Fluid Mechanics

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Lecture - 42
Pressure Drop through Packed Bed

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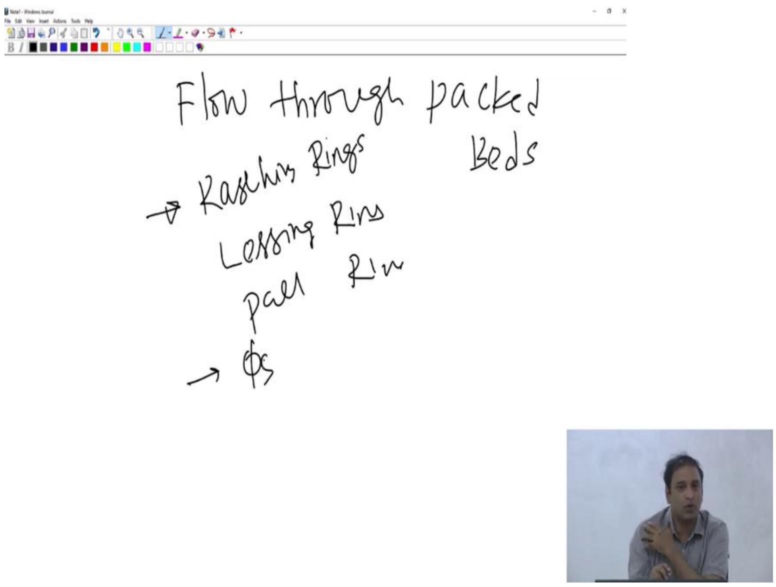

We will continue with flow through packed beds ok. So, any questions from the previous class? Any doubts that you have anything that you did not understand? So, just to sum up the previous class what we did was, we basically defined a few things in a packed bed right. We said what is a packed bed, what are the applications where they are used, we talked a little bit about what is called as a packed bed internals what does if you look inside a packed column what does it have, we talked a little bit about you know what are called as liquid distributors ok.

Which are which ensure that you know the channeling is kind of minimized and then also what are called as liquid collectors which are basically placed at different locations you know across the height of the packed bed. And the job of this liquid collectors is to ensure that you know you collect all the liquid that is coming down the packed bed at a particular location and then feed that into the distributor and distributor, does the job of uniformly distributing the fluid across the cross section of the bed right.

We talked a little bit about different types of distributors as well something called as a notch type, vein type you know and perforated you know channels can be used as distributors. Then we went on to talk a little bit a little bit about definitions right. We talked about what is a porosity ok. And then we talked about different ways of manipulating porosity right. We said you could use a packing material of different shapes you know; in general people use spherical particles or regular particles as packing.

Of course, you know you can go for more complex shapes we talked about using something called as a raschig rings right and lessing rings pall rings berl saddles you know things like that right. And what these different packing gives you is, it gives you a way of manipulating the porosity within the packed bed, it will also gives you a you know way of manipulating what is the total solid you know fluid area that is available for any you know process that you are trying to do.

Then we talked a little bit about you know other definitions like sphericity, you know and we talked about what is the definition you know and how does the you know sphericity depend on different you know shaped objects that is where we had stopped right.

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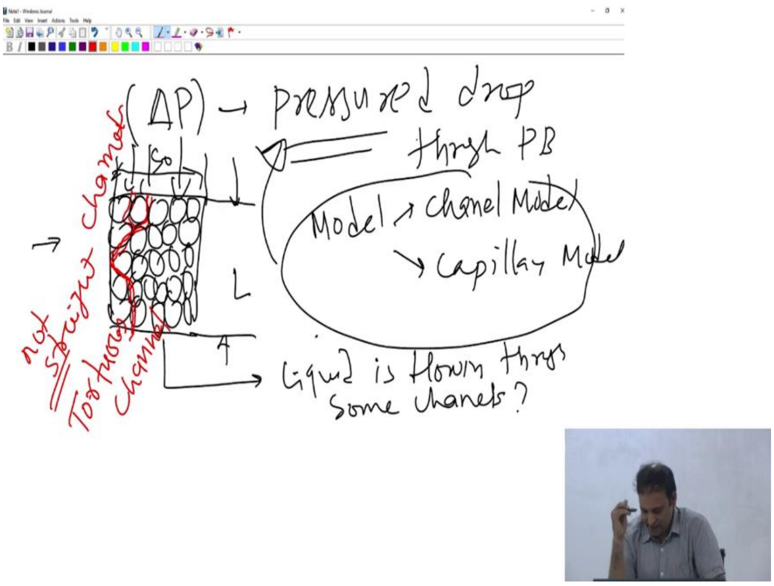

In today's class what we are going to do is to look at. So, we are interested in looking at pressure drop pressur drop through packed beds that is what we want to do you know for today's class.

