Fluid and Particle mechanics

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Lecture – 48
Filtration

So, this is going to be the last lecture on Filtration. What I will do; I will just do a quick recap of what we did in the last lecture.

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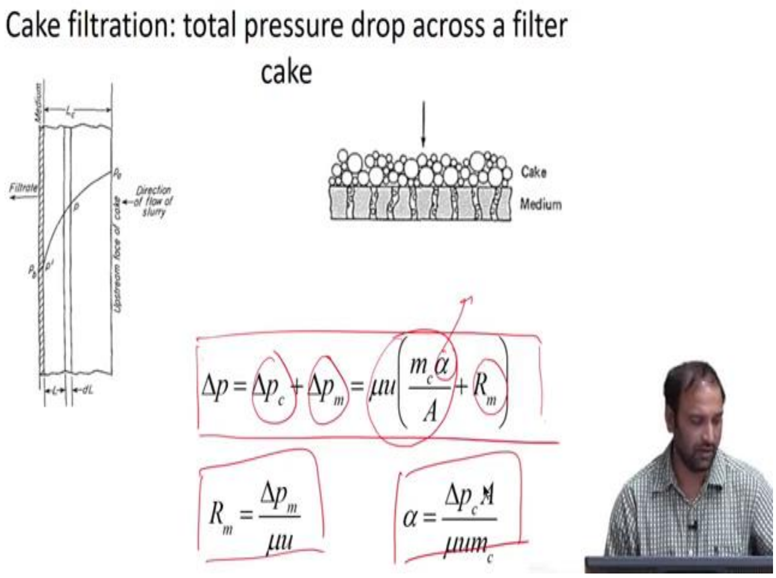

So, we were looking at developing an expression for what is the total pressure drop across a filter cake in the case of cake filtration. And what we did is, we ended up with this particular expression. We ended up with you know showing that the total pressure drop delta p, which is summation of the pressure drop because of you know the presence of the cake and because of the presence of the porous filter medium is equal to you know the contribution that comes because, if there is a that there is a term which is what is called as a specific cake resistance and R m is what is called as a filter medium resistance.

And they are kind of defined in this particular way right. That is what we are done in the in yesterdays in the last video lecture.

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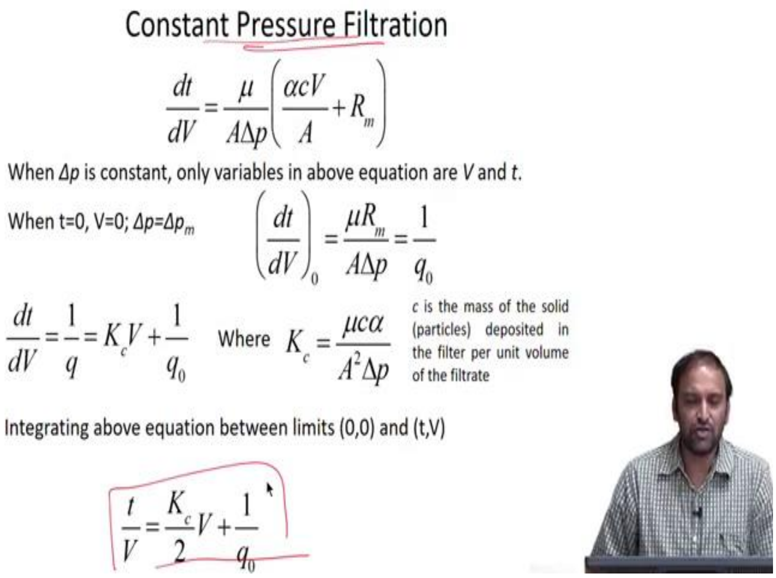

Constant Pressure Filtration

$$\frac{dt}{dV} = \frac{\mu}{A\Delta p}\left(\frac{\alpha c V}{A} + R_m\right)$$

When Δp is constant, only variables in above equation are V and t.

When t=0, V=0; $\Delta p = \Delta p_m$ $$\left(\frac{dt}{dV}\right)_0 = \frac{\mu R_m}{A\Delta p} = \frac{1}{q_0}$$

$$\frac{dt}{dV} = \frac{1}{q} = K_c V + \frac{1}{q_0} \quad \text{Where } K_c = \frac{\mu c \alpha}{A^2 \Delta p}$$

c is the mass of the solid (particles) deposited in the filter per unit volume of the filtrate

Integrating above equation between limits (0,0) and (t,V)

$$\boxed{\frac{t}{V} = \frac{K_c}{2}V + \frac{1}{q_0}}$$

And then, what we did is and we took that general expression and then we considered a case of a constant pressure filtration. Wherein, the objective is to relate the time of filtration and the volume of the filtrate that is collected during the course of filtration. So, we ended up with an expression which is t by V is equal to K c by 2 times p plus 1 over q 0 that is what we are done in the in the last video lecture.

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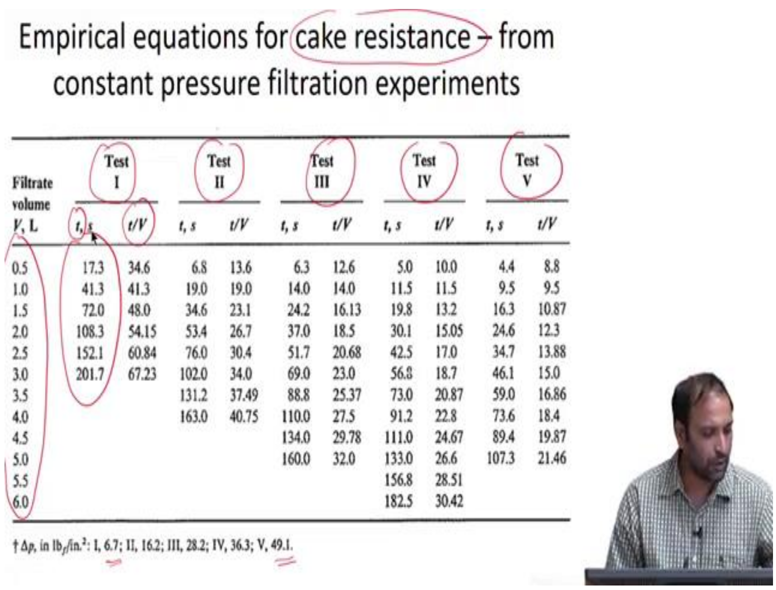

Empirical equations for cake resistance – from constant pressure filtration experiments

Filtrate volume	Test I		Test II		Test III		Test IV		Test V	
V, L	t, s	t/V	t, s	t/V	t, s	t/V	t, s	t/V	t, s	t/V
0.5	17.3	34.6	6.8	13.6	6.3	12.6	5.0	10.0	4.4	8.8
1.0	41.3	41.3	19.0	19.0	14.0	14.0	11.5	11.5	9.5	9.5
1.5	72.0	48.0	34.6	23.1	24.2	16.13	19.8	13.2	16.3	10.87
2.0	108.3	54.15	53.4	26.7	37.0	18.5	30.1	15.05	24.6	12.3
2.5	152.1	60.84	76.0	30.4	51.7	20.68	42.5	17.0	34.7	13.88
3.0	201.7	67.23	102.0	34.0	69.0	23.0	56.8	18.7	46.1	15.0
3.5			131.2	37.49	88.8	25.37	73.0	20.87	59.0	16.86
4.0			163.0	40.75	110.0	27.5	91.2	22.8	73.6	18.4
4.5					134.0	29.78	111.0	24.67	89.4	19.87
5.0					160.0	32.0	133.0	26.6	107.3	21.46
5.5							156.8	28.51		
6.0							182.5	30.42		

† Δp, in $lb_f/in.^2$: I, 6.7; II, 16.2; III, 28.2; IV, 36.3; V, 49.1.

As I mentioned so one of the nice things about doing this constant pressure filtration experiment is that, you know you can actually generate data with which I can say something about the cake resistance. And, I had mentioned that you could do a series of constant pressure filtration experiments. So, what you are looking at is data from for different filtration experiments.

So, test I, test II, test III, test IV and test V they are being done at different pressure drops, which are basically indicated here. So, 6.7 all they up to you know 49.1. And during each of these experiments, you basically collect what is the filtrate volume that is collected and the time ok. Therefore, I essentially have t versus you know t by V data.

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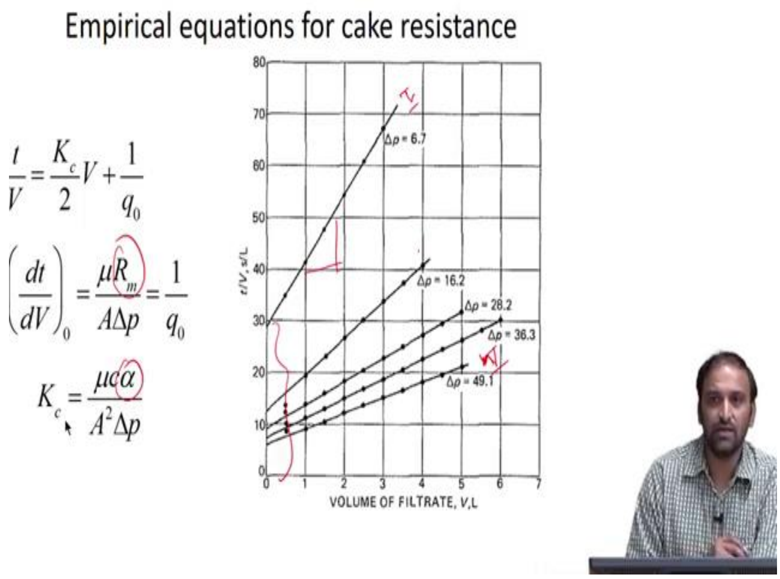

And what I could do with this data is I could actually plot them. These are again further five different trials. So, I have this is for trial I you know and then trial you know V right. And so, if you plot t by V versus V I get a straight line you know with particular slope and an intercept and from this slope and the intercept data I can actually calculate what is R m, which is the filter medium resistance and what is alpha which is the specific cake resistance.

