Fluid and Particle mechanics
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Lecture – 47
Filtration

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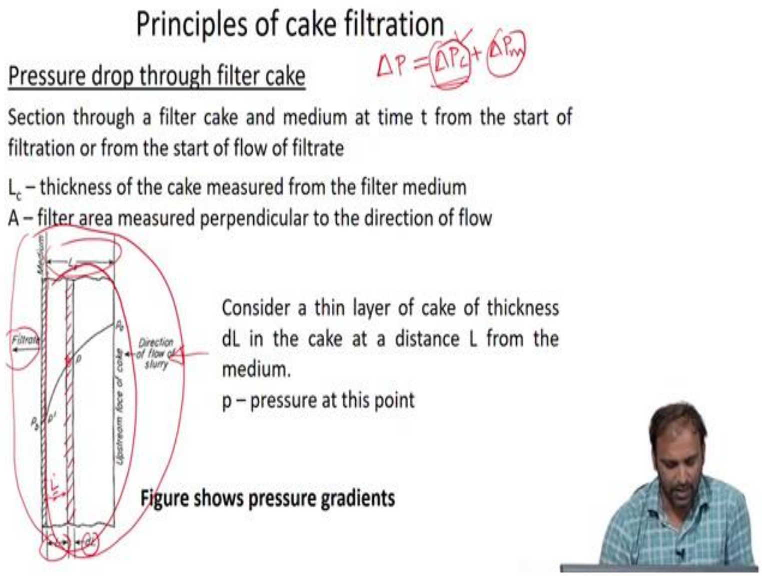

So, in the last class, we learned that if you look at the cake filtration. So, there is a the liquid that flows through two resistance there in series; that means, you know it. So, if initially you have the fluid that is flowing through a filter cake, plus it is also flowing through the filter medium. And we learnt that, the total pressure drop delta p is equal to the summation of the pressure drop across the filter cake plus the pressure drop across the filter medium right.

And, in this class what we are going to look at is, how do we think about expressing delta p c, that is a pressure drop across the filter cake and delta p m which is the pressure drop across the filter medium. In terms of some of the measurable properties such as you know the flow through the filtrate, flow rate and the viscosity of the in the filtrate the properties of the particles and things like that. So, first what we will do is we will start with thinking about the pressure drop through the filter cake that is, you know we are going to talk about how do we relate delta p c to the measurable parameters. What you are looking at is a diagram ok.

Where you visit this is basically a section through a filter cake; and the filter medium at some time t you know from the start of filtration ok. That is you know u at time t is equal to 0, you do not have; you do not have any filter cake developed; that means, you know your this part is not there right. So, only thing that you have is only the filter medium and during the course of filtration, the filter cake builds up at time t is equal to 0. What you are done is, you've taken a cross section and the figure basically represents that view ok.

And so in this diagram, so, that the direction of flow is basically marked there. So, that is a direction of flow of slurry and that is the you know if you want to call it as that is a upstream end right. And, then the cake builds up and you have a filtrate that is coming out that is the downstream. L c is the thickness of the filter cake and that is measured from the filter medium and A is the area available for filtration. So, what it essentially means is, you know if I have like say if this is my filter medium right.

And, if the filtrate is flowing in this direction, the cross sectional area that is available for you know for the filtrate to flow through that is A and what we are going to do is, we are going to look at a very thin section. So, what is drawn here is small section that is I am going to mark it now. So, that is a very small; thin section of dimension dL that is the dL is the thickness of the; you know the layer of filter cake that I am considering. And, this is basically located at a distance L for the distance from the filter medium to that layer is capital L. And, we are going to say that the pressure you know at that particular point is p ok. And, what is shown in the figure is the pressure gradients.

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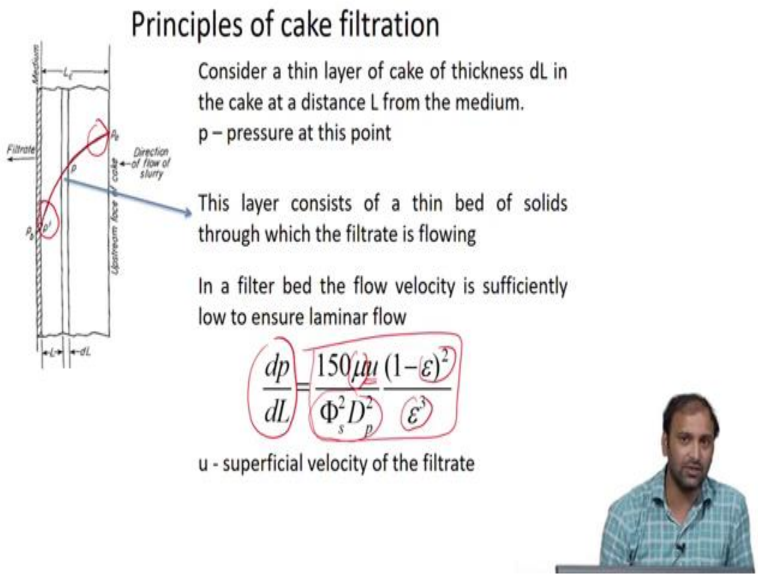

Now, if you look at this thin section ok, typically the velocity with which the filtrate flow through the filter cake are so low that you can essentially assume the flow to be laminar ok. So, therefore, you have so as I said right, we are basically borrowing the concept from flow through packed bed and we are trying to apply it to the flow through the filter cake. And, the pressure drop across the filter across the packed bed, we said you know it has two terms right one is the laminar flow term and the turbulent flow term.

