

Fluid and Particle Mechanics
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Lecture - 49

Tutorial - 07

Hello everyone, today I am going to solve the Tutorial of a Fluid and particle Mechanics and the topic is filtration. So, first I am going to solve the first question; the first question consists of first I am going to read the question laboratory filtration conduct at a constant pressure drop on slurry of calcium carbonate in S2O gave the data shown in the table.

Table is even here volume say time and t y V, the filtrate area is given 0.044 metre square, the mass of solid per unit of volume of filtrate is given 23.5 gram per litre, and temperature is given. The viscosity of water is 0.886 centipoise. And pressure drop is given in the question they have mentioned that the pressure drop is constant throughout the process, and the pressure drop is 46194.8 Newton per metre square. So, first I am going to convert all the data in the 1 units. So, here I am going to convert all the data in f foot pound system units.

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Handwritten calculations in a Notepad window:

$$\text{Filtrate Area} = 0.044 \text{ m}^2 = \frac{0.044}{0.3048^2} = 0.474 \text{ ft}^2$$

$$\text{Concentrate of filtrate} = 23.5 \text{ g/L} = \frac{23.5 \times 2.831}{454} = 1.42 \frac{\text{lb}}{\text{ft}^3}$$

$$\text{viscosity of water} = 0.886 \text{ cP} = 5.95 \times 10^{-4} \frac{\text{lb}}{\text{ft} \cdot \text{s}}$$

$$\Delta P = 46194.8 \text{ N/m}^2 = 31073 \frac{\text{lb}}{\text{ft} \cdot \text{s}}$$

Filtrate Volume	t	t/V
0.5	12.3	24.6
1	21.3	21.3
1.5	22.0	14.7
2	108.5	54.25
2.5	182.1	72.84
3	201.7	67.23

For constant filter press:

$$\frac{t}{V} = \left(\frac{K_c}{2} \right) V + \frac{1}{q_0}$$

slope intercept

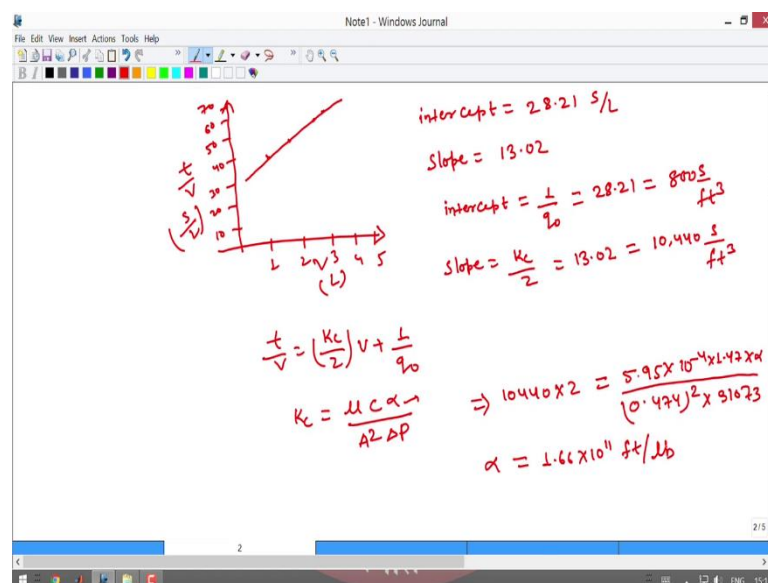
So, forth first I am going with area filtrate area; area equal to 0.044 metre square equal to 0.044 divided by 0.3048 square, it will be 0.474 foot square. Similar to that field concentrator filtrate is given 23.5 gram per litre in fpf it will be 23.5 in to 28.31 divided

by 454 is equal to 1.47 pound per feet cube, similar to 4 viscosity. Viscosity of water equal to 0.886 centipoises equal to 5.59 into 10 powers minus 4 pound per foot second.

And for the pressure drop also delta P equal to 46194.8 Newton per metre square, in foot pound system it will be 31073 lb per foot second. So, in the question that data table is given that with respect to volume, I am going to make the table also. Filtrate volume time second and t by V 0.5 1, 1.5, 2, 2.5 and 3.