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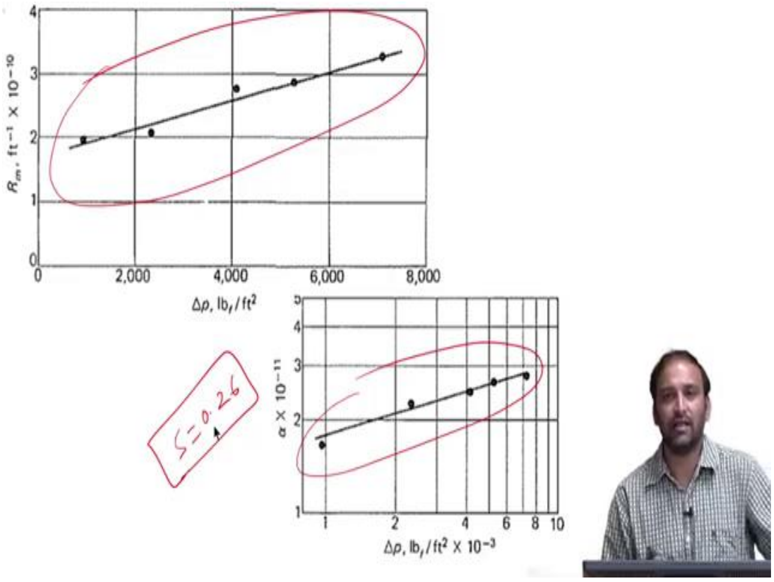

Now, for this particular example, if you look at how this R m and alpha vary as a function of you know for as a function of pressure drop. It turns out there you know both of them are not constant ok. Which is which the fact that R m also changes is little what surprising, but you know you could argue that look maybe the pressure drop that is you know applied across the during the filtration process is so huge that you know, there is there is some modification to the filter medium.

That means, you know there could be some pores that are clogged because of the clogged by the particles, because of the higher pressure drop that I have applied. That is why you know you see some variation of R m you know with respect to the pressure drop that I have applied. And, similarly alpha which is the specific cake resistance also changes ok.

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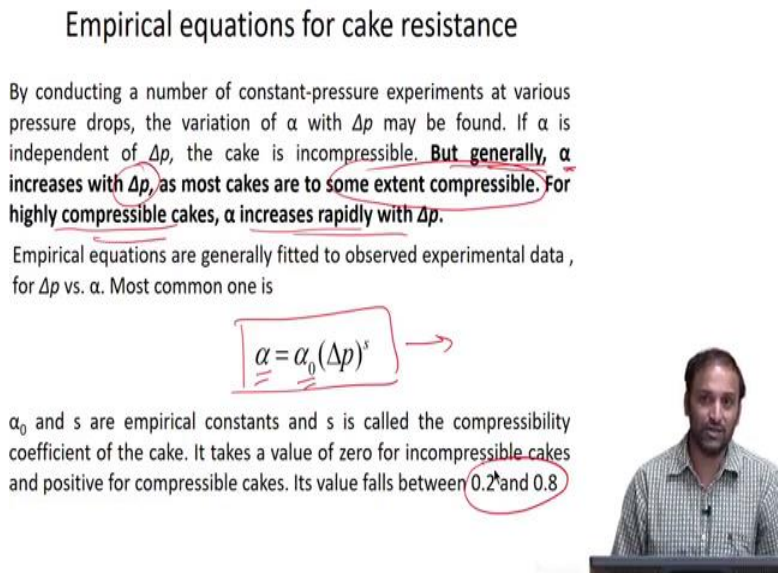

So, the point that I want to make is that ok in general alpha yeah can increase with delta p because, you know because, most of the cakes that people deal with they are to some extent they are compressible ok. And whenever, you have a compressible cake, you will have alpha varying with delta p for highly compressible cake alpha increases very rapidly with delta p, but you could have cases where you know there could be a or the variation could be may not be that steep. And, you could also have cases you know where you know you have a constant alpha as well ok.

It depends on you know, if you essentially; if you work with conditions where you know my delta p that I have applied is smaller plus if the slurry that I am working with is consists of rigid particles you know which do not kind of yield when you apply a pressure a particular pressure drop you know I could assume that the cake is incompressible.

So, most commonly what people do is, people use experimental data and they fit in you know power law kind of a you know expression of this sort where alpha goes as alpha 0 into delta p s to the power of s. And, depending upon you know what is the value of s ok, if it is closer to 0 its it is if it is 0 it is you know incompressible and if it becomes closer to 1 you know it becomes highly compressible ok. And, for the example that we looked at it turns out to know for this case you know the exponent s is about 0.26, which basically tells me that you know the cake that is being dealt with is you know slightly compressible right.

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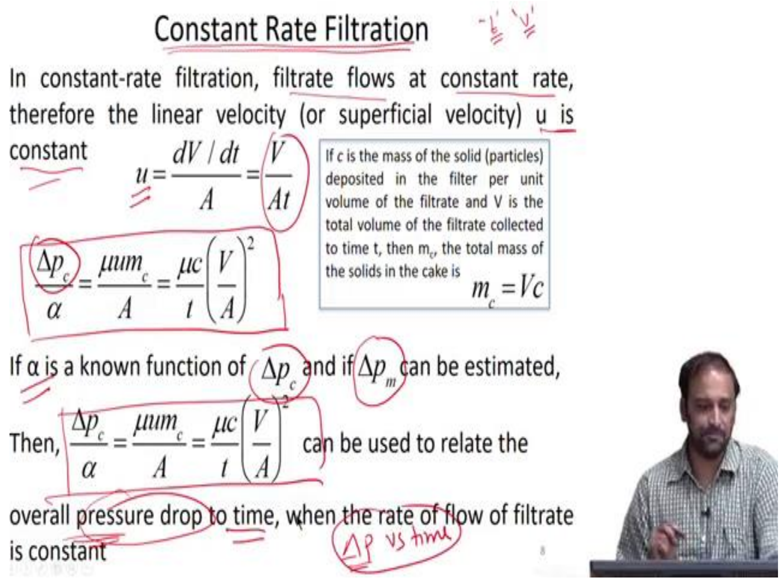

So, now that we have talked about you know the constant pressure filtration, we will move on to looking at constant rate filtration um. In the case of constant rate filtration; what is done is the filtrate flows at a constant rate ok. Therefore, you know your u ok, which is the superficial velocity is constant during the course of filtration. So, as I had mentioned that during the course of filtration, the cake develops because of the formation of cake there is going to be more and more resistance because of which ideally I should expect the flow rate to fall ok. That the filtrate should start coming down at lower and lower flow rates.

However, to maintain a particular constant you know superficial velocity or the flow rate of the filtrate, you carry out experiment by progressively increasing the pressure difference; that means, you know what you do is you because, you want to maintain a constant filtration rate you go on you start with certain delta p and then you progressively go on increasing delta p. So, that you know your filtrate flow rate remains constant during the course of filtration ok.

So, and because you know your u is dV by divide by A and that is constant for the case of constant rate filtration u becomes V by At. Now, the expression that you see here is the expression that we developed for the delta p c right. That is a pressure drop across the filter cake. And now, if you know the how delta p c and alpha are related and if I have some way of estimating what is delta p, you know then I can actually use this expression to relate the overall pressure drop to the time right.

$$u = \frac{\frac{dV}{dt}}{A} = \frac{V}{At}$$

$$\frac{\Delta p_c}{\alpha} = \frac{\mu u m_c}{A} = \frac{\mu c}{t}\left(\frac{V}{A}\right)^2$$

So, in the case of constant pressure filtration, we were interested in looking at an expression which will involve the time of filtration and the volume of the filter that I am collecting. In the case of constant rate filtration, what I am after is to obtain the relationship that relates the delta p versus the time that is what we are after.

