Fluid Mechanics

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Lecture - 44
Pressure drop through packed bed continue

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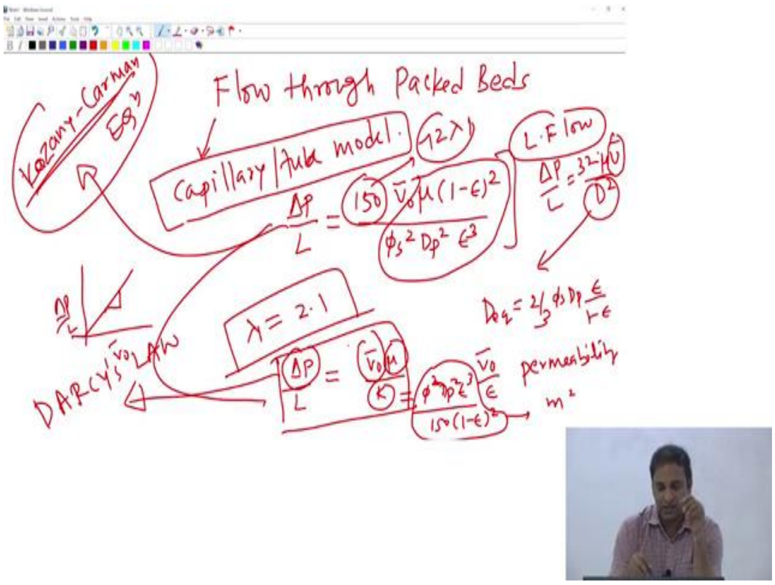

So what were you looking at in the last class is, we looked at a concept called capillary model or tube model right which is a methodology, which has been developed to basically correlate delta P by L which is the pressure drop per unit length of the packed bed to properties of the packed bed ok. In terms of V 0 bar which is the superficial velocity right of the liquid that is flowing through the packed bed mu which is the viscosity, 1 minus epsilon whole square divided by phi s square into D p squared times epsilon cube right.

That is what we had developed was that?

Yeah. So, what we said is that you know when we the way this was done as we and ah. So, we are basically looking at the case where you are looking at laminar flow conditions right laminar flow conditions and the starting equation was delta P by L is equal to 32 mu V bar divided by D square right and then what we did is, we use this capillary model and then we did the surface area balance and the volume balance and we said your you would have to replace your D by D equivalent which was something like 2 by 3 into phi s into D p into epsilon divided by 1 minus epsilon right.

Yeah and then this V bar which is the average velocity with which the liquid has flowing through the conduit right that we would have to replace by V 0 bar divided by epsilon right that is the modification that we did. And if when we work it out this the constant here it came out to be something like 72 times lambda right lambda 1 and we said that you know people have done a lot of empirical lot of you know experiments and they found out that you know this you know if you do an experiment of delta P by L versus you know if you have a data of delta P by well L versus V 0 bar right and if you are you know in the laminar flow condition, that is if you ensure that your Reynolds numbers are less than 1 you would get a straight line that is passing through the origin and if you know the other factors right this is basically your slope is going to be you know this time some constant right and people have found that you know that is of the order of 150 which basically gives you your lambda 1 as 2.1.

So, I was trying to discuss that yesterday said that this 2.1 accounts for the fact that the liquid is not flowing through straight pipe, but the pipes are tortuous ok. In essence how high this number is you know if it is much larger than you know 2.1 then will be much more tortuous ok, if it is closer to one you know you can say there it is like not too tortuous that is the you know you can think of. Now what you can do is I can actually recast this equation into something like this delta P by L is equal to I have this V 0 bar and mu at the top and at the bottom you can write it as kappa sorry K and K essentially from if you look at look this up K would be something like phi s square into D p square into epsilon cube divided by 150 into 1 minus epsilon whole square right.

So, I am basically I have taken this expression I just forgot to mention that this equation is what is called as a Kozany Carman equation that is because these people developed this equation. See these are the developments that happened in you know 1930s and 40s. The fact that you know these models are still used in predicting you know pressure drop through packed bed it is a testimony to the fact that these are very well thought out models and typically when people do this modeling in chemical engineering; that means, you take a particular process and then you kind of if you want to put across a very simple methodology to evaluate like say in this case the pressure drop per unit length the concept of you know the fact that you know this packed bed consists of you know tubes, you know and the concept of equivalent diameter ok.