So, the laminar flow term basically goes with you know, which is proportional to v or u in this case, which is the superficial velocity of the filtrate. And of course, there is going to be a u squared term that we are basically neglecting it because, we are assuming the laminar flow which is true because, the velocity with which the filtrate flows through the filter cake are sufficiently low that you can assume the laminar flow conditions.

So, the other interesting thing to notice; if you look at the pressure gradient either it is not linear right. So, it basically you know varies something like this right, that is the line that is basically drawn that is how the pressure drop varies right. Now, if you look at the conventional you know packed beds right so you would expect the pressure drop to vary linearly with the length of the packed bed right. You know if you; if you know because, you know if you are you know if you are using certain kind of packing so what happens is that you know you are the porosities are constants typically and then your mu which is the velocity, which is the viscosity of the fluid that is constant.

And, you know if you are working with rigid particles so essentially. So, all these terms are essentially constants and that basically gives rise to a linear variation of pressure with distance. However, that is need not that is need not be the case, in the case of you know flow through filter cakes because of which you essentially have you know not a linear variation, but a variation that looks more like you know quadratic ok. The reason for that is that if you look at regions of the filter cake close to the filter medium they are kind of subjected to greater compressive force compared to you know the region that is you know the closer to the upstream.

Because of which there could be a spatial variation of porosity that is one of the reason why there could be you know a non-linear variation. The other reasons could be there you know your view which is the filtrate the velocity with which the filtrate is flowing through the medium that also could be spatially varying you know depending upon you know what kind of system that you are trying to look at ok. So, therefore, this non-linear variation that you are seeing is essentially justified.

So, essentially what we are doing is, you know in this thin section, we are going to say that you know the delta p by L that is the pressure drop across this thin section. You know how does that vary with over the small distance dL and we are basically retaining only the first part of the Ergun's equation that is the Kozeny Carman equation right.

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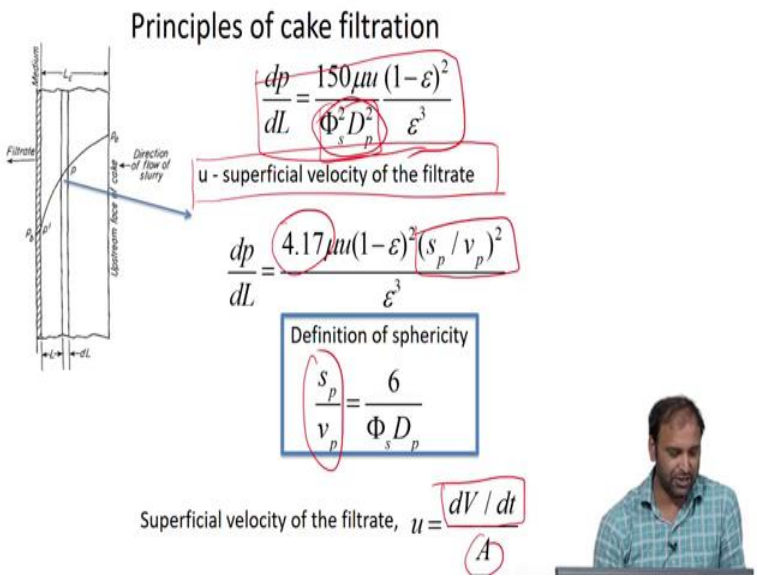

Now, what I can do is and as I said you know the u is essentially defined as the superficial velocity of the filtrate, exactly in line with you know how the v 0 bar was defined for the flow through packed bed. And what I can do is I can start with this expression, I can do some manipulations ok.

So, one of the manipulations that I can do is that instead of working with you know phi s squared D p square, where phi s is the sphericity and D p square is the; D p is the diameter of the particle that are part of the slurry which is being filtered or the diameter of the particle that constitute the cake, I can replace the sphericity in terms of s p by v p that is a specific surface you know surface area of the particle divided by the volume of the particle.

And if I do that, I can replace phi s squared D p square you know as s p by v p whole square divided by 6 square ok. And so, the 6 square 36 therefore, you know you have 150 divided by 6 square that leaves out you know 4.17 right. And the superficial velocity of the filtrate is essentially defined as, the rate at which the filtrate is collected at the downstream divided by the filtration area that is available.