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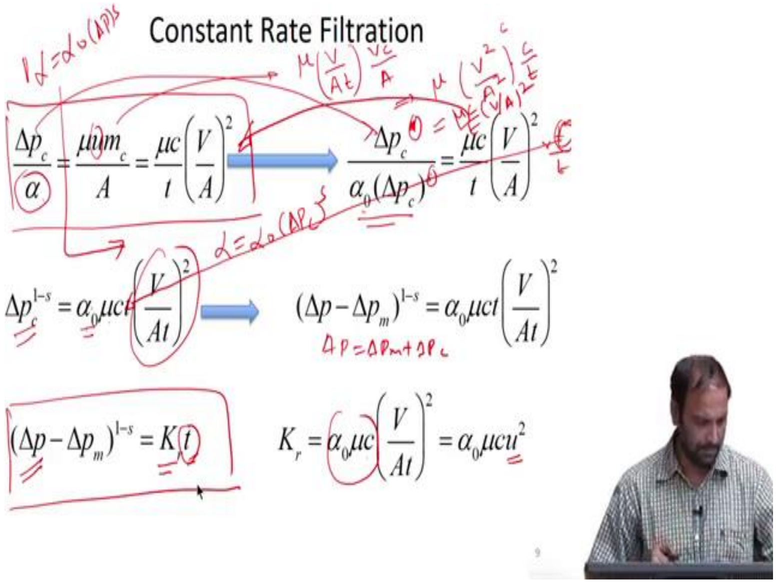

And, it can be done in a fairly simple way. So, what we are going to do is, I have you know this expression for delta p c that is delta p c by alpha is mu u mc divided by A and for u I am going to replace that u with. So, what I am going to do is, I am going to write mu instead of u I am going to write V by A times t that is for mu. And I have mc divided by A is what I have what I am going to do is I am going to multiply this by t and divide by t ok.

Therefore, what I essentially have is that that is mu and I have a. So, we also know that mc is V times c therefore, what I am going to do is this right let me just ok. I have mu here. So, that is and u and mc is V V times c right and divide by A. And what I am going to do is, I am going to multiply this by t and divided by t so, therefore, this becomes mu into V square by A square.

So, I do not have to do this multiplied by divided by t therefore, it becomes mu by V square divided by A square into c divided by t right. Therefore, this becomes mu times c divided by t into V by A whole square ok. That is what having right, but now if I so, we had said that you know there are empirical relationships that relate alpha to delta p c right. So, if I. So, we saw that you know your alpha is alpha 0 into delta p to the power of s.

Similarly, if you assume a relationship; similar relationship for alpha; so, I could write alpha is equal to alpha 0 into delta p c to the power of s ok. I could do that. Therefore, all I am doing here is I am retaining delta p c here right. And, then instead of alpha I am writing alpha 0 into delta p c to the power of s right. And therefore, if I take alpha 0 to the right hand side so, that I have alpha 0 multiplied by mu c t I can what I can do here is I can multiply by t and divide by t here.

Therefore, I can I have t that comes in the numerator that is this t. And I have V by A t whole squared right. And so, I because delta because delta p is equal to delta p m plus delta p c I can instead of delta p c I can substitute it for delta p minus delta p m to the power of 1 minus s because, you know I have the power 1 here right and I have the power s here.

So, therefore, if I take it to the numerator this becomes 1 minus s therefore, delta p minus delta p m to the power of 1 minus s is equal to some constant times t. And this constant K r essentially is alpha 0 into mu c multiplied by V by A t whole squared. And we know that V by a t is u therefore, your constant K r becomes alpha 0 into mu c u squared ok.

$$\frac{\Delta p_c}{\alpha} = \frac{\mu u m_c}{A} = \frac{\mu c}{t}\left(\frac{V}{A}\right)^2 \rightarrow \frac{\Delta p_c}{\alpha_0(\Delta p_c)^s} = \frac{\mu c}{t}\left(\frac{V}{A}\right)^2$$

$$\Delta p_c^{1-s} = \alpha_0 \mu c t\left(\frac{V}{At}\right)^2 \rightarrow (\Delta p - \Delta p_m)^{1-s} = \alpha_0 \mu c t\left(\frac{V}{At}\right)^2$$

$$(\Delta p - \Delta p_m)^{1-s} = K_r t\,; K_r = \alpha_0 \mu c u^2$$

So, this basically gives us the relationship between how this you know the how the delta what is the delta p that I would have to maintain, you know to basically achieve during the course of filtration that is you know as a function of time to basically maintain the constant rate of filtration. That is the working equation for the constant rate filtration.

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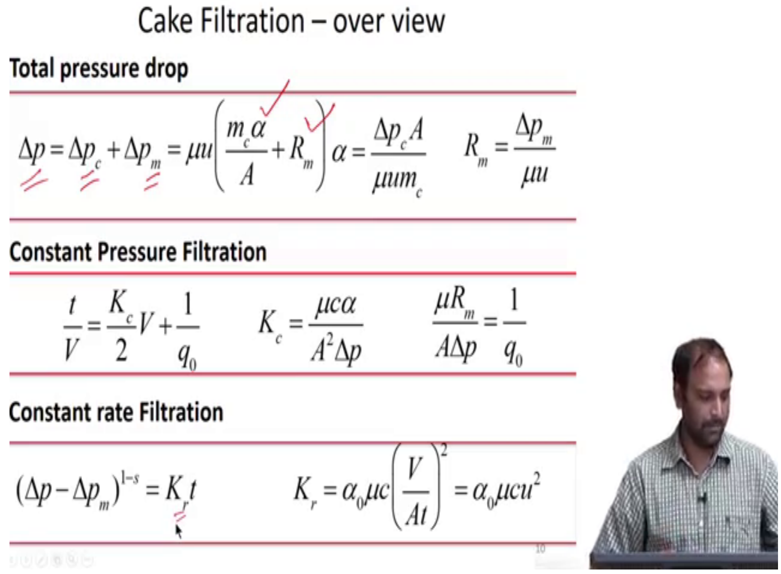

So, just to sum up what we have seen so far. So, the total pressure drop delta p is delta p c by delta p m and it is it can related to the specific cake resistance on the filter medium resistance. And for the case of constant pressure filtration, you have t by V is equal to K c into V by 2 plus 1over q 0. And, for the constant rate filtration you have delta p minus delta p m to the power of 1 over 1 to the power 1 minus s is Kr times t.

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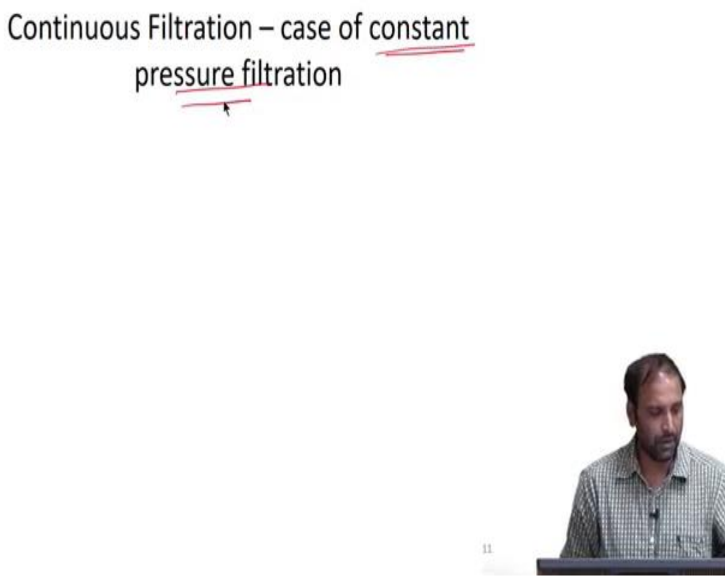

Now so, what I want to do, now is to look at continuous filtration. So, I would like to we would like to develop. So, we would like to consider the case of a constant pressure filtration, but; however, instead of looking at a batch filtration process that is what we looked at so far, but you know, but, but a continuous filtration process.

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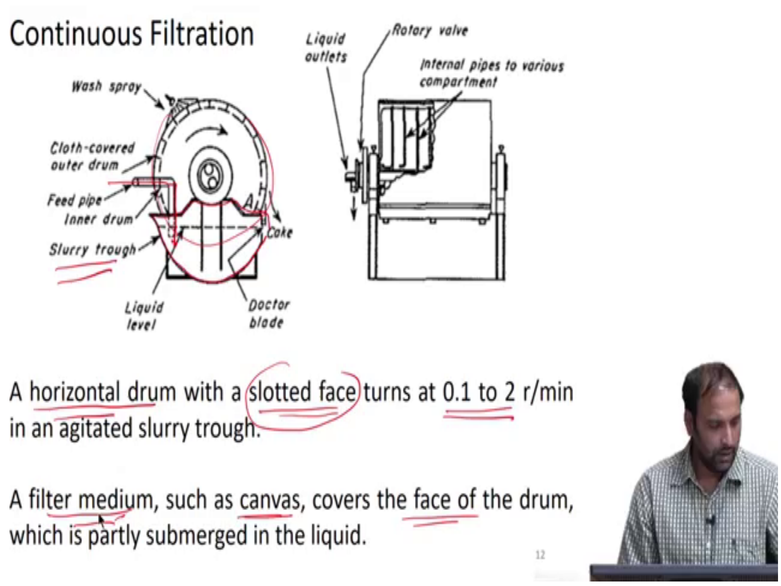

Before we do that I just would like to kind of take an example of a rotary vacuum process. So, and tell you little bit about what continuous filtration is all about. So, in the case of continuous rotary filtration ok, which is in this case its a in this case the pressure drop that is supposed to be maintained constant is done by applying a vacuum ok. So, in this case what you essentially have is that, you have a horizontal drum that is the horizontal drum. And so what you are looking at is if this is my drum, I am basically looking at it at a side view right.