And time 17.3, 1.3, 72.0, 108.3, 152.1, 201.7 and t by V is given from 34.6 sorry 34.6, 41.3, 48, 54.15, 60.84 and 67.23. So, in the question they mentioned it is a constant pressure drop problem. So, for that there is a equation it is, for constant filter process equation is t by V equal to k c by 2 into V plus 1 by q naught. So, if we plot t versus V versus this is a equation of a straight line where intercept is 1 by q naughty and k c by 2 is the slope.

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So, I am going to do a rough plot for this t by V versus volume 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 60, 70. So, if you plot this t versus V versus V it will give you a straight line. Similar to that this is the unit of this is the unit in second per litre.

This is whole imaging litre, later after getting the intercept and slope I will covert in the same foot pound in a system. So, from the graph you will get intercept equal to 28.21 second per litre and slope equal to 13.02. So, in your intercept is normally is equal to 1 by

q naught from that equation and equal to 28.21 in foot pound system it will be about 800 second per feet cube. Coming to the slope part slope equal to k c by 2 equal to 30.02 in foot pound system it will be 10440 second per feet cube. And now I am going to apply the same constant pressure formula.

So, the equation was $t \text{ by } V \text{ equal to } k c \text{ by } 2 \text{ into } V \text{ plus } 1 \text{ by } q \text{ naught}$ where k c is equal to $\mu c \alpha \text{ area square to pressure drop}$. Here μ is the viscosity and c is the concentration. So, now, everything is known except this alpha; alpha is a specific k a k resistance that we have to find out alpha equal to $\text{area square } \Delta p \text{ k c by } \mu \text{ into c}$.

So, just put the slop part k c by 2 equal to 10440 into 2 equal to 5.95 into 10 power minus 4 into 1.47 into alpha divided by 0.474 area square into pressure drop 31073. So, you will get a specific cake resistance equal to 1.66 into 10 power 11 foot per 8 pound this is the first part.

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The image shows a screenshot of a Windows Journal window titled 'Note1 - Windows Journal'. It contains handwritten calculations in red ink. The calculations are as follows:

$$\frac{1}{q_0} = 800 \frac{s}{ft^3} = \frac{\mu R_m}{A \Delta p}$$

$$\Rightarrow 800 = \frac{5.95 \times 10^{-4} R_m}{0.474 \times 31073}$$

$$\Rightarrow R_m = 1.98 \times 10^{10} ft^{-1}$$

Below the calculations is a table with three columns. The first column is labeled '2)' and the other two are labeled 'Pressure drop', 'Rm', and 'α'.

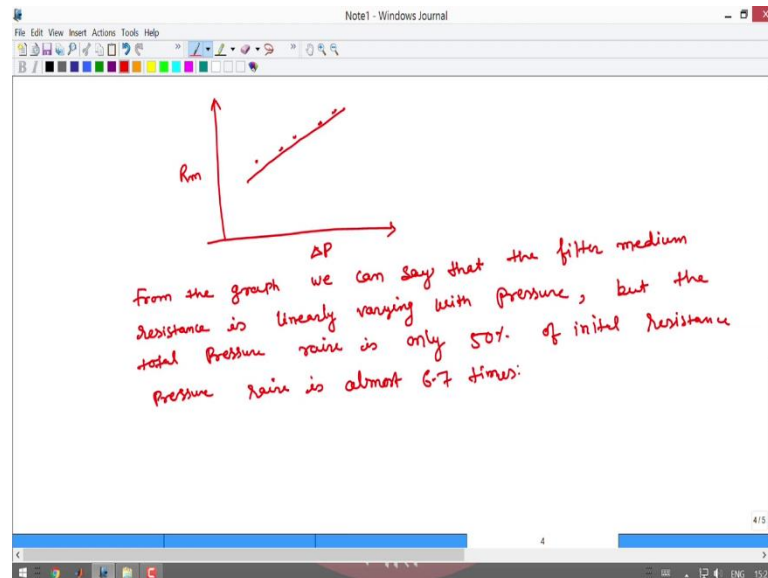
2)	Pressure drop	Rm	α

In the second part I have to find cake resistance that is from the slope we got the slope equal to sorry intercept intercept equal to 800 second per feet cube and equal to this one μR_m cake resistance into area into pressure drop.