$$\frac{dp}{dL} = \frac{150\mu u}{\Phi_s^2 D_p^2}\frac{(1-\epsilon)^2}{\epsilon^3}$$

$$\frac{dp}{dL} = \frac{4.17\mu u\,(1-\epsilon)^2\left(\frac{s_p}{v_p}\right)^2}{\epsilon^3}$$

$$Sphericity: \frac{s_p}{v_p} = \frac{6}{\Phi_s D_p}$$

$$u = \frac{\left(\frac{dV}{dt}\right)}{A}$$

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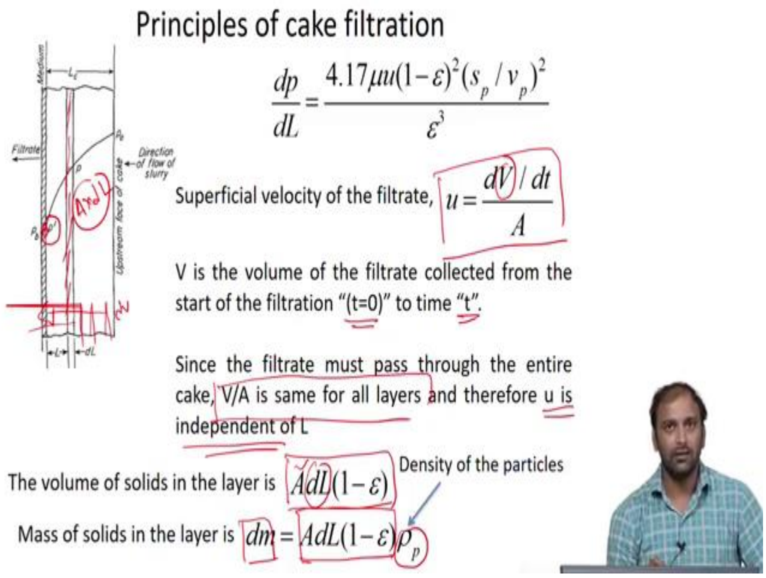

So, in this definition u is equal to dV by dt divided by divided by A. V essentially refers to the volume of the filtrate collected from time t is equal to 0 until time t is equal to you know t. And, once a particular volume of filtrate enters the filter cake, essentially the same volume you know flows through the entire filter cake.

That means you know your whatever dV that enters the same amount of volume is essentially flowing through the entire you know if I basically spread them into different strips right does to the. So, dV essentially is same across all the strips that basically constitute the filter cake. Therefore, you know your V by A is same for all layers. Therefore, I can assume that u is independent of the length across which the filter you know the filtrate is essentially flowing. And the volume of the solids in the layer, I can write this as A is the filtration area multiplied by delta L.

That gives me the volume of the solid plus the liquids you know in the filter cake right in the thin cross section right that is so, essentially A multiplied by dL gives me the volume of this thin section that multiplied by 1 minus epsilon will give me the volume of all the solids in the inner thin section. And, if I want the mass that is dm, that is the mass of the particle in the thin section; you know which is essentially this the volume of the solids in the section multiplied by rho p; where rho p is the density of the particle that constitute that make up the you know of the solids in the filter cake right.

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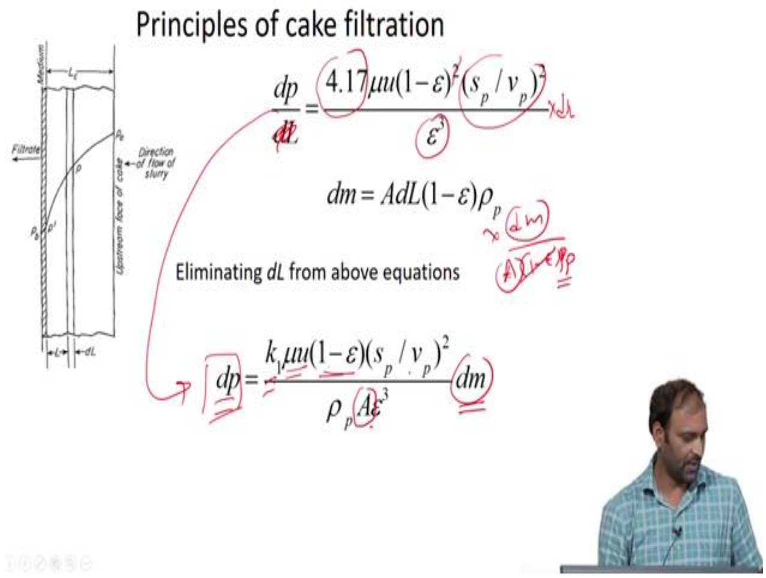

So, therefore, I can again simplify this equation further. So, I have delta p here and I am going to take dL to the right side and I can write dL ok, I had dL here so if I want to write it as you know this term multiplied by dL here right. So, therefore, I can express dL as dm divided by A times 1 minus epsilon into rho p right.

So, one of the epsilon essentially cancels with this. So, I am basically left with 1 minus epsilon here, there is k 1 there is mu u here. So, I am basically taking this as a constant k 1 right. And of course, s p by v p remains here, and I have dm that comes in and there is A you know that is here. And, essentially epsilon cube remains here and you know rho p. So, essentially I am basically expressing delta p in terms of delta m right.

$$dm = A\,dL\,(1-\epsilon)\rho_p$$

$$dp = \frac{k_1 \mu u\,(1-\epsilon)\left(\frac{s_p}{v_p}\right)^2}{\rho_p A \epsilon^3}\,dm$$