So, therefore, I basically see a circle that is what you know this circle that you are seeing is all about. Now, this drum it actually has a slotted face; that means, you know there are pores on the outer drum. And this drum rotates at a slow speed ok. They are typically point one to two you know revolutions per minute and if you look at the diagram carefully.

So, what you do is, you this particular drum is immersed partially right in you know a slurry trough. And there is a feed that basically helps us with feeding the slurry that needs to be filtered into the slurry trough. And as you can clearly see that, there is a portion of the drum that is immersed in the slurry trough. And over this slotted phase ok, you essentially have a filter medium, you know it could be a canvas it could be, you know a kind of a cloth for example. And that covers the face of the drum ok, the entire drum is essentially covered with you know the filter medium.

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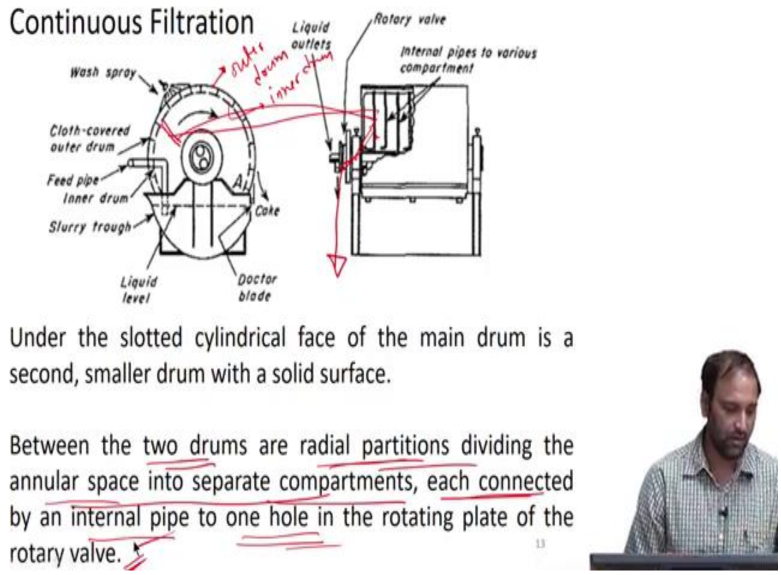

And so, now, so, that is the outer drum. Outer drum and there is also an inner drum ok. And the essentially the inner drum is actually is it is a solid it does not have pores; however, it has some slots which are essentially kind of shown in this diagram ok. These slots are essentially for carrying out the filtrate liquid into a rotary valve which is over here right. And from there I can actually take out the filtrate liquid.

And between the two drums, there are these radial partitions thus these are the radial partition that I was talking about ok. And they divide the space between the outer and the inner drum into separate annular compartments ok. And each of this is connected through an internal pipe that is these pipes into a one hole in the rotating plate to the essentially so all these are connected to the rotary valve and from there the fluid is kind of taken out.

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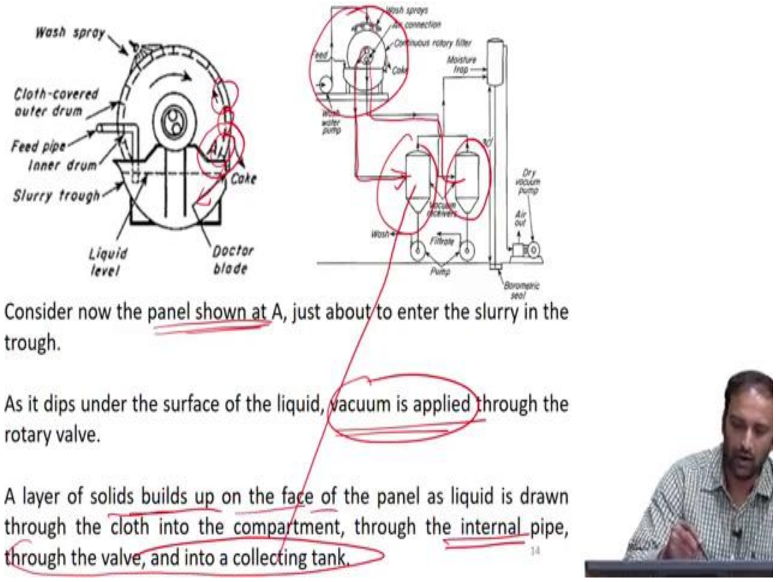

Now, what we are going to look at is. So, if you look at So, I am going to start with looking at this region A that is that is you know we going to look at the panel A that is shown here. And this particular region is about to enter the slurry um. That is present in the trough. Now, as soon as this region enters the slurry, what is done is a vacuum is applied you know through the rotary valve ok.

So, you have the capability of applying vacuum whenever we want and you also have the capability of releasing the vacuum whenever we do not want it. So, now, as soon as the vacuum is applied a layer of solid builds up right, because of you because you are applying a vacuum right and the liquid is sucked into you know these radial tubes ok. And whatever solids that you have in the slurry, they basically are deposited on the filter medium.

And of course, the filtrate is collected. So, it basically it comes to the wall and is collected in the collecting tank oh in this case. If you look at this particular operation you know, this is a the rotary filter that we are talking about. It basically has two collectors so one is for the washed liquid which you are going to talk a little bit later and one is for the filtrate. Basically filtrate gets collected in one you know container and the washed liquid that is typically used for washing the filter cake is collected in a separate drum.

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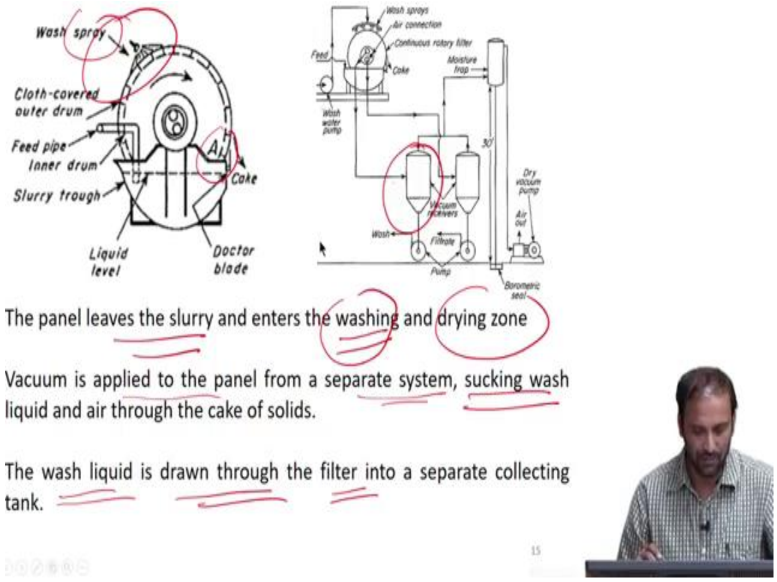

Now, so as soon as the panel A leaves the slurry, it enters you know the washing and the drying zone. So, this is basically where the washing is done. So, you essentially have a spray that is basically sprayed on to the cake that is formed. And that ensures the removal of the residual filtrate liquid. And vacuum is applied to the panel from a separate system. So, essentially once you have used a washed liquid of course, I would like to drain out whatever washed liquid has also entrained within the filter cake.

So, again I would have to suck all the washed liquid. So, for that you have a separate arrangement to do that. And once the washed liquid is drawn through the filter into the separate collecting tank that is you know that is your washed liquid that is a tank for collecting the washed liquid.

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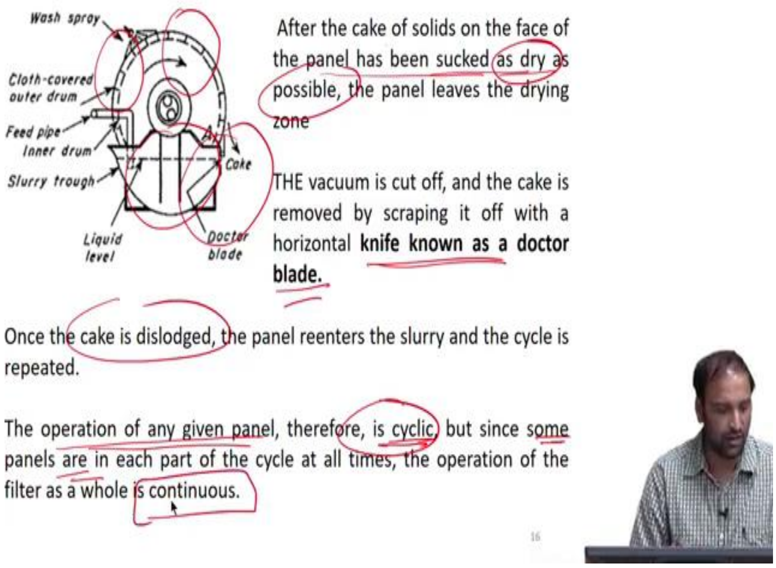

Now so, after the cake of solid on the face of the panel have been sucked so; that means, so the objective of this is to make sure that the cake is as dry as possible. So, once that is done it basically leaves the drying zone. And once the it leaves a drying zone ultimately, you remove the cake by using a knife which is what is called as a doctor’s blade that is doctor blade. Which is essentially used for scraping the cake that is formed on the filter medium and then ultimately the cake is dislodged.