So, everything is known here just the value 800 equal to is 5.95 into 10 power minus 4 R m divided by 0.474 into 31073. So, you will get R Rm equal 1.98 into 10 power 10 foot inverse, this is the first part of first question.

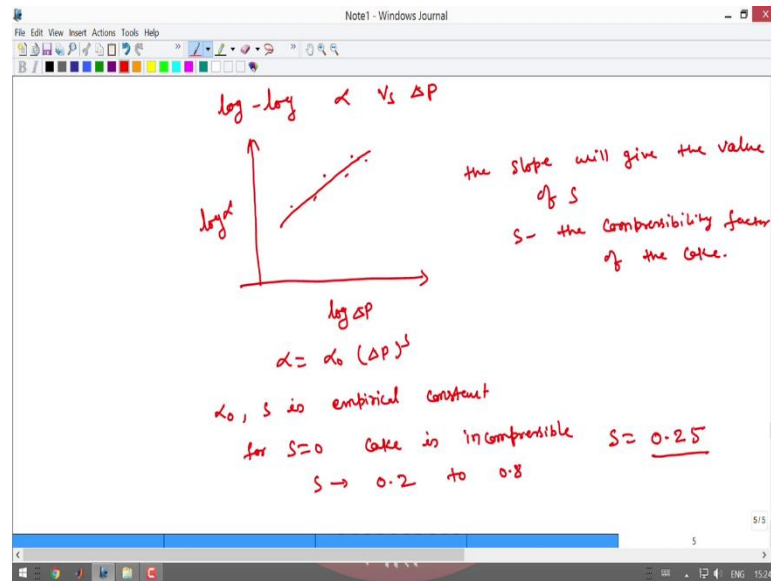
Now, I am coming to the second question second question is the just the extension of the first question. Here in the second question only the pressure drop is not constant pressure drop is varying with respect to specific resistance. So, in the in this question there is table is given, pressure, in this question table is given second question versus pressure drop filter resistance and a specific filter resistance alpha, this table are given, so ok.

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So, I am going to plot the graph R_m versus ΔP , it will show how the cake resistance vary with the pressure. So, if you plot this graph it will come to the straight line, from this graph from the graph we can say that the filter medium resistance is linearly varying with pressure. This is the and, but the total pressure raise; but the total pressure rise is only 50 percent of initial resistance. But in this case if you see the table pressure raise is almost 6 to 7 times from the initial value.

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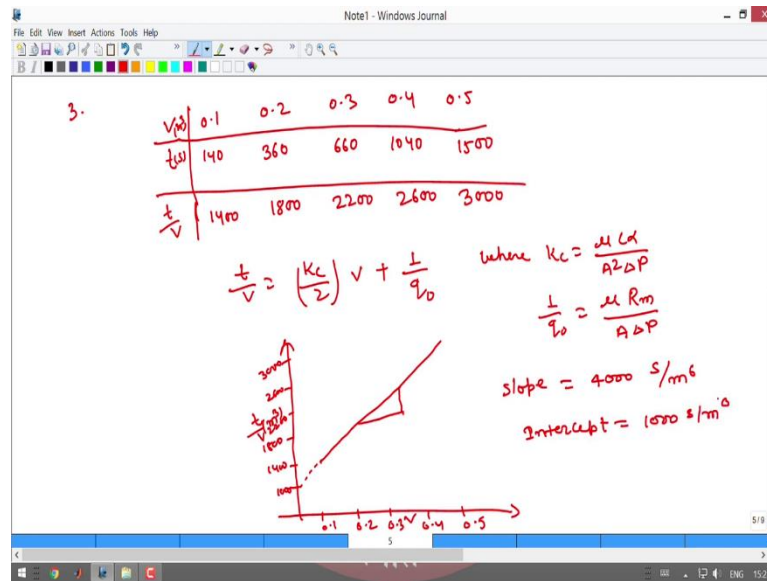
In the second part the from the table I am going to plot the log; I am going to plot log log plot of alpha versus pressure drop. So, I am going just giving the rough plot here, log alpha log delta P.

So, it will come similar to a straight line if you fit the point. So, from this equation there the slope from this plot the slope, will give the value of S . For varying pressure drop the cake is not incompressible I am sorry ok. Oh sorry this S is the compressibility factor of the cake ok.