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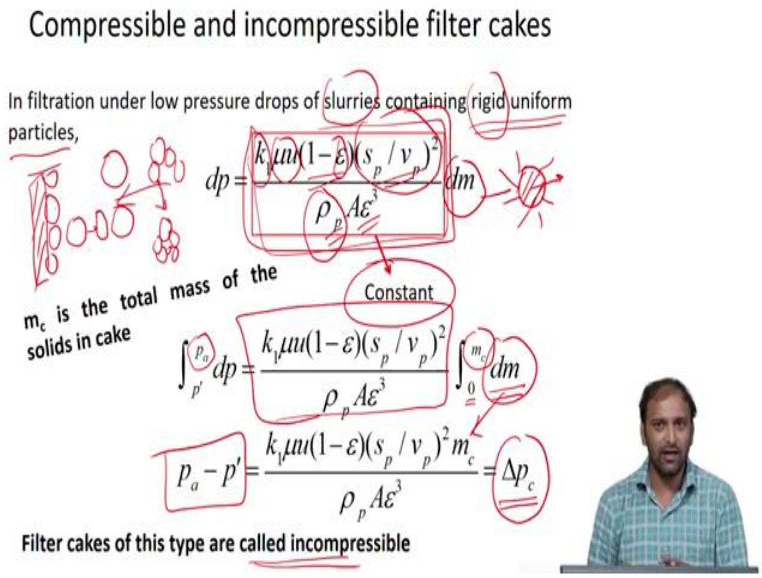

Now so, if you work with; if you are doing filtration operation and if you ensure that you know you are carrying out filtration operation under conditions where the pressure drops are very very low. And, if you are working with a slurry which contains rigid uniform particles ok; all the pre factor including k 1 mu u 1 minus epsilon s p by v p rho p you know all of these are our constants ok. And the only variable that I have is dm ok.

And you could ask a question as to, what are the conditions under which you know you could have you know this you know the each of the parameters that I am mentioning as constants could vary, you can consider several cases. One of the example could be the you know if I have like say, I am working with a filtration process in which I instead of having rigid particles that constitute the you know the slurry, if I have like say emulsion droplets which are essentially liquid droplets right.

And we know that you know as the filtration you know occurs. So, I have you know the filter medium and once these liquid droplets start depositing on to the filter medium right. When you know the further filtration happens you know, these droplets could deform right instead of them being perfectly spherical you know they could take some shape like that right. So, therefore, there could be a variation of s p by v p which we may have to worry ok. If the particles that I am working with are not rigid ok, but similarly if you have you know the slurry which contains loose flocks, you know for example, you know if the slurry consists of you know some kind of aggregates of particles.

So, what could happen is, during the filtration process there could be some compaction of these aggregates which could lead to changes in the porosity ok. And of course, you could also have cases you know, where you could. So, essentially if you are working with rigid particles you know, you could assume the rho p to be constant ok. However, if you are working with you know something like micro gel particles which you know in such cases, your rho p itself will not be a constant, you know you micro gel particles are particles where you have a rigid core you know plus some hairy brush kind of a thing there could be spatial you know variation of density.

So, therefore, there are cases where, you know you need not have the pre factor constant ok. However, if you are working with slurries which contain rigid particles which do not deform during the course of filtration, I can assume you know all the these terms could be constant. Therefore, what I can do is I can get an expression for the pressure drop across the filter cake just by integrating this equation starting from so, I my pressure at.

So, if I go back to the right. So, if you have. So, p prime is the pressure at the filter medium right. So, therefore, I can integrate this expression going from p prime to p a. Where p a is the pressure at the upstream end therefore, I can integrate from p prime to p a D p is equal to so all of this is constant if I am working with slurries containing rigid particles. And the mass of the cake that is that you know deposited to begin with the 0.

Then when the pressure drop is you know, when the pressure is reached and you know and I have a total of mc mass deposited. Therefore, I am going to have the integrations from 0 to m c. Therefore, this essentially gives rise to p a minus p prime, which essentially is your delta p c which is the pressure drop across the filter k and so therefore, this becomes mc right.

$$\int_{p'}^{p_a} dp = \frac{k_1 \mu u \,(1-\epsilon)\left(\frac{s_p}{v_p}\right)^2}{\rho_p A \epsilon^3} \int_0^{m_c} dm$$

$$p_a - p' = \frac{k_1 \mu u \,(1-\epsilon)\left(\frac{s_p}{v_p}\right)^2 m_c}{\rho_p A \epsilon^3} = \Delta p_c$$

Now, so in this case I have assumed that the filter cake is incompressible ok. And as I said that is only true, if I am working with rigid particles right.

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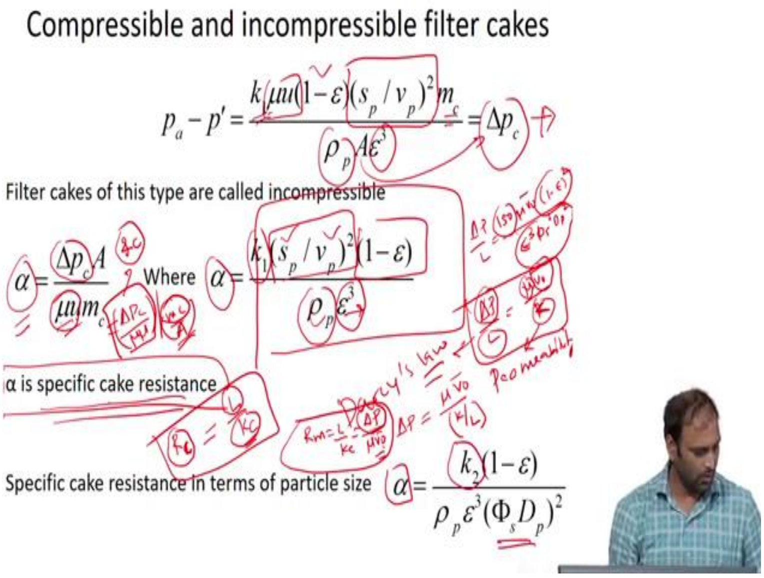