So, this not easy to come up with right and the fact that they were able to do it you know in the 1930s and 40s and people are using these correlations even to date in the industry that is because the fact is a very well thought out model and any model that kind of you know stands for over 10 years is considered to be really good model, but the fact this is been there for more than you know 50- 60 years; that means, you know this is one of the really nicely developed a methodology for basically getting this delta P by L ok. And this expression where delta P by L is equal to V 0 bar into mu divided by K and this K is what is cause of permeability ok. And if you look at this expression this K bar has units of meter square right because you know your phi s is a is non dimensional D p square has units of meter square and other things are all you know non dimensional right.

$$\frac{\Delta P}{L} = \frac{\overline{V_0}\mu}{K}$$

So, so, and this equation is also called as Darcy’s law which essentially states that you know you are the velocity with which a liquid is flowing through a porous media is basically proportional to delta P which is the pressure drop and it is inversely proportional to mu which is the viscosity of the fluid ok. And if you go back and look up people in other disciplines especially if you look at civil engineering you will always come across cases where you know people are looking at porous rocks, you know people are looking at flow through you know you know sand for example, ok.

So, everywhere you know people kind of use these the flow through packed bed concept really you know extract some meaningful parameters. We also want to look at cases where we are going to say that these expression that we have developed this Kozany Carman equation you can also use it for basic characterization of the particles itself which we are going to look at a little later ok. That is for the laminar flow conditions if you want to look at the you know the turbulent flow conditions.

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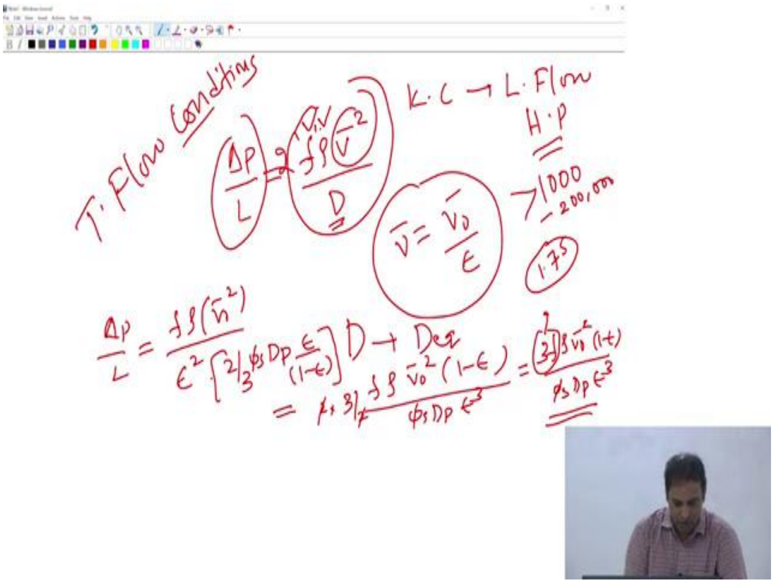

So, this is the Kozany Carman equation that we would have developed as I said its only for laminar flow conditions. So, if you want to develop a similar kind of you know expression for delta P by L for the turbulent flow conditions again we like we started with you know Hagen Poiseuille equation you will again have to start with a similar expression, but now for the turbulent flow conditions right.

So, it turns out you know delta P by L for the flow of liquid through conduit in the turbulent flow condition it goes something like f that is a friction factor times rho that is the density of the fluid and V 0 bar or say V 0 just put as V 0 at this point V 0 square divided by just want to where is that anyone remembers that is D right ok. That is your expression for a pressure drop per unit length through the pipe turbulent flow conditions that is a friction factor density of the fluid, average velocity with which the fluid is flowing through the packed bed and D is your diameter of the conduit right.

$$\frac{\Delta P}{L} = \frac{f\rho\bar{v}^2}{D}$$

So, again we like into exactly the same exercise you replace your V bar by V 0 bar divided by epsilon that is the velocity with which the liquid is flowing through the packed bed right that is the actual velocity right that is the average velocity which are the critical onto the pipe and then D we are going to replace by D equivalent right can you just do that and let me know what you get for delta P by L for the turbulent flow conditions. It is for turbulent flow conditions right. So, it is going to be delta P by L you are going to have f there rho and your V 0 bar is V 0 bar square divided by epsilon square at the bottom and your t equivalent is going to be 2 by 3 into phi s into D p into epsilon divided by 1 minus epsilon right.

Therefore that becomes f into rho into V 0 bar square and this 1 minus epsilon goes up divided by you have phi s D p into epsilon cube at the bottom and that is going to be 3 by 2 right is that. That basically goes something as I have missing 2 somewhere oh there is a 2 here this its actually 2 times f rho V bar square by D ok. So, there is a 2 here. So, this this gets cancels. So, you essentially get 3 times f rho V 0 bar square 1 minus epsilon divided by phi s D p into epsilon cube.

$$\frac{\Delta P}{L} = \frac{3 f \rho \overline{V_0}^2 (1-\epsilon)}{\phi_s D_P \epsilon^3}$$

So, that is and again you know we do not know what this f is right again f depends on your Reynolds number and stuff like that ok. Again people have done a lot of experiments and they have found that you know this 3 times f factor is of the order of 1.75 again this is based on several experiments that people have done on packed beds of you know different materials they fill ah cylinders in the packed bed, they fill bull saddles you know spherical balls and then you make sure that you know your Reynolds number conditions such that your Reynolds number is greater than 1000 right.