So, that is a schematic of the packed bed say that this is a column whose length is like say L and say that you know it could be any cross section. So, let us say for the simple case let us take a cylindrical column, let us say that the you know the surface area or the cross section area of the channel is something like s0that is a cross sectional area of the column in which the packing is being placed let us say the let us write the packing ok. So, the packing is something like this right some spherical particles, there are put in a fluid sorry put in a column and you want to find out what is the pressure drop through the packed bed.

People typically think about two concepts, one is if I have a way of calculating what is the frictional losses come that come because of individual particles in the packed column. If I have a way of calculating that and sum that up for all the particles that form the packed bed you have you can come up with a way of finding out what is the frictional losses because of the presence of particles in the column. That is one way of doing it.

The other way of doing this is that you know actually flow the liquid is flowing through the column. And it is flowing through the space that is available for it and basic it essentially flows over the solid surface right you have a particles that form the bed and there is a space that is available and when the liquid is flowing through it, basically flows over a solid surface ok. If you have a way of calculating what is the total drag because of the fact that you know the part the liquid is flowing over this solid surface that is another way of you know thinking about you know calculating the pressure drop ok.

So, what we are going to do is to look at a very famous model which is what is called as a channel model it is also called as a capillary model for you know which is which have been developed to basically predict what is the pressure drop for the flow through the packed bed that is what we are going to do ok.

The concept is fairly simple so, what we are going to do is we are going to say that you have this packed bed right. Now if you imagine that the liquid is flowing through this column and for a minute forget about the particles ok. Let us say that the liquid is flowing through this column and of course, we said that you know the let us put it this way ok. So, can I say that so, because of the fact that you have a packed bed. So, can I say that the liquid is flowing through some channels is that a fair thing to say?

Right because you know of course, so wherever the liquid is flowing through so, I could if I were to maybe trace like say let us say I am going to trace the. So, let us say there it was it was wherever you know the fluid is flowing through right that could be the you know it could be flowing like that can I say that. Of course, it is flowing through the gaps between the particles right it is flowing through the gap between the particles right and of course, it could so, happen there you know some other stream from some other location may come and join right. It is it try to say that?

So, I can say that the liquid is essentially flowing through a channel; of course, these channels are not straight channels right; these are not straight channels right. This is a what are called as tortuous. These are tortuous channels because you know that is you know liquid is basically not taking a straight path it is through basically going through a column which can you know turn you know you know there could be change in the direction of the fluid as the liquid flows to the column ok.

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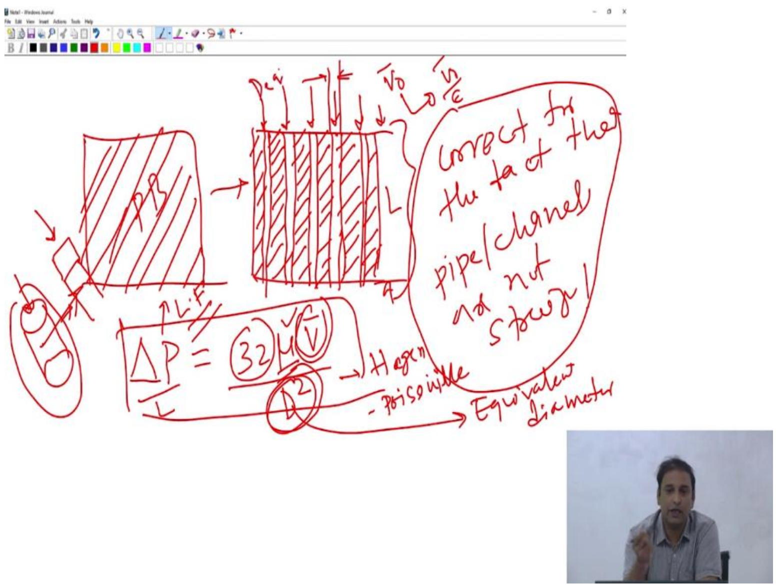

Now, the way we are going to work out this channel model is that, we are going to say that I am going to replace a complicated packed bed with a very simple configuration where I am going to say that there is a channel for the liquid to flow through and you have this the hatched region that I am that I am drawing these are the solids that formed the packed bed; of course, is a very simplified model right.