Now, so, what I can do is, I can define a particular quantity, which is what is called as specific cake resistance ok. Which essentially is I am basically manipulating this expression ok. I club lot of constant together. So, that is k 1 there ok. s p by v p whole square that is here ok, 1 minus epsilon that is here ok. And, rho p into epsilon cube all of these things. So, if you look at these parameters carefully, these are all the parameters that are specific to the kind of particles that your that are present in the slurry.

You know and essentially the these are these are properties which depend on the on the packed bed right. So, you know epsilon is a porosity of the bed right, and Sp Vp and rho p are the properties of the particle. So, all these constants to get put together are kind of clubbed together and that is your alpha. And essentially you are left with delta pc there ok. And your A from here goes to the numerator divided by you have mu u and mc.

$$\alpha = \frac{\Delta p_c A}{\mu u m_c} \; where \; \alpha = \frac{k_1 \left(\frac{s_p}{v_p}\right)^2 (1-\epsilon)}{\rho_p \epsilon^3}$$

So, the other way to think about you know this specific cake resistance is that if you look up the flow through packed beds right. So, we had developed this expression delta p by L goes as 150 mu v 0 bar into 1 minus epsilon whole squared divided by epsilon cube into phi s square D p square right. Now, I can plug a lot of these constant that are there right. So, all these constants I can plug them into you know one constant, I can write this as delta p by L is equal to mu v 0 bar divided by something called as k ok. k is what is called as a permeability, k is what is called as permeability which basically tells you in a way one of the way to think about this would be that you know how much permeable you know the packed bed is for the flow of fluid ok.

$$\frac{\Delta P}{L} = \frac{\mu V_0}{K}$$

And so, if you look at this expression, this is what is called as a Darcy's law ok which essentially states that, the flow or the superficial velocity essentially is directly proportional to the pressure drop across the bed and it is inversely proportional to viscosity right. Now, I can rework this I can say that your delta p basically is mu v 0 bar divided by K by L rights or is a ok. So now, now what I can do is in flow through packed bed. So, you know you define resistance ok, which basically goes something like this.

So, your L by L is the length of the packed bed. Divided by this permeability in this case if because, we are talking about flow through you know the cake I can write this as K c that is the permeability of the cake. That essentially is defined as some kind of resistance ok. I am going to call it as resistance because of the filter cake R c ok. So, therefore, the resistance because of the filter cake is defined as the length of the you know the packed bed or in this case the length of the cake divided by K c, where K c is the permeability of a cake right.

Now, so therefore, your R m which is L by K c basically goes as delta p divided by mu into v 0 bar ok. So, just look at the analogy of this expression with this right. So, I have delta p c there, I have mu into u ok. Therefore, I can write this as delta p c divided by mu into u whole divided by mc divided by A ok. This is the cake resistance that is R c and because, you have a flow through you know filter cake here. So, I would have to worry about what is the mass of the cake that has you know that is kind of deposited per unit area of the filter ok.

So, therefore, your alpha which is the specific cake resistance is essentially defined as R c, which is the resistance of the cake which essentially is delta p divided by mu v 0 bar in the case of flow through packed bed. So, in this case is going to be delta p c which is the, the pressure drop across the filter cake divided by mu times u where u is the superficial velocity divided by mc divided by A which is the mass of the cake developed per unit area of the filter ok. That is you know the concept of you know the specific cake resistance.

Now of course, you know I can express you know alpha also in terms of you know phi a squared D p squared that is basically going back to another starting equation only thing is instead of k 1 I am going to have some other constant k 2 ok.

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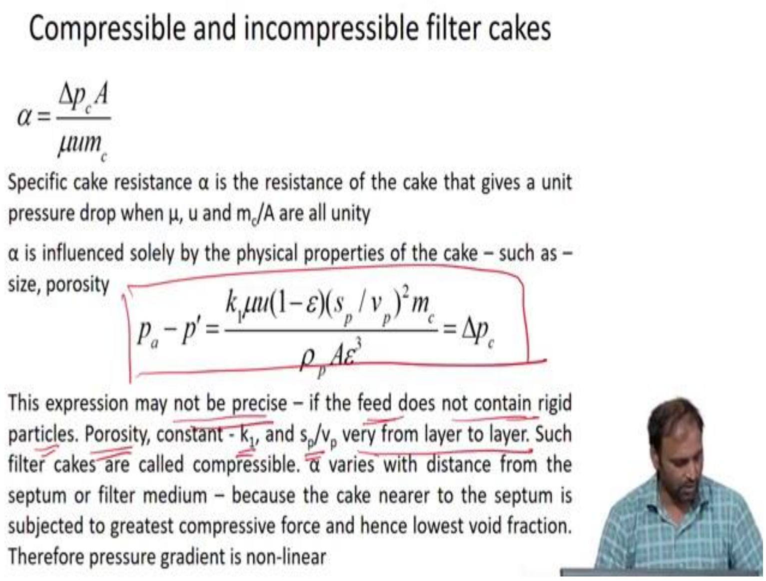

Now, So, therefore, so this is the expression that we have kind of developed right. And as I said right also so this expression that we are looking at it may not be precise ok. If the feed does not contain rigid particles ok; therefore your porosity the constant k s p by v p, they may vary from layer to layer right as I said there you know if you look at you know the region close to the filter medium ok. If there is a large cake that is developed, the region close to the filter medium, there could be much more closer packing of particles compared to the region closer to the upstream right.