So, we said for the flow of fluid flow of fluid around particle, we said you know anything greater than 1000 up to about 200,000 was the last number range for which the conditions are turbulent right, but therefore, ah. So, now, this 1.75 if it turns out that if you go back and look it up.

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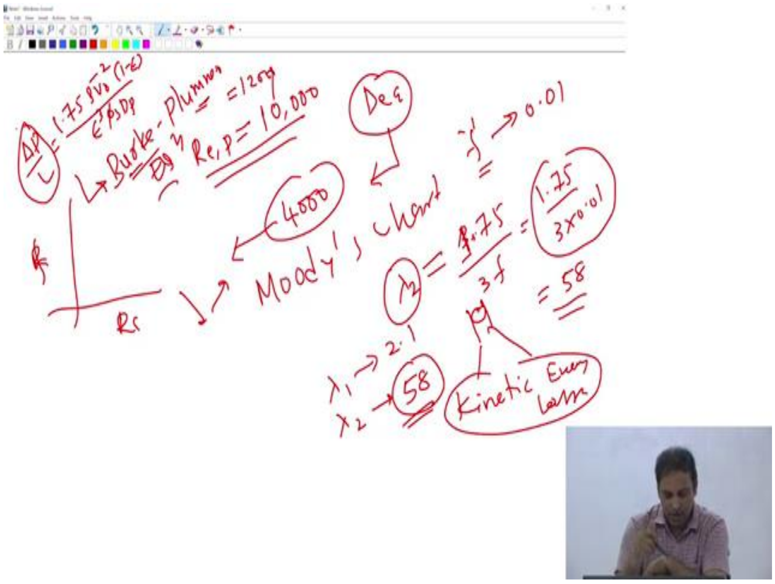

So, say that you know I have a Reynolds number condition of the order of like say 10000 ok. Now what you do is now I know what is your D equivalent you use that and you back calculate what is the Reynolds number that corresponds to flow of the fluid through the pipe ok.

Now, it turns out you know the Reynolds number would be of the order of 4000 right. Now from this you can go back to what is called as a Moody's chart right m o o d y Moody's chart right you can go back and look up you know corresponding to a Reynolds number of 4000 what is your friction factor and that friction factor comes up to be something about 0.01 all of you know Moody's chart right its basically your Reynolds number your friction factor versus Reynolds number plot right. I think you are going to have an assignment where you are supposed to write something about this if I am not wrong ok.

So, so, the point is now if f I can go back to f and I can say that your lambda 2 which is the tortuosity factor now it is actually 3 times 1.75 that is a the factor that I have got from experiments divided by 3 times f. So, therefore, if you do it with 3 times 0.01 this comes to be 58 much to0 larger right. We character this lambda 1 to be 2.1 for the laminar flow conditions, lambda 2 which is the tortuosity factor for the turbulent flow conditions it is about 58.

So, therefore, so, the fact now this lambda two is much much larger; that means, to say that you know it is not that the pipes are more tortuous it is just that the pipes are as tortuous as the laminar flow conditions; however, because of the fact that there is a turbulent flow condition what is going to happen is there is going to be a lot of changes in the area you know; that means, you know in some channels the fluid is going to be you know flowing in a channel of particular area and there could be expansion of the liquid at some locations plus the fact that you know liquid is going to turn the direction in which it is flowing all that leads to much more larger kinetic energy losses that leads to a larger you know you can think about that in terms of the larger value of tortuosity yeah go ahead.

I just said that you know. So, basically ok. So, when we looked up here right. So, this is when I when we did this right 3 times f rho V 0 bar square 1 minus epsilon divided by phi s D p and epsilon cube that is for a straight pipe right, but of course, you know I would have to have a tortuosity factor lambda 2. So, what I mean these are all based on some of the experiment that people have done look after this these you know development in terms of you know correlating your delta P by L two different parameters were developed now you know what people did is, they did a series of experiments under different Reynolds number conditions and then they matched this theory with the experiment and then that is how this 1.75 and you know 150 numbers have been obtained.

Yeah. So, that is what 3 f times lambda 2 is 1.75 and therefore, that is why you know in the next this thing what we did is I want to estimate what is this lambda 2 value therefore, you know it is 1.75. So, if you assume a particular you know Reynolds number you know it will be different right depends on you know what is the Reynolds number condition that you have in the system; that means, you know.