So, all I have to do is I would of course, I would have to correct for the fact that pipes or channels or not straight. Of course, I would have to correct for this thing, but I can assume that for the for the simplest case that I can say that you know I can assume a packed bed to be consisting of channels through which the liquid is flowing. And I am going to assume that they are straight channels and of course, later on I am going to correct it for the fact that you know the channels are tortuous ok; that means, they are not straight that is what we are going to do yeah.

Now whenever you want to do that ok. So, again one of the concept that we are going to follow is we are going to go back and look up what we are done in the previous lectures ok. Do you all remember this formula delta P is equal to? Sorry it is not 13 is a 32 then we just check, if that is right yeah I remember this formula? That you guys are done what is that any thoughts what is that? What is that? I am going to write down the name of this equation Hagen Poiseuille equation right you remember this, what do you right?

$$\frac{\Delta P}{L} = \frac{32\mu\bar{V}}{D^2}$$

So, when we looked at flow through conduits; flow through circular pipes, you had developed an expression for you know the average velocity through the packed bed. You had developed expressions for what is the pressure drop ok. The pressure drop basically goes as 32 times you know mu which is the viscosity of the fluid which is flowing through the conduit and v bar is what is called as a average velocity with which the liquid is flowing through the column and D is the diameter of the column through which the liquid is flowing right.

Whenever you want to kind of look at analogy of flow between you know pipes of different dimensions or different kinds what you do is, you would have to come up with a way of actually relating the parameter that you that are involved to the actual quantities in a particular flow problem

So, for example, say that you have you know flow through a circular pipe ok. Say that you know I would like to use you know whatever equations that you are developed for a flow through a circular pipe, if you want to extend that to flow through a rectangular channel. So, one of the concept that people do is, they instead of so, wherever you have dimension D which is the diameter of the column they replace that with what is called as a equivalent diameter.

You replace D which is the dimensions of the pipe through which the liquid is flowing through in this case to what is called as a equivalent diameter that is one thing you would have worry about and this v bar is the average velocity with which the liquid is flowing through the pipe.

In the case of packed bed we said that you know they something called as a v 0 bar which is what is called as a superficial velocity or the empty tower velocity that is the velocity before the liquid enters the column. Accordingly the velocity with which the liquid is flowing through the column is v 0 bar by epsilon right that is the actual velocity right.

So, therefore, what I am going to do is, I am going to take this expression that has been developed for pressure drop for flow through pipes and we know that this is only valid for laminar flow conditions ok. I am going to use that, I am going to modify it and then yeah you have somebody has a question yeah. What is that?

Oh yeah delta P by L I am sorry with that yeah that is true right. So, this is delta P by L that is the pressure drop per unit length right that is equal to 32 mu v bar divided by D square right yeah that is right.

Now, how do we go about calculating the equivalent diameter? So, if you have a simple geometry like you know going from a circular pipe to a you know a square conduit or a rectangular conduit, you know you can there are simple ways of getting what is the equivalent diameter you know. But; however, for this case it is a little bit complicated right.

What do you think that one should be doing to get what is the equivalent diameter? So, I am just going to say that you know the length of the tubes is same as the length of the packed bed and I am going to say that this diameter of the pipe through which the liquid is flowing through is say D equivalent.

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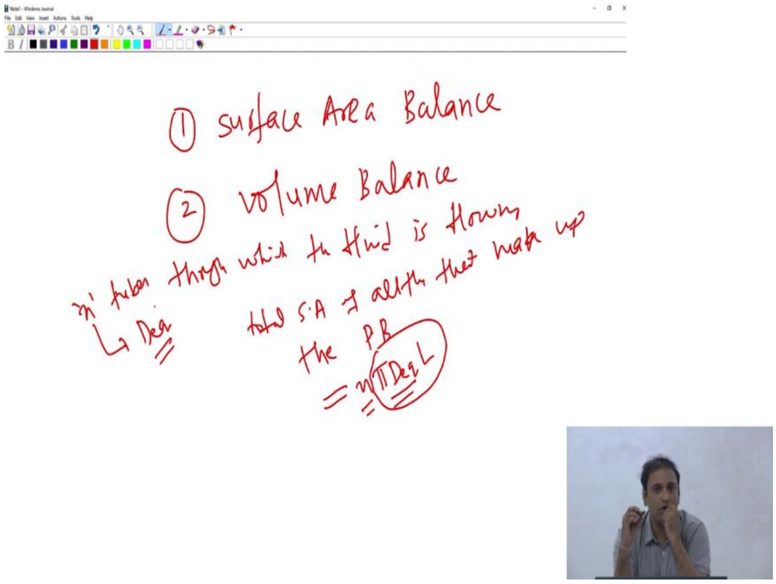

So, what do we do? So, for doing that we do two things one is we do what is called as a surface area balance and number 2, what is called as a volume balance ok.