So, this equation will be suitable alpha equal to alpha naught into delta P raise to power S , here S alpha here alpha not and S is empirical constant. Normally S for S equal to 0 cake is in incompressible and for normally the value of S vary from 0.2 to 0.8 for S is being positive the cake will be compressible.

So, from this plot you will get S equal to 0.25. So, from this we can conclude that the slope of the above plot is S and the this is slightly compressive. The value of S is about 0.25 the so, the cake is not that much is slightly compressible.

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Now, I am going to solve the 3rd question in this tutorial. In the 3rd question the filter has an area of 0.5 metre square and operate at a constant pressure drop 500 kilo Pascal. The following test result were obtained for slurry in water which gave rise into a filter gate the table is given here, volume versus time. 0.1, 0.2, 0.3, 0.4, 0.5, 140 second meter cube 360, 660, 1040, 1500 this is also a constant pressure programme. So, we are going to apply the same equation as in the first one.

So, for that we need the value of t versus V for plotting the graph. So, if you calculate the t versus V, you will get this one is 1400 this one you will get 1800, 2200, 2600, 3000. So, we are going to apply the same formula t versus V equal to k c by 2 into V plus 1 by q naught.

Here k c is equal to mu c alpha divided by A square delta p and 1 by q naught equal to mu R m by A into delta P. So, I am going to do a rough plot for this. So, this one is a volume this is time versus time by volume. 3000 of volume 0.1 0.2 0.3 0.4 0.5 so, here if you plot the graph you will also you will get the same straight line. So, from the graph the slope if you calculate the slope you will get slope equal to 4000 second per meter 6 and intercept equal to 1000 second per meter cube.

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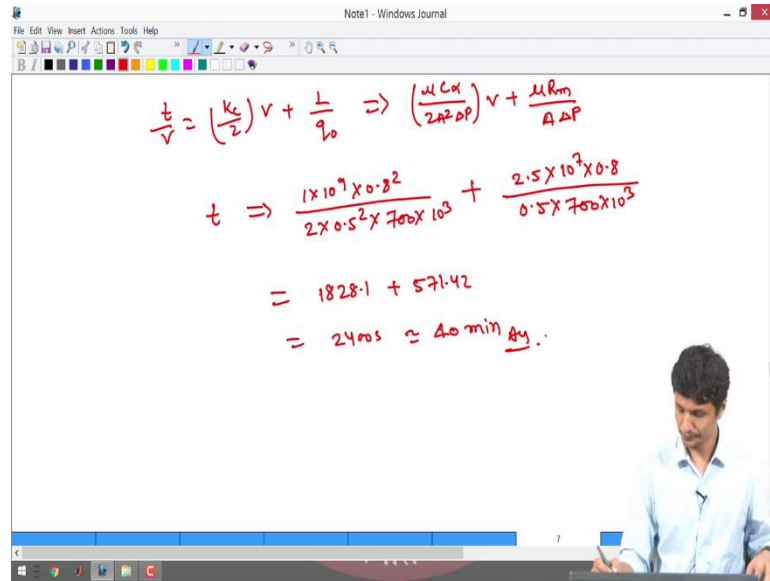
Area = 0.5 m^2
 Constant Pressure drop = $500 \text{ kPa} = 500 \times 10^3 \text{ Pa}$
 $\frac{1}{q_0} = \frac{\mu R m}{A \Delta P} \Rightarrow \mu R m \Rightarrow 1000 \times 0.5 \times 500 \times 10^3$
 $\mu R m = 2.5 \times 10^7$
 From the slope part
 $k_c = \frac{\mu C \alpha}{A^2 \Delta P}$
 $\Rightarrow \mu C \alpha = 4000 \times 0.5^2 \times 500 \times 10^3$
 $= 1 \times 10^9 \text{ Pa.s/m}^2$
 $\Delta P = 700 \text{ kPa} = 700 \times 10^3 \text{ Pa}$

So, for the first I even I am flowing the question area is given to 0.5 meter square and in the first case the constant pressure being dropped is, 500 kilo Pascal equal to 500 into 10 power 3 Pascal.