So, therefore, if I were to take a you know Reynolds number of the order of like say 1200 ok. Of course, you know your lambda 2 that you all have got to have been lower right that is because you know again the Reynolds numbers conditions are such that you know maybe kinetic energy losses or in that cases are much lesser compared to the case of ten to the power four Reynolds number so on.

So that is the this expression is what is called as a I will go back to the next expression. So, this delta P by L which is 1.75 into rho V 0 bar square into 1 minus epsilon divided by epsilon cube into phi s into D p right that is the expression that we developed right in the previous; in the previous slide right that is here rho V bar square 1 minus epsilon phi s D p epsilon cube and multiplied by 1.75, this is what is called as a burke b u r k e plummer p l u m m e r equation again this is the two folks which came up with this expression you know again in the 1940s . Now so, we have kind of developed expressions for delta P by L for laminar flow conditions and turbulent flow conditions and someone called ERGUN e r g e n or g u n Ergun's equation.

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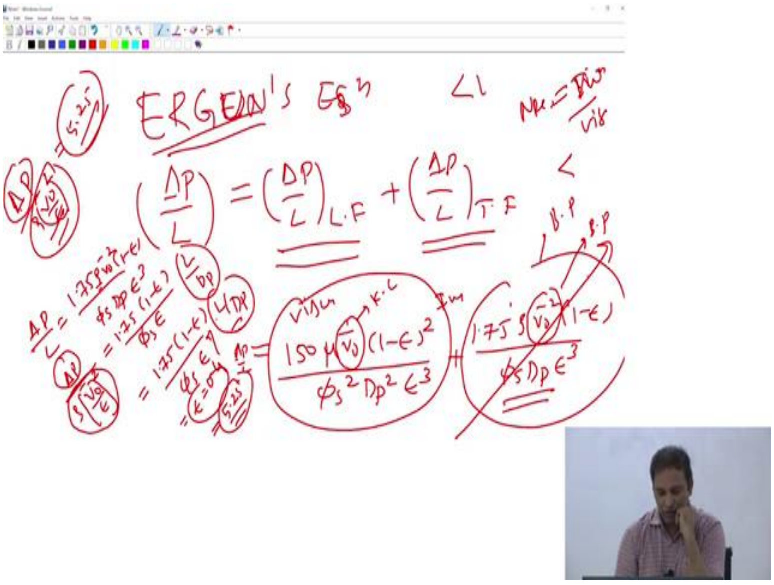

So, what he did? What he did is he said the delta P by L for the entire you know range it basically is delta P by L for the laminar flow conditions plus delta P by L for the turbulent flow conditions ok. Again the concept is this right you know if you look at a Reynolds number right this is the you know inertial force divided by viscous force right that is your Reynolds number.

Now, the pressure drop that you have when the Reynolds number is lower it is kind of dictated more by the viscous flow viscous term and in the case where in a where you have you know high Reynolds number, it is kind of dictated by the inertia term right you know. So, therefore, all he did was he clubbed this Kozany Carman equation and Burke Plummer equation and he said that you know if you really are interested in the pressure drop per unit length for the entire you know regime of flow all you have to do is sum up these two equations ok.

So, therefore, Burke Plummer's equation go something like it. So, 150 into mu V 0 bar 1 minus epsilon whole square divided by phi s square into D p square into epsilon cube plus 1.75 rho V 0 bar square right 1 minus epsilon divided by phi s D p into epsilon cube that is the overall the you know that is the Ergun equation which basically captures the pressure drop per unit length for the entire flow regime going from all the way from laminar to turbulent.

$$\frac{\Delta P}{L} = \left(\frac{\Delta P}{L}\right)_{Laminar} + \left(\frac{\Delta P}{L}\right)_{Turbulent}$$

$$\frac{\Delta P}{L} = \frac{150\mu\overline{V_0}(1-\epsilon)^2}{\phi_s^2 D_P^2 \epsilon^3} + \frac{1.75\rho\overline{V_0}^2(1-\epsilon)}{\phi_s D_P \epsilon^3}$$

So, the point that I want to say is that you know all this basically assumes you know if you look at laminar flow conditions, this is the term which basically dominates the pressure drop, if you go to the turbulent flow condition this is a term that dominates the pressure loss. However, if you have any you know Reynolds number in between, you know it there is a contribution of both is the both of these terms to the pressure drop. Ah Now if I look at this expression right yeah go ahead.