Or, if you could have a case you know where you could have, if you have applied up a larger pressure drop across the filter medium you know there could be compaction of the cake which may lead to variation in the spatial variation in the porosity ok. Therefore, I may have to worry about non-linearity in such cases; however, for the case of incompressible cake I can assume that there is a linear variation of pressure drop across the length.

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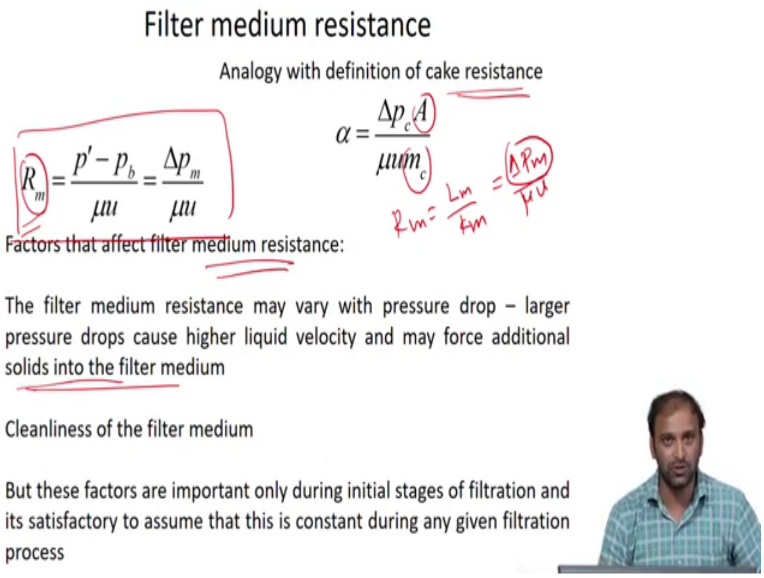

So, similarly in analogy with you know the definition of the cake resistance, we can also define the filter medium resistance ok. So, we worried about m c by A ok. That is because you know we were talking about a specific cake resistance ok. Which ok; that means, I know I would have to worry about what is the mass of the cake develop per unit area of the fit you know filter, but; however, in the case of filter medium resistance I do not have that term does because this is this is defined as the filter medium resistance ok.

Which is R m essentially goes as the length of the filter medium that is you know if you if you have a you know if L m is the length of the filter medium, divided by what is the permeability of the medium that essentially goes as delta p m which is the pressure drop across the filter medium divided by mu and u alright.

$$R_m = \frac{p' - p_b}{\mu u} = \frac{\Delta p_m}{\mu u}$$

So, but of course, you know the delta p m if you look up you know the delta p m may also varying right because, you know if you could have a case; if you have if you have if you have applied a larger pressure drop across the filter during the filtration process.

What could happen is? The larger you know the pressure drop can may force some of the particles through the filter medium some of the solids to the filter medium. And depending upon the cleanliness ok, if you have like some of the pores that are going to be blocked because of the you know presence of solids ok. So, these factors may change delta p m, but you know if you maintain the cleanliness you know if you are working with a reasonably smaller pressure drops, you can assume delta p m to be constant during the entire course of filtration process. However, delta p c would varied in the course of filtration ok

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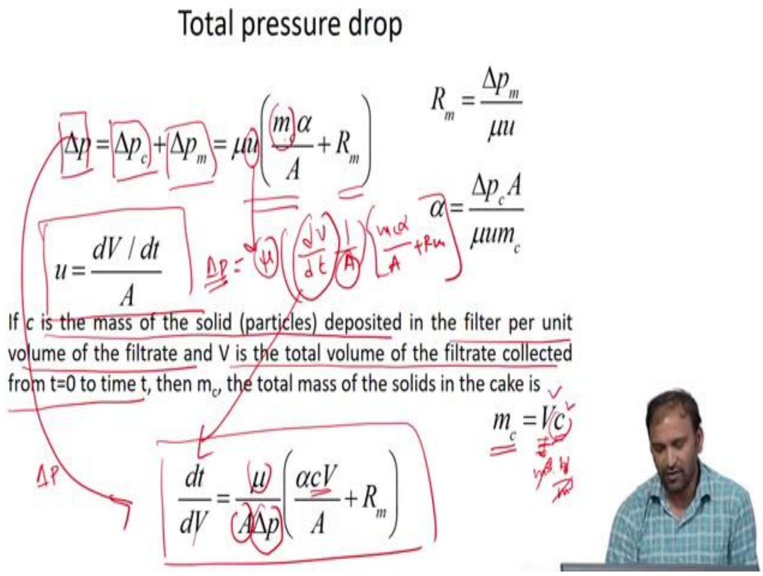

So, therefore, I can write the total pressure drop that is delta p is delta pc plus delta p m ok. As the contribution from the pressure drop because of the cake and the contribution that basically comes because of the presence of the filter medium. Now, we had defined u which is the superficial velocity as dV by dt divided by A. You can relate mc which is the mass of the filter; mass of the cake that is deposited that is mc. in terms of V which is the total volume of the filtrate collected from time t is equal to 0 to time t ok. And c which essentially is the mass of the solid particles deposited in the filter per unit volume of the filtering.