Now, if you look at the operation for at anytime it is cyclic ok, but that what I mean to say is that. So, some of the panels in each part there you in any given time. So, they will always they will always be. So, what I mean to say is that the operation of any given panel. So, what I mean by any given panel is you know so each compartment right that is the each panel ok. So, for the operation of any given panel, you know it happens in a cyclic way, but since some panels in each part of the cycle at all times, the operation of the filter as a whole is a continuous process ok.

That means, you know there are some panels you know in which the filtration is going on, there is some panels you know where there is washing going on, there is some panels you know where there is a drying going on ok. And, some panels where there is a the scraping of the you know of the filter cake is going on ok. In that sense you know. So, the whole operation occurs in a continuous way.

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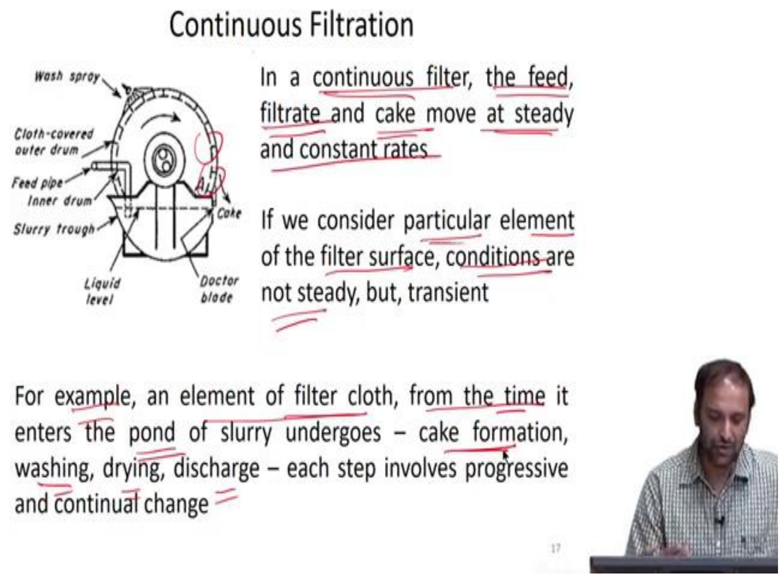

So, therefore, what we can do is, you can actually modify. So, if you look at the continuous filtration process, the feed the filtrate, the cake they all move at a steady and constant during constant rate during the entire course of filtration. So, if you consider the particular element of the filter surface; that means, you know if I take different you know these panels, the conditions are not steady, but they are transient right.

Because, you know some are undergoing you know the cake formation process some are undergoing the washing process you know some are going the cake removal process. So, therefore so for example, if you take an element of the filter cloth ok, from the time it enters the point of slurry it undergoes the cake formation the washing, drying discharge and each step involves progressive and continuous change right.

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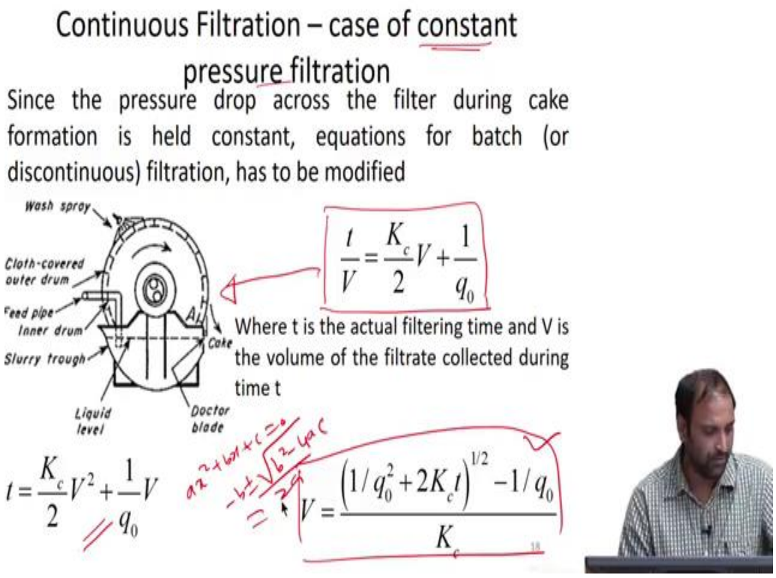

So, what we are going to do is, to develop the working equation for the constant pressure filtration. For the continuous filtration process, what we are going to do is. We are going to start with whatever expression that we developed for the constant pressure filtration that does t by V is equal to K c divided by 2 times V plus 1 over q 0. We are going to modify that to essentially suit the need of the continuous filtration process that happens in a vacuum rotary filter right.

So, the way we do this is so now, this expression that you see that is a quadratic equation in V right. So, therefore, it has two roots. So, if you have you know ax squared plus bx plus c right, if that is your quadratic equation, we know that the roots are minus b plus or minus square root of b square minus four ac divided by 2 a right. Now, if you if I get there two roots one of them is positive that is what this root is you can get that just by you know by this particular formula.

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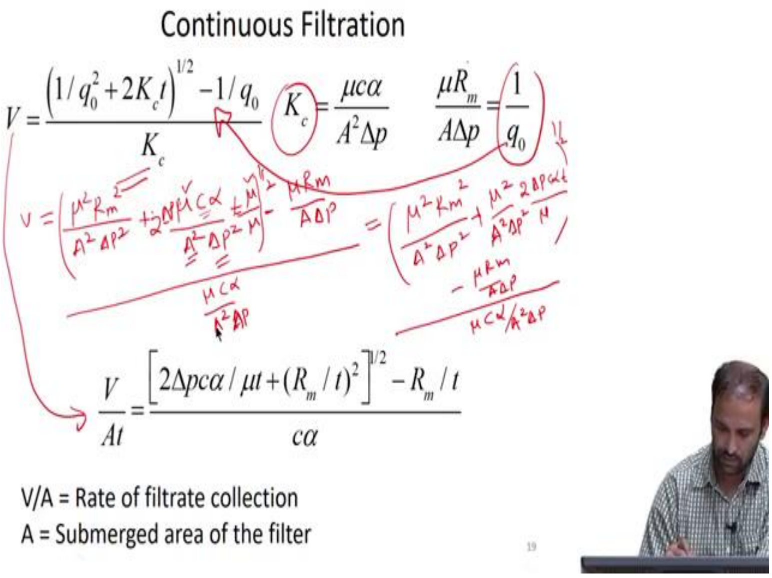

Now, so once I have this. So, I would like to because I know what is K c and you know 1 over q 0, I would like to substitute for these in this expression and then I would like to reduce this expression from this into this. So, the way to do that would be what I can do is, its just a simple algebraic manipulation. So, what I can do is instead of 1 over q 0, I am going to replace that with.

So, I am going to write it as V is 1over q 0 is mu square R m square divided by you know A square into delta p square that is for 1 over q 0 square plus 2 excuse me two times K c I can write it as mu c alpha divided by A square into delta p. And of course, I have the t there that is to the power of 1 over half minus again 1 over q 0 is mu R m divided by A into delta p divided by instead of K c, I have mu c alpha divided by A square delta p right.

Now, what I can do is um; I can I can do some simple manipulations or what he can do is, I can multiply and divide this by delta p. So, therefore, what I am going to do is I have 2 there which is which was there earlier as well ok. I am going to write delta p here and then I am going to make this as delta p square right. So, then I have just multiplied and divided by delta p, this is the second term in the in the parentheses um. Then what I can also do is, I can multiply and divide this also by mu ok.

So, what I am going to do is again I am going to rub this off ok. I am going to multiply this by mu divided by mu oops sorry about that ok. Divided by mu and I have 1 over 2 right. So, what I can do is. So, I have mu square there right I have mu square and R m square divided by A squared delta p squared plus, I have mu mu here that basically makes it mu square right. I have A and delta p square divided by A square delta p square. And I am going to write two times ok. Delta p that is this I have c times alpha into t divided by mu whole to the power 1 over 2 minus mu R m divided by A delta p whole divided by mu c alpha divided by A square into delta p right. So, I can do that. Now, what I am going to do is rub this all off ok.

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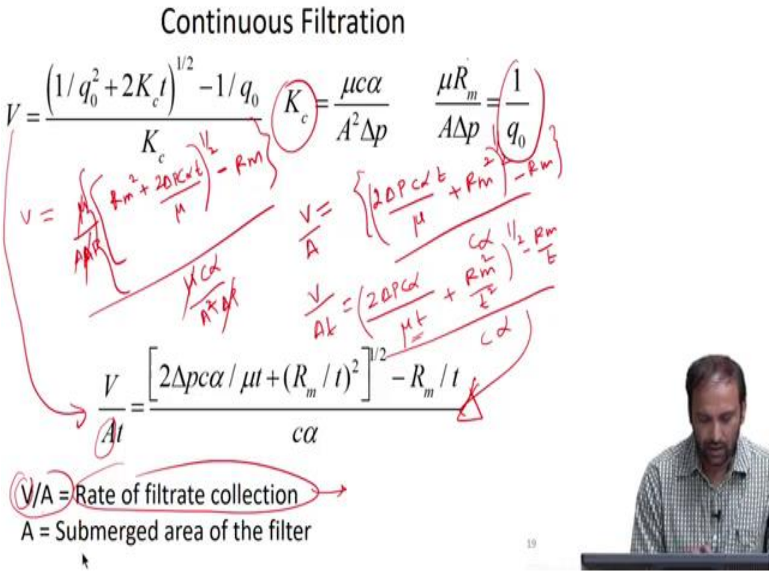

So, what I am going to do is, I am going to take mu divided by A delta p is constant. I am going to take it out of the. So, therefore this; so therefore, this becomes R m square plus and this is also out this is also out ok. So, that that becomes two times delta p c alpha t divided by mu to the power of 1 and a half minus. Again I have taken mu A delta p outside that becomes R m divided by mu c alpha divided by A square into delta p.