Now, let us say that there are n conduits or n tubes through which the fluid is flowing right and let us say that is diameter of each of tube is D equivalent. Can I write an expression for the total surface area of all the tubes that make up the column that make up the packed bed ok? That is total surface area of all the channels or tubes or capillaries that make up the packed bed. What would that be? That is going to be n times pi D equivalent times L is it because you have tubes you are right your perimeter right times the length it is going to give me what is the total surface area that is available for the fluid to flow through that multiplied by n is going to give me what is the total surface area of all the channels put together is that, yeah.

$$Total\ S.A = n\pi D_{eq}L$$

Now, let us think about expressing. What do you what can you say about the volume of the solid particles are present in the packed bed? What can I say about the volume of the ok?

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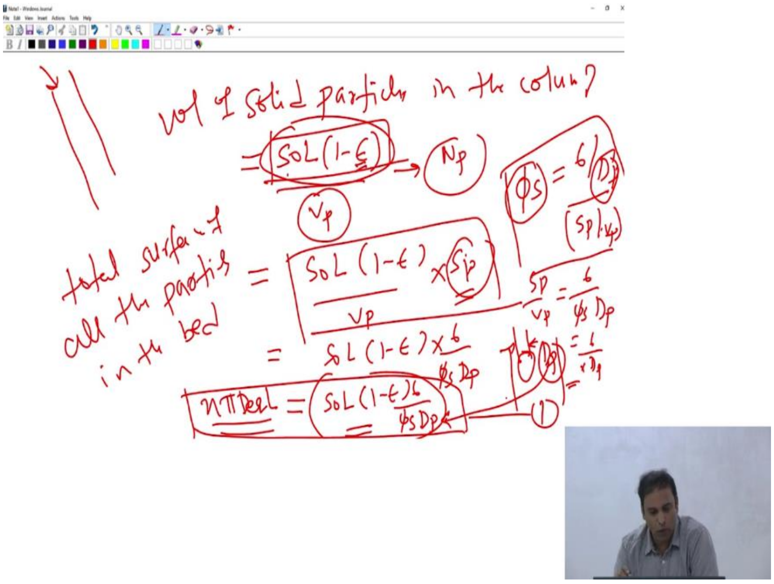

Let me just go back here what is the volume of solid particles in the column, what is that? It is going to be S 0 which is the cross section area times L that is going to give me the volume of the packed bed that multiplied by 1 minus epsilon is going to give me the volume of the solids in the is that. That is going to give me what is the total volume of all the solid particle that you have in the packed bed.

$$Volume\ of\ solid\ particles = S_0 L(1-\epsilon)$$

Now, if I divide this right you have you have a doubt somebody has a doubt? If I divide this by V P which is the volume of the single particle, what is it going to give me? It is going to give me the number of particles right is going to give me the number of particles that are present in the bed, is it ok?

$$N_P = \frac{S_0 L(1-\epsilon)}{V_P}$$

So, now if I ask you what is the total surface area of all the particles in the bed is S 0 into L into 1 minus epsilon divided by V P multiplied by S P right where S P is the surface area of every particle that you have is it okay? Now, you know that when we talk about sphericity phi s right it is 6 by D P divided by S P by V P right that is a definition of sphericity therefore, I can write this S P by V P as 6 by phi s into D P. I can as a substitute here S 0 into L into 1 minus epsilon multiplied by 6 by phi s into D P right yeah ok.

$$Total\ S.A\ of\ all\ particles = \frac{S_0 L(1-\epsilon)}{V_P} S_P = S_0 L(1-\epsilon)\frac{6}{\phi_s D_P}$$

Now, the fact that you know the channels are basically made up of the walls right. So, you have these channels right these channels through which the liquid is flowing ok. Now the fact that you know these channels are basically made up of the walls that arise because of the particles present in the fluid ok. So, what I am going to do is, I am going to say that the total surface area of all the channels in the column is equal to the total surface area of all the particles that I have in the system ok. Therefore, what I can do is I can write n times pi D equivalent times L that is the total surface area of all the channels in the column is equal to S 0 into L into 1 minus epsilon into 6 by phi S into D P that is the total surface area of all the particles that I have in the column ok.

$$n\pi D_{eq} L = S_0 L(1-\epsilon)\frac{6}{\phi_s D_P}$$

Therefore this is basically the surface area balance ok. All I am doing is I am calculating what is the total surface area of all the columns that I have in the system and that has to be equal to the total surface area of all the particles in the system ok. This equation is yeah if you have any doubts we can discuss that the only ideally there is some small correction as well right because you know what is happening is that you know if you have particles there are in contact right there is a small contact area that is available I am basically neglecting that ok.