So, I was I mean I just took a some case that I was actually given in the book ah, but the idea is this right you assume a. So, if I want to get what is the f right which is the friction factor, if I want to use a Moody's chart I should know what is the Reynolds number right for the flow through pipe right. Now what you do is I want to get this Reynolds number. So, what you do is, if I say that you know the flow is occurring in turbulent flow conditions in the packed bed I find out what is the Reynolds number for that condition ok.

From that I go to this you know the pipe concept and I back calculate what is the Reynolds number is, you know if the if the fluid at similar Reynolds number would to would flow through the pipe and that turns out to be know in this case about 4000 and I went to the Moody chart Moody's chart and for the Reynolds number you know again you know it depends on your smoothness and stuff like that ok. Typical value of f for that case you know is about 0.1 that is how it was right.

Now, if you look at this Burke Plummer equation right what you can do is I can actually rearrange this a little bit ok. So, if I say delta P by L right I have delta P by L is equal to you know 1.75 rho V 0 bar square into 1 minus epsilon divided by phi s into D p into epsilon cube what I can do is I can rearrange this a little bit ok.

What I will do is I will have delta P here I am going to get this V 0 bar you know to the other side and let us say V 0 bar square right and then what I am going to do is I have rho here I am going to get rho was to this side and that is equal to 1.75 into 1 minus epsilon divided by phi s, I am going to take l to the other side I am going to write as L by D p and so, I have what I can do it let me just do this way I am I have taken also epsilon square to the other side I am going to take V 0 bar square divided by epsilon to this side right.

So, therefore, I am going to end up with something like this 1.75 into 1 minus epsilon into phi s into epsilon into L by D p ok. Now this is the pressure losses and this is the can you identify this head losses right the correction head losses right. So, therefore, this basically now that is divide by 2 right.

So, now, what you do is if I substitute for typical values of you know ah. So, what can you say about L by D p? L is the length of the packed bed and D p is the diameter right and I can say it what is that? It can it can tell you something about the number of layers of particle that you have right if you had if I have a total bed of height L and D p is your diameter, it will give you typically it will give you the number of layers of particle that you have right.

Now, if I. So, now, if I take some value of epsilon for example, if I take epsilon to be something like say 0.4 and if I substitute in this. So, what I get is a value which is something like have it or not 5.25 this is for L by D p is equal to 1; that means, this 5.25 is the. So, basically what you are doing is. So, in basically turns out to be delta P by rho V 0 bar square by 2 right is of the order of 5.25 ok that is for every layer of for every layer because I am taking L by D p as 1 it will basically telling you what for every layer of the particle that I have in the you know packed bed, what is the pressure losses compared to the head losses right. And the fact that you know this pressure drop is 5.2 times you know this head losses; that means, the kinetic energy losses in the case of you know the turbulent flow conditions are much much larger.

So, if you were to do a similar kind of analysis for flow through pipes, you will you know you will see that you know this delta P this ratio will be much less than 1; 1 or less than 1 the fact that it is a much larger number that basically tells us that the kinetic energy losses in the packed bed because of the fact that there is a change in the cross section area of the conduits through which the liquids are flowing plus the fact that you know there is a change in the direction of the fluid that leads to much higher kinetic energy losses in the case of flow through packed beds. So, that is that is all what I wanted to say.

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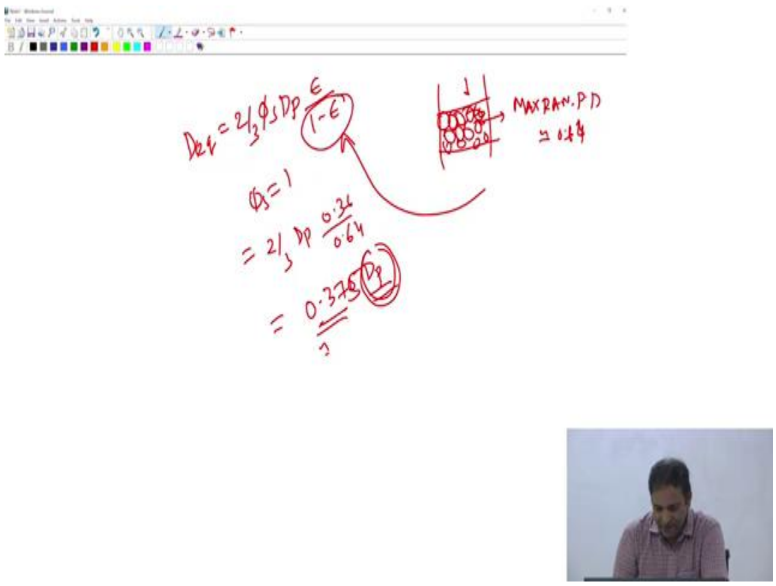

Now, it turns out that any questions so, far. So, we you can also think about doing a now that we know what is this D equivalent right which is you know 2 by 3 into phi s into D p into epsilon divided by 1 minus epsilon right. Now if you take this expression let us say that you know I have a packed bed and see there is no I have filled it up with spherical particles and say that there is a random you know packing and we know that you know the maximum no random packing density is of the order of 0.64 right.