So, I am going to cancel mu here I am going to cancel the delta p as well and one of the A also gets cancelled. Therefore now, so what I can do? So, therefore, essentially what I have is V is equal to A times, I am going to write the second term first delta p into c times alpha t divided by oops um; divided by mu plus R m square to the power of half minus R m divided by c times alpha right. I can take A to the other sides that becomes divided by A. So, I am going to get rid of; I am going to get rid of A here right.

So, now what I can do is I can divide by t you know on both the sides. So, therefore, V divided by A times t becomes 2 times delta p c alpha t divided by because, I am taking it inside you know there is a to the power of one and a half there. So, therefore, it becomes mu into t square plus R m by t square R m square by t square to the power of half minus R m divided by t whole divided by c times alpha right. And one t gets cancelled here so I have this ok.

$$V = \frac{\left(\frac{1}{q_0^2} + 2K_c t\right)^{1/2} - \frac{1}{q_0}}{K_c}; K_c = \frac{\mu c\alpha}{A^2 \Delta p}; \frac{\mu R_m}{A\Delta p} = \frac{1}{q_0}$$

$$\frac{V}{At} = \frac{\left[\frac{a\Delta p c\alpha}{\mu t} + \left(\frac{R_m}{t}\right)^2\right]^{1/2} - \frac{R_m}{t}}{c\alpha}$$

So, essentially that is what u is an end up with right. So, that is this expression. So, it is basically a simple mathematical manipulation will essentially lead you to this expression. Therefore, what we have done is we have kind of converted this into V by A t is equal to 2 times delta p c delta p into c times alpha divided by mu t right. That is this is mu here right; mu t plus Rm by t whole squared to the power of one and a half minus R m by t divided by c times alpha.

Now, So, in this expression this V by A is the rate of filtrate collection right. That is a because this is the volumetric flow rate right volumetric flow rate divided by A will give you what is the rate at which the filtrate is being collected. And in the case of batch filtration, A was the filtration area; however, in this case this A is going to be the area of the drum that is submerged in this slurry.

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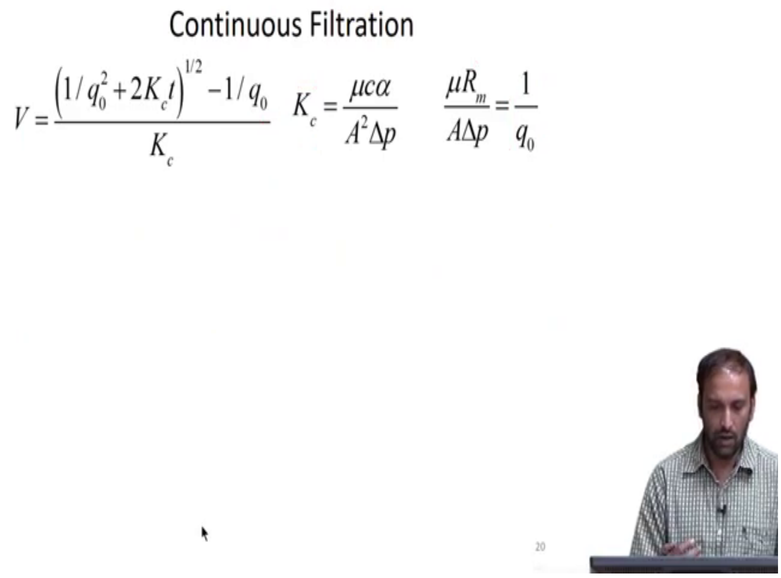

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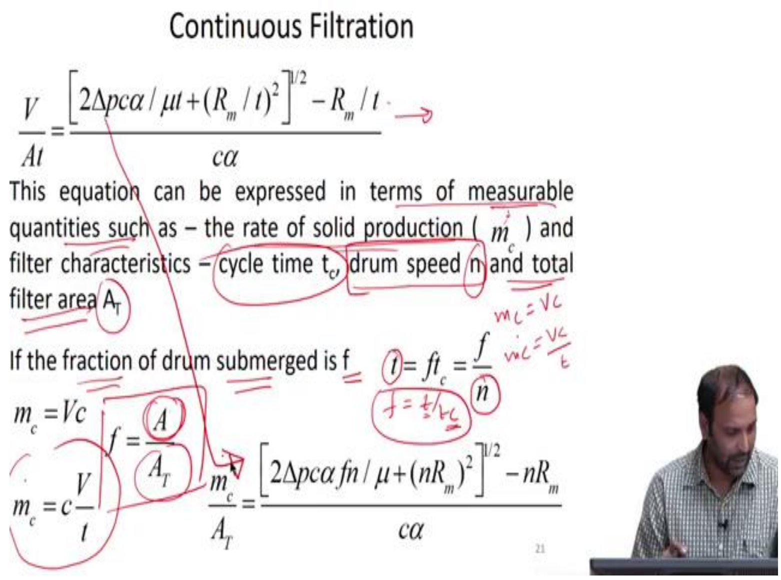

So, now what we are going to do is, we are going to start with this and then do some simple manipulations. So, now, that you have this expression. So, I would like to kind of modify this expression to bring in the; the experimental variables that are typically used in the course of you know continuous filtration. That means, I would like to express this in terms of measurable quantities such as the rate of solid production ok.

So, this m dot m dot c is the rate at which the. So, essentially we talked about mc being V times c right and if I want the rate of production of the solid which is m dot c essentially becomes you know V times c divided by t right. That is what I have here. So, I would like to express this in terms of the rate of solid production. And if you look at the filtration process, so we worry about what is called as a cycle time tc and also the speed at which the drum is rotating that is n and of course, the total filter area right.

So; that means, I have the entire drum that is available for you know you know I have a drum area which is which is At. However, only a part of it is available for filtration that is what is A right. Therefore, if the fraction of the drum submerged is f. I can actually relate this as t by t c right. That means, that is a total cycle time and that is a time for which the drum is in contact with the slurry and this essentially depends on the rotation speed right.

Depending upon what is the speed at which the drum is being rotated, if it is rotating very fast ok, you know that is you know; you know the drum is in contact with the fluid for a shorter period of time. However, it is rotating very slowly that is the drum is in contact with the slurry for a longer period of time. So, therefore, so I have these expressions which is the fraction of this you know area that is submerged, you know goes as A divided by At. That is A is the area that is available for filtration.

And At is a total area. And again I can express the same thing in terms of t by t c. And of course, the filtration time basically depends on you know is f divided by n, where n is the speed at which the drum is rotating. So, therefore, what I can do is I can use all this and then recast this x this equation into this form ok. Again it is a simple manipulation. So, if you are interested you know if you know you can you can try and do it you know. So, all you have to do is maybe I will just do that quickly ok.

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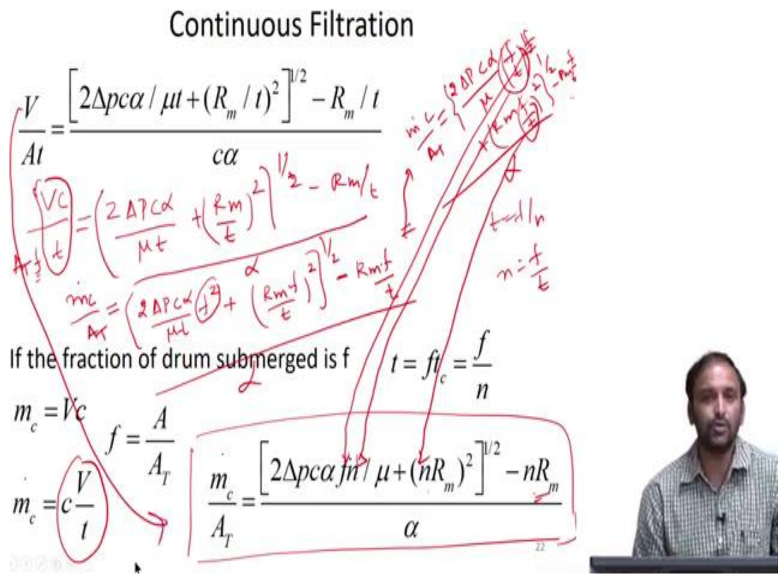

So, I would like to develop this right that is what I am after. So, what I will do is, I will start with this expression right. I have V divided by At now what I can do is I can take this c to the to the other side therefore, it becomes V c is equal to 2 A 2 delta p c times alpha divided by mu t plus R m divided by t whole square to the power of one and a half minus R m divided my t right.