So, coming to the intercept part $1 \text{ by } q \text{ naught}$ equal to $\mu R m \text{ A into } \Delta P$. So, from this equation I am going to calculate $\mu R m$ equal to $1 \text{ by } q \text{ naught}$ was $1000 \text{ into area is } 0.5 \text{ meter square into pressure of } 500 \text{ into } 10 \text{ powers } 3$. So, you will get $\mu R m$ equal to $2.5 \text{ into } 10 \text{ power } 7$. So, from the slope part k_c equal to $\mu C \alpha \text{ by } A^2 \Delta P$.

So, from this equation I am going to calculate $\mu C \alpha$ $\mu C \alpha$ equal to $4000 \text{ into } 0.5 \text{ square } 500 \text{ into } 10 \text{ power } 3$. It will give the value of $1 \text{ into } 10 \text{ power } 9 \text{ Pascal second per meter square}$. In the question they have asked to calculate the time need to collect $0.8 \text{ meter cube of filtrate at a constant pressure drop of } 700 \text{ kilo Pascal}$. So, again I in this question they have asked to calculate with the constant pressure drop of $700 \text{ kilo Pascal to } 700 \text{ into } 10 \text{ power } 3 \text{ Pascal}$. So, we will apply the same formula.

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$$\frac{t}{V} = \left(\frac{k_c}{2}\right)V + \frac{1}{q_0} \Rightarrow \left(\frac{\mu C \alpha}{2 A^2 \Delta P}\right)V + \frac{\mu R_m}{A \Delta P}$$

$$t \Rightarrow \frac{1 \times 10^9 \times 0.8^2}{2 \times 0.5^2 \times 700 \times 10^3} + \frac{2.5 \times 10^7 \times 0.8}{0.5 \times 700 \times 10^3}$$

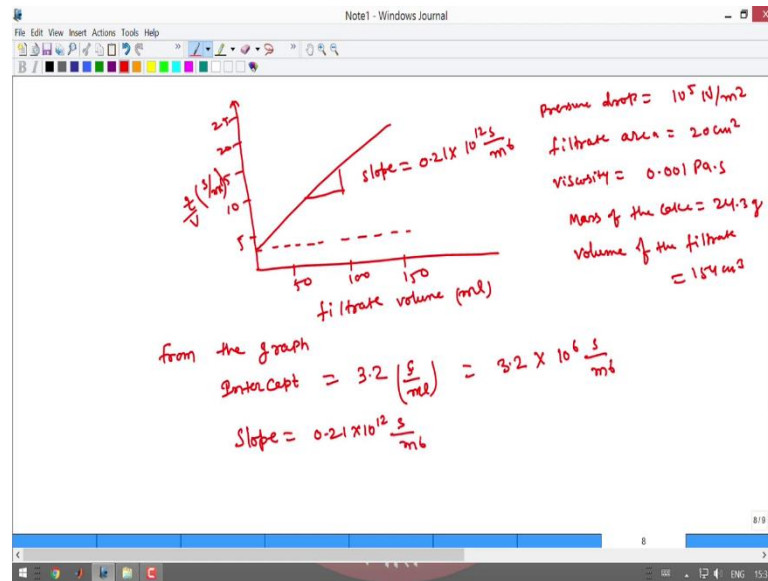
$$= 1828.1 + 571.42$$

$$= 2400s \approx 40 \text{ min approx.}$$

t versus V equal to k c by 2 into V plus 1 by q naught equal to mu C alpha by 2 A square delta P just I am putting the value of k c here, and value of q naught mu alpha R m into A into delta P.

So, now everything is know just we need to calculate the time, to calculate the filtrate of 0.8 meter cube 1 into 10 power raise to 9 into 0.8 square 2 into 0.5 square into 700 into 10 power 3 plus 2.5 into 10 raise to 7 into 0.8 0.5 into 700 into 10 into 10 raise to 3. So, it will about 1828.1 plus 571.42 is equal to 2400 second equal to 40 minutes, this is the time required to calculate the about 0.8 meter cube of all the filtrate.