Now, if I substitute that value back here right and if I say they know I am working with spherical particle your phi s is 1. So, you are 2 by 3 into you know D p times 0.36 divided by 0.64 right that is your 1 minus you know that is a packing density. So, it turns out there is the number that you get is of the order of 0.375 D p. This is a good kind of exercise to do what it basically tells you is there you know that the diameter of the channels through which the liquid is flowing these are much smaller than the dimensions of the particle itself ok. In this case is of the order of 0.34 0.38 or something like that right that is the; that means, the diameter of the channels through which the liquid is flowing through in the packed bed it is you know typically it is always less than the diameter of the particle that make up the column.

So, that is the it does make sense right because it you know of course, it has to be less than because you know the liquid is basically flowing through a gap that is available between the particles right therefore, it is expected that you know if this has to be less than the diameter of the particle. So, maybe I am just going to end whatever we were discussed on the packed bed looking at some applications.

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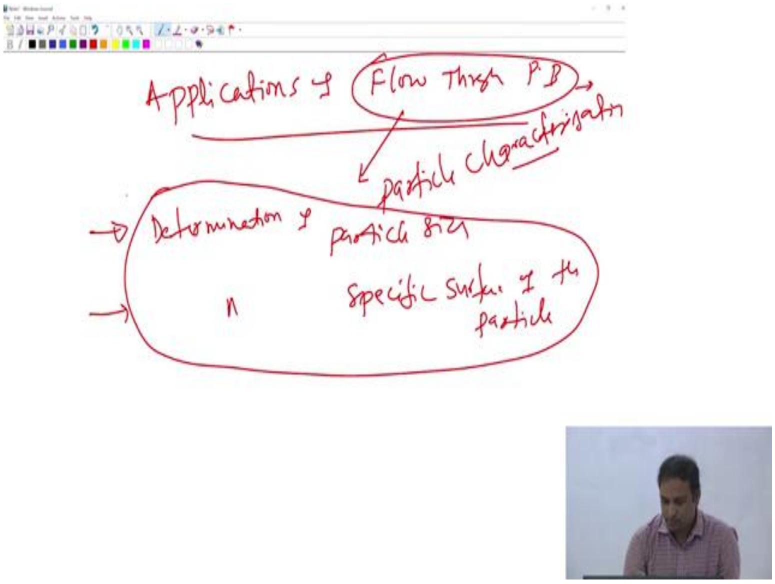

Of course I mean in terms of the applications, we have said you know they are kind of used in you know all kinds of chemical you know unit operations plus they are used in chemical industries, they are used in you know as adsorption columns, you know they are used in distillation, you know they used in you know reaction engineering in terms of catalytic bed reactors and things like that right.

But flow through packed bed concept also has been exploited for actually what is called as a particle characterization itself. So, couple of applications I am going to talk about one is a determination of particle size other one is determination of what is called as a specific surface area of the particle right. Again you have a assignment coming up right the next assignment is going to be on particle characterization and stuff like that. So, you will have to write up something about you know different methods of particle characterization. So, this could be one of the method.

So, basically what you. So, in doing these particle characterization, what is actually exploited is that if you look at you know this Ergun equation right if you look at Ergun equation. So, this is the Korzony Carman equation and that is the Burke Plummer.

And if you look at this expression your delta P by L right it goes linearly you know with the superficial velocity and it goes as the square of the velocity you know in the turbulent case, but if you work with you know if you work with conditions where Reynolds number is less than 1, I can actually neglect this right therefore, if you were to plot therefore, if you were to plot delta P by L if you were to do an experiment where I measure the pressure drop per unit length across the packed bed as a function of the superficial velocity, what I should get is I should get a straight line passing through the origin and the slope of this line is going to be 150 times mu eps 1 minus epsilon square divided by phi s square D p square into epsilon cube.