Now, I know that f is A divided by at therefore, a is f times At. Where At is the total area. Therefore, I am going to replace; I am going to replace A with At times f right. Now, therefore, I have V c by t and we know that V c by t is m dot therefore, I have m dot c divided by At. I am going to take f to the right hand side therefore, and because I am going to and of course, I have divided by alpha here, because I am going to take f inside the you know the bracket here.

So, therefore, essentially I have 2 delta p c times alpha mu into t multiplied by f squared. Sorry multiplied by yeah of course, f squared, plus R m into f divided by t whole squared to the power of half note that you know I have f square here and of course, f square here and to the power of one and a half it basically gives me f that I have taken to the other side right minus R m into f by t right.

Now, let me just, now that is the divide of course, I have it everything is should be divided by alpha. Sorry of course, I have t here everything is divided by alpha. Now, so I know that because t is equal to f by n therefore, t is equal to f by n like therefore, my n is equal to f by t right ok. Therefore, what I can do is I have m dot c divided by At is equal to 2 times delta p c alpha into mu into f by t into 1 over t. Sorry I have f here right that is f square plus R m into f by t whole squared one and a half minus R m into f by t, whole divided by alpha right.

Now, f by t is n I have that here and then another f which basically is this f. And this f by t is n right. So, therefore, this becomes n times R m whole squared and this becomes n times R m. So, essentially we have kind of recasted this expression essentially into this form that is what we have done.

$$\frac{m_c}{A_T} = \frac{\left[\frac{2\Delta pc\alpha fn}{\mu} + (nR_m)^2\right]^{\frac{1}{2}} - nR_m}{\alpha}$$

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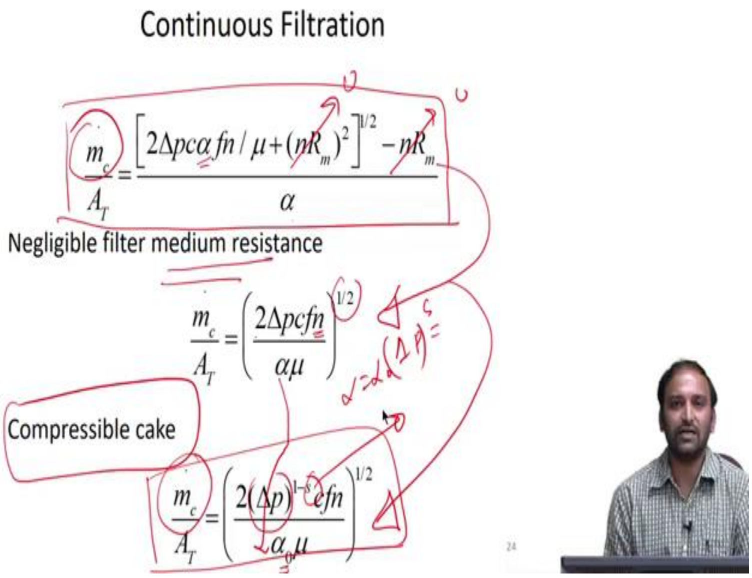

So now so, that is the equation, that we have recasted. Now if you assume that you know you had working under cases you know, where the filter medium resistance is negligible there I can essentially kick this term out and also this term out. So, the only thing that will be remaining is m dot divided by At is equal to 2 times delta p times c into f times n divided by you have mu here you have alpha here. So, therefore, that is that is what you will I think of course, you know you had alpha here.

So, therefore, it would become alpha divided by alpha square because, you know if I take it to the there is one and a half there is therefore, one gets canceled therefore, in the numerator you should have alpha right. That is what so that expression is right ok. And of course, you know I can say that if you are working with a compressible cake, I know that delta p is equal to. So, alpha is equal to alpha 0 into delta p to the power of s right. So, therefore, I can replace for alpha in terms of alpha 0 into delta ps therefore, alpha 0 remains in the you know denominator and you know the numerator becomes delta p to the power 1 minus s.

$$\frac{m_c}{A_T} = \left(\frac{2\Delta p_c f n}{\alpha\mu}\right)^{1/2}$$

$$\frac{m_c}{A_T} = \left(\frac{2(\Delta p)^{1-s} c f n}{\alpha_0\mu}\right)^{1/2}$$

Therefore, so you basically have a framework by which I can actually find out, what is the rate at which the solids are being deposited onto the filter cake. And, you know for a case, where both the cake resistance and the medium resistance becomes important. And, I can recast this you know expression to really look at cases where the filter medium resistance becomes negligible.

And of course, you know I can also kind of modify the expression to those to suit the need, where I have a cake that is compressible; that means, I am going to I have taken the empirical expression that was developed to essentially get an expression in terms of expression that relates m dot to the coefficient that basic basically tells me something about, how much compressible the cake is.

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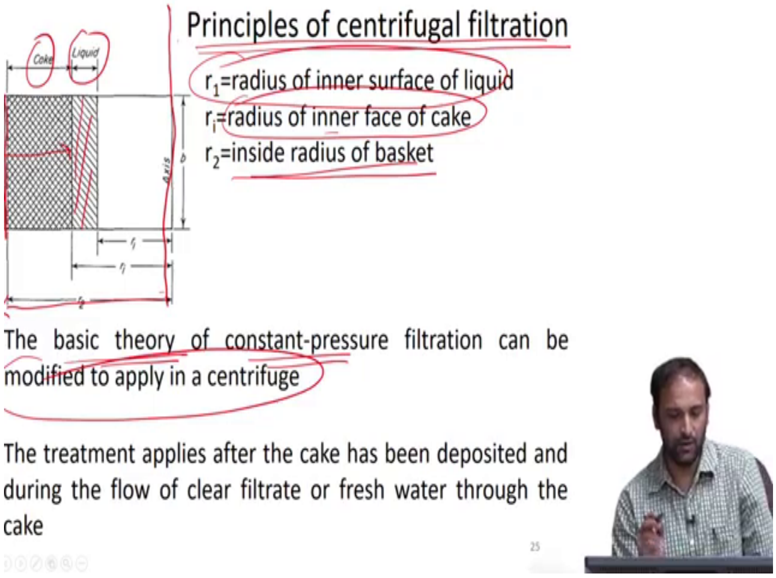

So, the last topic that I want to discuss is to look at something called as the principles of centrifugal filtration um. So, I mean if you so the way the centrifugal filtration works is you have a container right. Maybe you can think about it a cylindrical drum and it is basically rotated right. And so, what is so what you are looking at here that is the outermost that is a inner surface of the drum ok. And this inner surface of the drum of course, would be it will be perforated that means there are pores ok.

And on top of it you essentially put a filter cloth. And because of the centrifugal action so what will happen is, the slurry is kind of forced to come in contact with the filter medium and the solids basically get deposited onto the filter medium and that at least the formation of a filter cake. And of course, you would have a liquid with the particles you know in the inner region of the you know the centrifugal filtration set up.

So, if you think about this as a axis of rotation um. So, this r 2 essentially refers to the inside radius of the basket or the drum that basically rotates. And, r i refers to the radius of the inner face of the cake that is you know, that the cake is formed until this distance. And ri is the radius of the inner face of the cake. And r one is the radius of the inner surface of the liquid right and of course, you know if once the filtration is complete, if all the you know the all the slurry is kind of consumed. So, you really would not have this liquid. What will be left in the end would just be the fill the filter cake.

And what we can do again in this case you know, I can whatever a basic theory that we have developed for constant pressure filtration. You can also apply the same kind of you know formalism, but; however, you would have to modify that to suit the need of centrifugal filtration.

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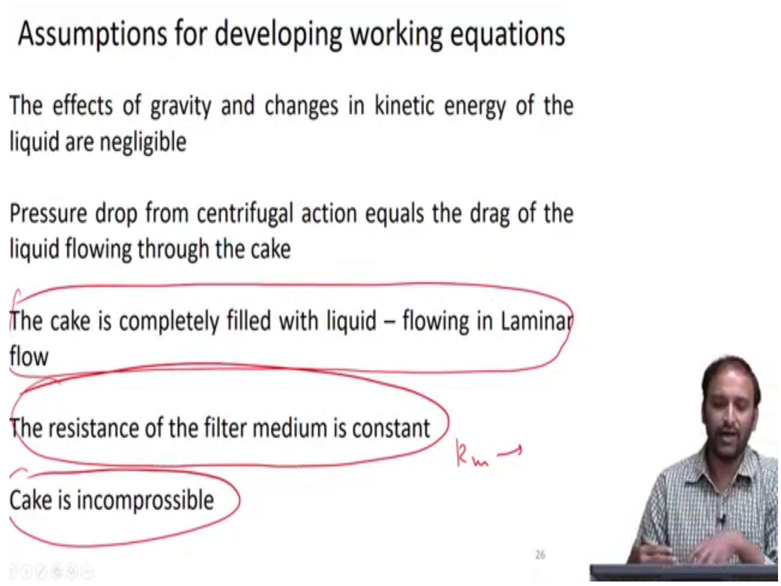

Of course there are some assumptions that are kind of important when you develop these things. So, we are going to assume that the cake is incompressible, the resistance of the filter medium is constant; that means, you know. So, your R m is not changing during the course of filtration. And, you know we are going to assume that the cake is completely filled with liquid such that you know the flow is laminar right. So, you have all these assumption that that have gone in terms of developing these expressions.