If I know these two quantities, mc essentially is the product of V times c. So, this will have units of like say meter cube ok. And c will have you know itself something like kg per meter cube therefore, mc essentially will have units of mass that is the mass of the solid particular and basically deposit onto the filter cake per unit volume of them right. So, now, so therefore, instead of mc I can replace mc with c times V right. And so, now, what I have done is here and of course, I have u here right. So, u I am going to write it as dV by dt multiplied by 1over A right. And because I have delta p right delta p is equal to. So, I have delta p is equal to dV by dt multiplied by mu into mc alpha A plus R m right I have this.

Therefore, I can get this you know dV by dt into the other side that becomes you know dt by dV right. I am going to get delta p to the to the right hand side that will have delta p in the denominator. And mu remains you know wherever it is and A also remains in the denominator. So, essentially I am recasting this equation you know as dt by dV is equal to mu divided by A delta p times alpha c V divided by A plus R m right.

$$\Delta p = \Delta p_c + \Delta p_m = \mu u \left(\frac{m_c \alpha}{A} + R_m\right); \;\; R_m = \frac{\Delta p_m}{\mu u}\; ; \alpha = \frac{\Delta p_c A}{\mu u m_c}$$

$$u = \frac{\frac{dV}{dt}}{A}\; ; m_c = Vc$$

$$\frac{dt}{dV} = \frac{\mu}{A\Delta p}\left(\frac{\alpha c V}{A} + R_m\right)$$

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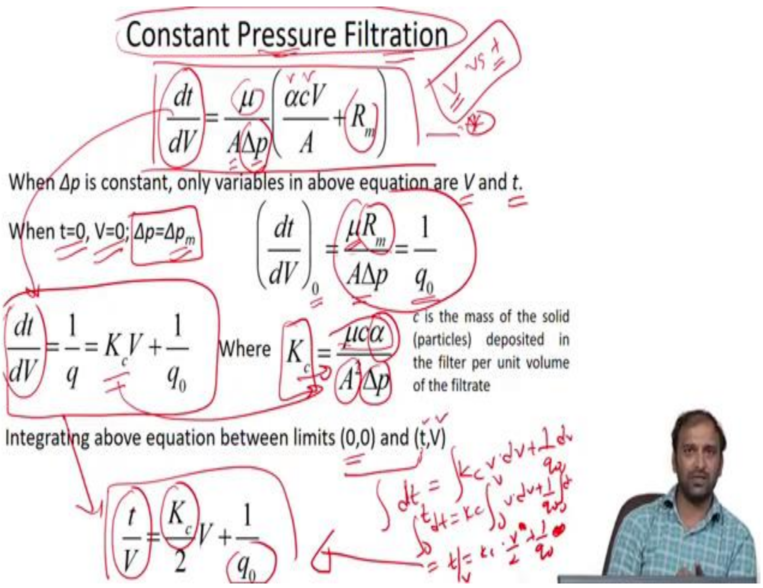

Now, so now, now I think I have an equation, which is in a particular form which I can further reduce to suit the need of you know so we were talking about the constant pressure filtration and the constant you know rate filtration right. So, in the case of constant pressure filtration, what is done is you basically maintain a constant pressure drop across you know the filter filtration process. And then you basically look at how does the volume of the filtered that I collect you know varies as a function of time ok. Therefore, I am basically interested in to develop an expression in which t and V are my variables.

So, therefore, I would. So, what I have is this expression ok. So, what I know is you know because, I am working at a for a constant pressure filtration. My delta p is going to be a constant. Therefore, in this expressions only variables are V and t, but I know the initial condition at time t is equal to 0. I do not have any filtrate coming down; that means, you know your V is equal to 0. And the only pressure drop that I have is the pressure drop that comes because of the presence of the medium. Therefore, my delta p is essentially is equal to delta p m therefore, I can. So, therefore, my delta t by delta V at time t is equal to 0 is equal to mu multiplied by R m divided by A times delta p ok. Plus the other term is going to be constant is equal to 0 because V essentially is 0.

Therefore and I call this as 1 over q 0 ok. So, therefore, I can recast this equation as dt by dV is equal to one over q. That is equal to K c times V. So, I have defined a constant K c which basically is your mu multiplied by c alpha that is the numerator and divided by A square into delta p and because, delta p is a is constant that basically is kind of included in the in this constant K c.

Therefore, dt by dV is equal to 1 over q is equal to K c times p plus 1 over q 0 ok. I can integrate this equation further and then if I put the limits that at time t is equal to at time t is equal to 0, but you know. So, I have the coordinates t 0 0 and t V ok; that means, you know at time t is equal to 0 I do not have any filtrate collected and you know at time t is equal to 0 the total volume with the filter is essentially V ok. Therefore, this expression essentially becomes t over V is equal to K c by 2 times V plus 1 over q 0 right. So, what essentially done is your dt goes as K c into V into dV plus 1 over q 0 into d V right.