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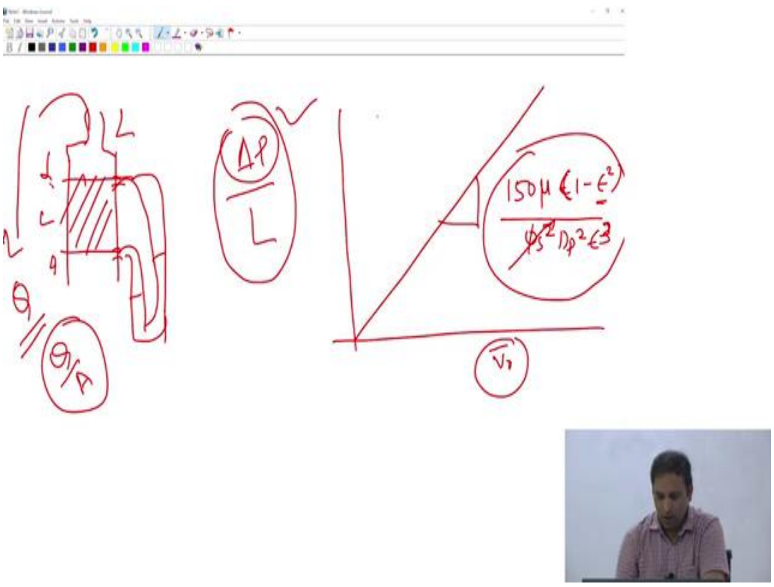

So, if you work with you know conditions where your Reynolds number is less than 1. So, this is what should happen. Now if I have you know a way of you know measuring delta P by L in my experiment and of course, I can also if I have a way of measuring what is the flow rate typically what is done is if you have a packed bed filled with packing material what you do is, you basically have two pressure tappings one at the top one at the bottom you connect it to a manometer that basically gives you what is the pressure drop right you can get this. I know what is the length of the packed bed that is your L.

I have a way of getting this and once the liquid actually comes out of the packed bed I can basically collect it right I can measure Q which is the volumetric flow rate and volumetric flow rate divided by the cross sectional area will give you what is the superficial velocity.

So, you have a way of measuring this and this and then once you estimate the slope if you have some way of measuring your epsilon we kind of discussed up a couple of methods right and of course, if you are working with the spherical particles your phi s is going to be 1, from the slope I can actually directly get what is the diameter of the particle ok. That is one of the applications of you know Korzany Carman equation where if you do experiments under laminar flow conditions I can actually use this delta P by L versus V 0 bar plot to get the dimension of the particle.

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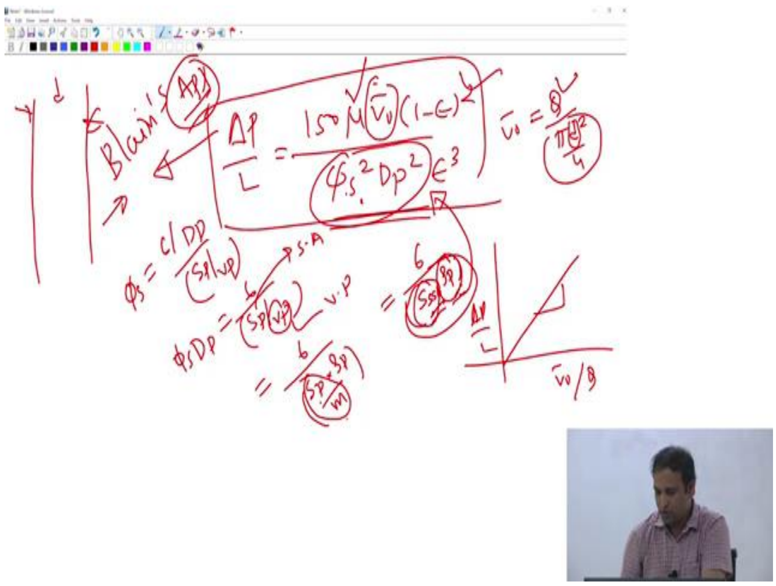

Now, again if you again will go with again the same expression 150 times mu V 0 bar sorry that is wrong right that is your that is one minus epsilon whole square. Now if you look at this right your phi s square into D p I will go back to the definition of sphericity where we said your phi s 6 by D p divided by you know S p by v p right that is what we did we said therefore, your phi s into D p is going to be 6 divided by S p by V p right where s p is the surface area of the particle and V p is the volume of the particle right.

Now, I can write this as 6 divide by S p into volume I can write it in terms of the mass the density. Therefore, it becomes multiplied by density of the particle divided by mass of the particle whole square right that and your S p by m which is the this surface area of the particle per unit mass that is what we define it as a specific surface area right.

So, therefore, it goes as 6 divided by say s may be specific you know surface area times rho p whole square right is it ok? No its no there is no whole square right sorry I was talking about this. So, there is no right is it ok? So, basically I have kind of rework this. So, essentially again it is the same concept right. So, all you are doing is you are instead of phi s square into D p square I am basically expressing that in terms of the density of the particle and the specific surface area again I can plug it back into this equation.

$$\phi_S = (6/D_P)/(S_P/V_P)$$

$$\phi_s D_P = \frac{6}{\left(\frac{S_P \rho_P}{m}\right)} = \frac{6}{S_{SSA}\ \rho_P}$$

So, once I do that if I were to do a again a similar experiment of course, I can think about you know this V 0. So, what people do is the moment you do experiments in a column of a particular diameter right say this is d right what you do is I can basically express this V 0 bar also in terms of the volumetric flow rates divided by the area right. So, pi d square by 4.