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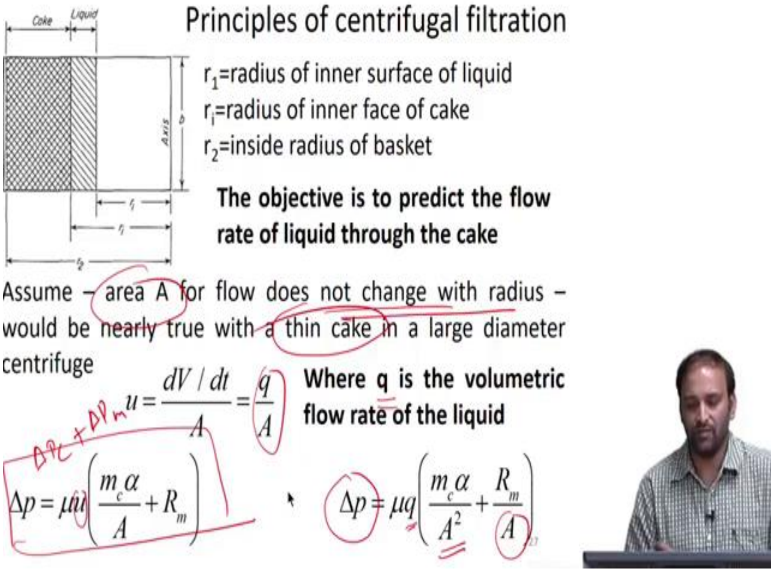

So, again we will start with the same working equation the total pressure drop that is delta p is you know delta p c plus delta p m right that is what we had we had mentioned and this u which is the superficial velocity, with which the filtrate is flowing. So, we are going to write that as q by A that you know where q is the volumetric flow rate of the liquid. So, essentially I have replaced u by q times A. Therefore, I have q here. And I have taken A to the in the parenthesis therefore, because I have here. So, I have a square here and of course, you know I have divided by A that comes into the picture.

$$\Delta p = \mu q \left(\frac{m_c \alpha}{A^2} + \frac{R_m}{A} \right)$$

And so, we are going to assume that area A that is the area available for filtration, does not change with radius ok. Of course, that is this is only true if you are really working with the drum that is really-really large and whatever filter cake that is formed it is very very thin ok. Only in such cases I can assume that you know the area A is constant during the course of filtration; however, you can of course, modify your working equation to also suit the need, where the area could also be changing as well.

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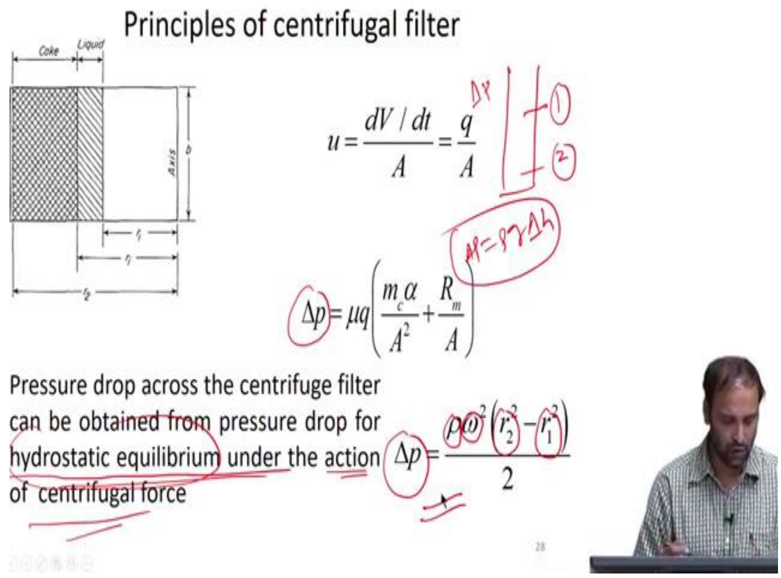

So, now if I want to solve this further I should know what is delta p ok. And this delta p which is the pressure difference that occurs during the course of centrifugal filtration, you can actually obtain by bringing this concept of hydrostatic equilibrium under the action of centrifugal force ok. What it essentially means is this right if you have a liquid column right and if I want to get what is delta p the pressure difference across like say if I have a liquid column.

So, if this is say the position 1 and position 2. We know that you know delta p is equal to is you know rho g delta h right that is the you know that you kind of derive it from the that basically is the hydrostatic equilibrium right If you have a vertical column of liquid. Similarly if you have a fluid in a centrifuge and if you are rotating at a at a at a speed omega right that is the rotation speed. And if rho is the density of the fluid and I can actually similar to relating the pressure drop to you know, the difference across two locations. If I have if I am at some radial positions r 1 and some other radial position r 2.

The pressure difference delta p essentially goes as rho times omega square into r 2 square minus r 1 square divide by 2. You can actually derive this by doing a very simple force you know force balance. Now, so, therefore, so I am going to you know instead of delta p I am going to you know use this expression that comes from the hydrostatic equilibrium.

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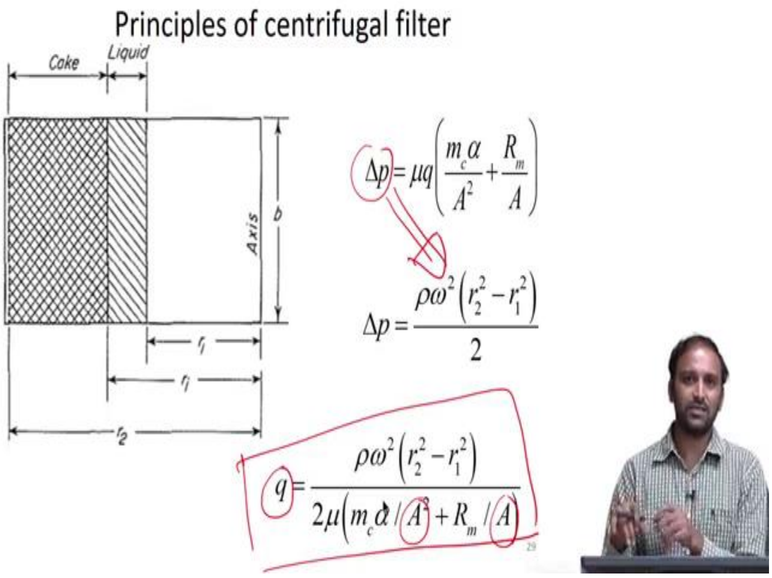

So, therefore, from this I can actually get what is q that is, what is the rate at which the filtrate is coming out you know from the during the you know from the centrifugal filtration process. Again a simple you know rearrangement leads you to this expression. And of course, I have assumed that you know A is essentially a constant right that as I said will only be true if you are working with drums which are sufficiently large in diameter or cases where the cake that is developed is very very thin right.

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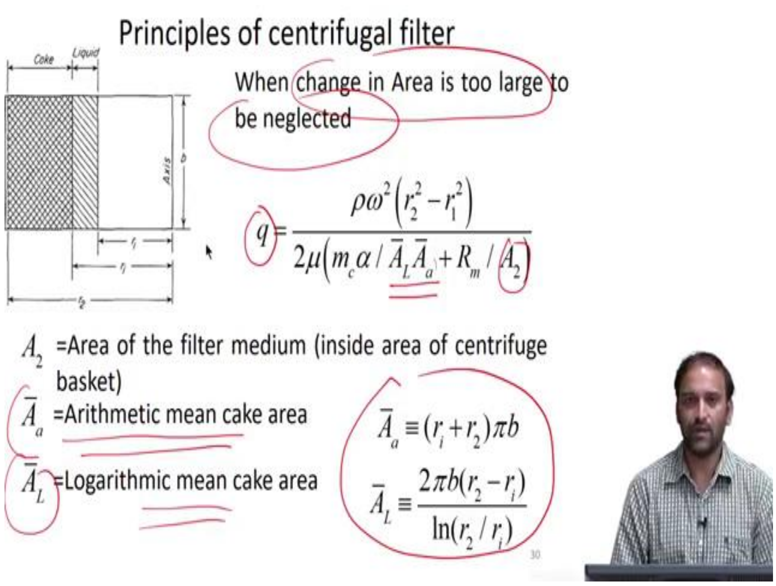

However, if you have cases where, you know the there is a change in the area is too large ok, if you cannot neglect the change in the area. All you have to do is you would have to replace you know this A square that you had here right with AL bar times A a bar and the A that you had here right with A 2 that is the inner area of the drum itself right. A 2 is the inner you know area of the drum that is available for filtration right. And A a bar is what is called as a arithmetic mean cake area and AL is what is called as a logarithmic mean cake area ok.

$$q = \frac{\rho\omega^2(r_2^2 - r_1^2)}{2\mu\left(\frac{m_c\alpha}{\overline{A_L A_a}} + \frac{R_m}{A_2}\right)}$$

$$\overline{A_a} = (r_i + r_2)\pi b$$

$$\overline{A_L} = \frac{2\pi b(r_2 - r_i)}{\ln\left(\frac{r_2}{r_i}\right)}$$

So, these areas better capture you know quantitatively what how do the q changes during the course of a filtration process ok. So, that is you know how one would deal with centrifugal filtration process. So, with that I would end the topic on you know, centrifugal filtration and you will have some assignment based on this in this particular week yeah.

Thanks.