So, therefore, if I integrate this what I get is your 0 to t dt is equal to K c into 0 to V V into dV plus 1 over q 0 into 0 to V dV right. So, that is becomes your t is equal to K c into V square by 2 plus 1 over q 0 into V ok. Therefore, I can divide everything by, this becomes t over V and 1 we essentially goes and this also goes. So, therefore, you basically end up with an expression which is t by V is equal to K c by 2 into V plus 1 over q 0 ok. So, therefore, we are going to developed working equation, which tells us you know how the dependence of you know p and V during the process of constant pressure filtration.

$$\left(\frac{dt}{dV}\right)_0 = \frac{\mu R_m}{A\Delta P} = \frac{1}{q_0}$$

$$\frac{dt}{dV} = \frac{1}{q} = K_c V + \frac{1}{q_0}\ ; where\ K_c = \frac{\mu c \alpha}{A^2 \Delta p}$$

$$\frac{t}{V} = \frac{K_c}{2} V + \frac{1}{q_0}$$

One of the nice things about this expression is you know, I can actually if I perform experiments under constant pressure; conditions I can actually estimate what is K c and what is q 0. And you can if you go back and look at what is you know K c and q0 depend on, you know the K c depends on what is alpha which is the specific cake resistance. And 1 over q 0 basically depends on you know is related to R m which is the filter medium resistance. Therefore, what I can do is, I have a framework to essentially calculate what is the filter medium resistance under specific cake resistance?

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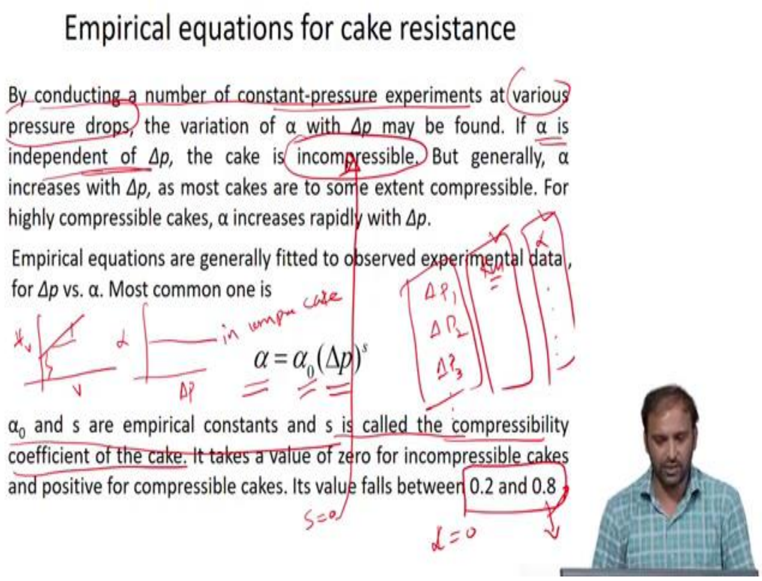

And you can also exploit the constant pressure filtration, this is to say something about what is the type of the cake; that is kind of formed during the course of filtration. So, what I can do is, I can conduct a number of constant pressure experiments; that means, you know I basically maintain various pressure drops that is you know I do experiments at delta p 1, delta p 2, delta p 3 like that ok.

And then I calculate what is R m and what is alpha that is essentially basically doing I do a constant pressure filtration I do t versus V versus where t by V versus you know V right. And, then I get a straight line I look at what is the slope and the intercept and from the slope and the intercepts so I can actually calculate these things. Therefore, for a set of delta p conditions, I can calculate what is alpha and R m. And, if alpha is independent of pressure drop; that means, no matter what I am what is the pressure drop there is maintained.

If alpha remains constant then I can say then you know the cake that I have there is form is incompressible. However, that need not be the case; that means, you know if I were to plot. You know alpha for delta p if it remains constant; that means, it is a incompressible cake ok. However, that need not be the case and one of the empirical equation that is being developed to kind of capture, how the how does the alpha which is the filter which is the specific cake resistance base is a function of pressure drop is alpha goes us alpha 0 into delta p to power s, which is an empirical equation which is obtained by doing a lot of experiments.

$$\alpha = \alpha_0(\Delta p)^s$$

And, this alpha and s are empirical constant. And s is something called as a compressibility coefficient ok, it tells you how compressible or how incompressible the cakes are if s is equal to 0; that means, it is a incompressible cake. However, typically people have found that you know the alpha takes a value from 0.2 to 0.8 depending upon what pressure drop you have maintained. And more closer this value is to 1; that means, it is highly compressible, more closer it is to 0 that mean it is you know less compressible.

And, for you know alpha is equal to 0 I can say that the cake is completely in incompressible. Therefore, this particular framework gives you a way of calculating both the filter cake resistance and the filter medium resistance, which in turn can be used you know as a method for identifying the nature that filter cake that is essentially formed during the course of filtration ok. maybe I will stop here. So, I will continue with talking about the constant rate filtration and the continuous filtration in the in the next video.

Thanks.