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Now in the fourth question; in the fourth question there is 1 plot given plot is there 50, 100, 150, 5, 10, 15, 20, 25. This is filtrate volume ml and this is time versus volume in second per ml. So, in this question the given data are pressure drop; pressure drop is given $10^5 \text{ Newton per meter square}$ filtrate area area equal to 20 cm square , viscosity is given $0.001 \text{ Pascal second}$. Mass of the cake 24.3 gram volume of the filtrate equal to 154 c m cube there was what is the filter medium resistance and filter cake resistance. So, this is also a constant pressure drop questions. So, I am going to apply the same equation here also.

So, from the graph, in intercept is about 3.2 second ; second per ml in this I unit it will be $3.2 \text{ into } 10^6 \text{ second per meter}^6$. And slope will be if you calculate it it will above 0.21 in a sign it will be $10^{12} \text{ second per meter}^6$ from the slope 0.21 into 10 to the power $12 \text{ second per meter}^6$.

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From the intercept Part

$$\frac{1}{q_0} = 3.2 \times 10^6 = \frac{\mu R_m}{A \Delta P}$$

$$R_m = \frac{3.2 \times 10^6 \times 20 \times 10^{-4} \times 10^5}{0.001}$$

$$R_m = 6.4 \times 10^{11} \text{ m}^{-1}$$

↳ filter medium resistance

Coming to slope

$$\frac{k_c}{2} = \frac{\mu C \alpha}{A^2 \Delta P} \Rightarrow$$

$$0.21 \times 10^{-2} \times 2 = \frac{0.001 \left(\frac{24.3}{1.54} \right) \times 1000 \times 1}{(20 \times 10^{-4})^2 \times 10^5}$$

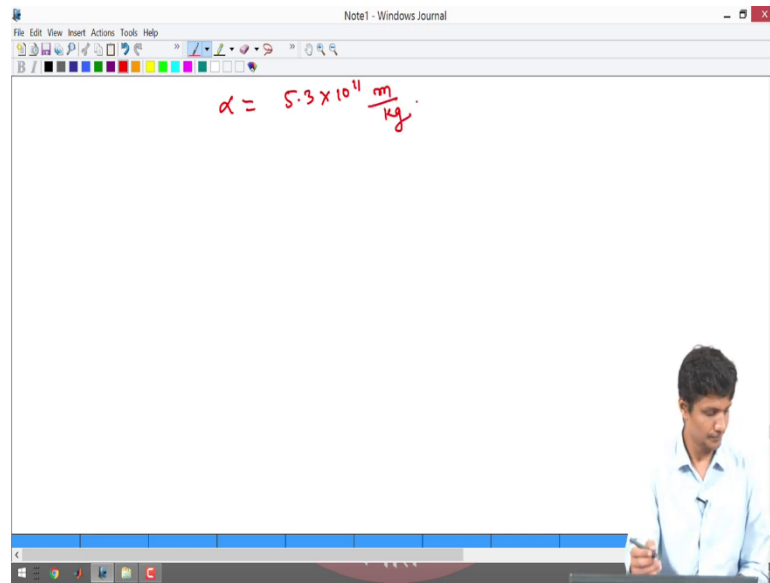
So, from the intercept part $1/q_0$ is equal to 3.12×10^6 for a (Refer Time: 29:38) it was 3.2×10^6 equal to $\mu R_m / A \Delta P$. Here everything is known, we have been known viscosity just I have to calculate the filter medium resistance R_m .

So, R_m is equal to 3.2×10^6 into area is given 20 in meter it will be 20×10 power by minus 4 into delta 10 power raise to 5 divided by μ will be 0.001 . So, R_m will come 6.4×10^{11} meter inverse this is filter gradient resistance.

Coming to slope, slope was given $k_c / 2$ equal to $\mu C \alpha / A^2 \Delta P$ where A square delta P slope we have got from the graph is $0.21 \times 10^{-2} \times 2$ equal to everything else is given 0.001 C is mass of the cake is given 24.3 .

And volume of the filtrate is from that we can get the concentration in a sign at you have to multiply it by 1000 into alpha divided by area square 20×10 raise to the power minus 4 square into pressure drop to 5 .

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$$\alpha = 5.3 \times 10^{11} \frac{\text{m}}{\text{kg}}$$

So, from the calculation it will come alpha equal to 5.3 into 10 raise to 11 meter per kg.

Thank you.