So, basically you know I can subsequent I can basically you know recast this expression in terms of the volumetric flow rate of the liquid that you know that is flowing through a packed column I can recast it in terms of the diameter of the column you know that makes the packed bed and also this phi s and D p in terms of you know your specific surface area and density of the particle if I have a measure if I have a way of measuring, if I know that I am using a fluid of a particular velocity if I have a measuring way of measuring rho p again we discussed how to get rho p right in one of the classes using what is the solution densitometry technique.

So, again a similar plot you know k delta P by L as a function of either V 0 V bar or you know Q if you want to do it. So, again from the slope I can actually back calculate what is a specific surface area again which is one of the useful quantity if especially if you are looking at you know applications where you know catalysis or something where you know the surface area becomes important one of the parameter that for you know for which you would have to characterize the particle is for this specific surface area and flow through packed bed concept gives you a way of you know calculating what is the specific surface area.

So, now commercially people there are a lot of instruments, there is something was a blains apparatus; blains apparatus which is a commercially available instrument which is for measuring specific surface area where in the concept that we discussed flow through packed bed under laminar flow condition that is what is basically used for getting the specific surface area ok. You have an experiment in the next semester in which you are going to use this concept for measuring the specific surface of a particle. So, any questions with that I am going to stop with flow through packed beds. We are going to look at some examples maybe the tutorial on Friday to look at a few problems where in we are going to look at flow through packed bed.

So, the next concept that I am going to look at is something called as a fluidized bed. Any questions that you have? Yeah.

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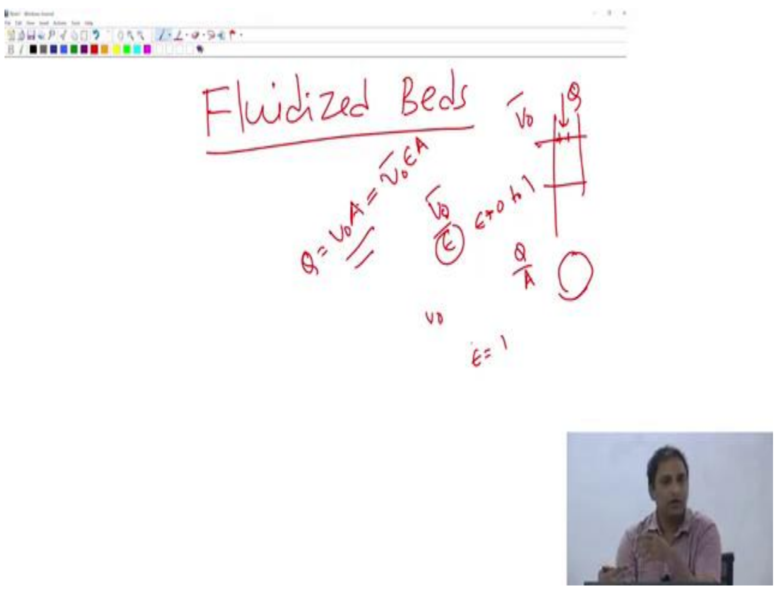

No when you say see V 0 bar was the superficial velocity. Correct. So, all you do is you have a you know packed bed you know this if Q is a volumetric flow rate and if is a circular conduit you know you have your cross sectional area going to be a circle right and Q by A is your you know superficial velocity right. Now the fact that you know your the entire cross sectional area is not available for flow, but only part of the you know area is available. So, therefore, what do you do that is V 0 by epsilon is what how it defined right.

So, because of the fact that there is a less area available I can it just basically comes from the continuity equation right.

So, your Q is equal to you know some velocity times you know A and the same has to again go through the this is before it enters the column the same has to go through the packed column that is going to be some you know say if the s is V 0 here right say that sees this is V 0 now if I say the velocity through the column is V 0 bar that has to be epsilon times A right because the volumetric flow rate has to be you know same across the column.

No. So, V 0 bar. So, the epsilon is the; is a void fraction right. So, what is the. So, now, this epsilon can go from 0 to 1 right. When you see epsilon is equal to 1 that is when you do not have a packed bed right both the velocity is exactly the same right. Now the moment I have some porosity the average velocity is always going to be higher than the velocity with which the liquid is coming you know just above a packed bed right. So, basically epsilon captures the factor in the velocity goes up with a liquid you know enters the packed bed that is the.