity because that is what is going to interest us ok. So; that means, we need to define a mean velocity from an exact velocity.

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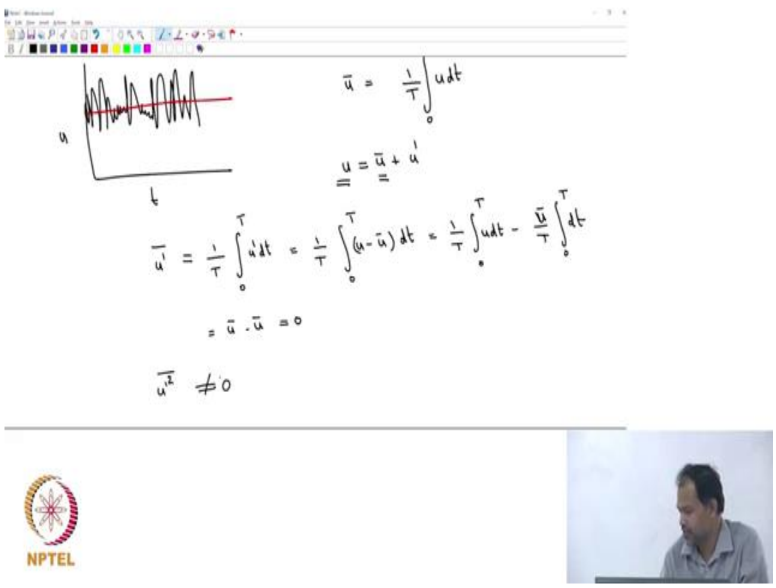

So, we have u as a function of time which is highly fluctuating we said all right.

We will define a mean velocity and worry about a mean velocity how do we define a mean velocity? u let us say mean is equal to I will say I will integrate it from some 0 to t u dt and then I will divide it with the time interval right. So, basically I have taken a time t or sufficiently long interval that I will keep on adding the velocities and then I add up all of them and divide with my time period I am going to get my average velocity right. So, that is the u bar that I will have. So, therefore, one says that ok. So, therefore, my total velocity at any point is going to be my mean velocity plus a fluctuating velocity correct. So, u is the one that satisfies the equations what we are interested in calculating is u bar and the difference between u bar and u is what you call the fluctuating velocity that is all right.

$$\bar{u} = \frac{1}{T}\int_{o}^{T} u\, dt$$

$$u = \bar{u} + u'$$

Similarly, you can define a v bar w bar p bar everything I mean pressure mean velocity and so on what would be u prime bar? So, what is u prime bar? Mean of fluctuating component what do you think that would be? It should be 0. So, let us prove that is equal to by definition is 1 by T integral 0 to T u prime dt that is equal to 1 by T integral 0 to T u minus u bar d t that is equal to 1 by T integral 0 to T u dt minus 1 by T integral 0 to T. So, u bar is a constant. So, I can write it down outside u bar dt that is equal to u bar minus u bar is equal to 0.

$$\overline{u'} = \frac{1}{T}\int_{o}^{T} u'\, dt = \frac{1}{T}\int_{o}^{T} (u - \bar{u})\, dt = \frac{1}{T}\int_{o}^{T} u\, dt - \frac{\bar{u}}{T}\int_{o}^{T} dt = \bar{u} - \bar{u} = 0$$

So, as expected it comes out to be 0 how about u prime square bar ? So, I am taking the square of the fluctuating velocity and me is you know averaging it will that be 0 or nonzero? It will be nonzero and this distinction actually becomes crucial ok. So, let us see where it leads to any doubts by the way till so far.

$$\overline{u'^2} \neq 0$$