But if you leave that you know if you assume that you know these are point contacts, if you assume that you know there is no significant reduction in the surface area because of these contacts. This is a good equation or a good approximation to say that the total surface area of all the columns that you have in the you know in the in the bed is equal to the total surface area of all the particles that you have the column right is that right.

All that we did in obtaining this right hand side was that we said that the total volume of the bed that or the total volume of the particles in the bed is S 0 into L into 1 minus epsilon, epsilon is the void fraction right. That gives me the total volume of solid divided by the volume of the particles will give me the number of particles in the bed multiplied by S P which is a surface area of every particle that basically give me gives me the total surface area and all we are saying is the total surface area because of the column you know I should be equal to the total surface area of them you know all the particles right yeah. No this S P by V P this is we know that this is a characteristics of the particles that we using right what is that?

D P is basically diameter of the particle right if you have a packed column if I am saying that you know I have you know particle of some dimension ok. If I were to be working with spherical particles if I say that you know your diameter of the particle is D P that is your the D P here that is the diameter of the particle that are that make up the column that is D P.

No the fact that I am using sphericity concept, it need not be for a spherical particle right. It could be for any particle as long as you know the dimensions of the particle is D P that is all yeah I mean all we are trying to do is. So, if I take this to be for a spherical particle we know there is basically 6 by D P itself right, but; however, you know your S P by V P which is 6 by phi s into D P it is going to be just 6 by D P for the case of spherical particle; however, if you are working with particle of any other dimension you would have to work with have in this pre factor which is sphericity that is all yeah. Now, what we will do is we will look at the volume balance.

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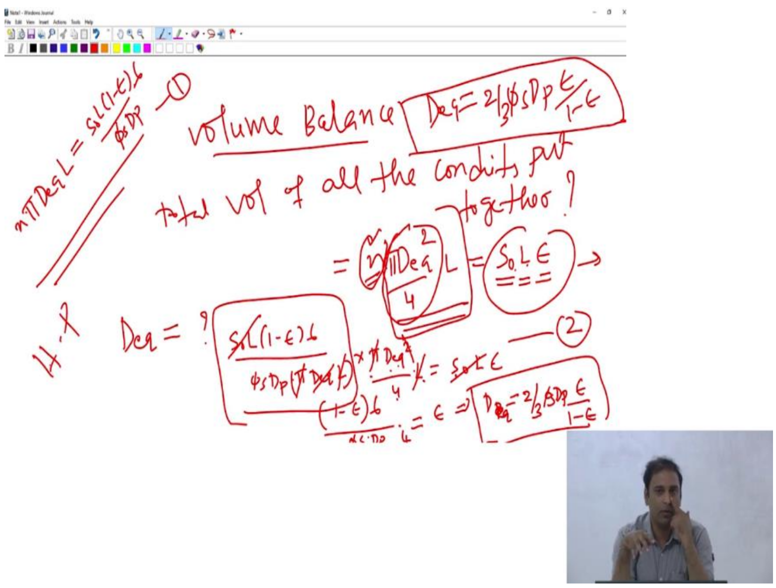

Now what we are going to do is, we are going to say that we talked about the total surface here that is available for the liquid to flow right what about the total volume? Volume of all the conduits put together what is the volume of all the conduits put together that is going to be equal to n times right D equivalent square into pi divided by 4 that is you know that is the right multiplied by your L right there is a total volume of. So, that is the volume of the cylinder right you pi D equivalent square divided by 4 multiplied by L is going to be the volume under the pipes itself multiplied by n which is the number of pipes that I have that basically is the total volume of all the conduits put together right.

And of course, that has to be equal to the total volume of the liquid that you have in the system right that is going to be S 0 into L into epsilon right S 0 is your cross section area L is your length and epsilon is the porosity and this is the volume balance ok. All that we are doing is we are saying that the total volume of the liquid you know in the system or the liquid is flowing through the system is basically equal to the volume that is available for them to flow in this case, the total volume that is available for them to flow is going to be the number of channels multiplied by the volume of the each channel right ok.

$$V = \frac{n\pi D_{eq}^2 L}{4} = S_0 L\epsilon$$

Now, what you are going to do is you are going to use this equation and the equation that we wrote up here that is 1 and 2 you are going to use that, and you are going to work out and then get what is your D equivalent ok. So, let us do that.

So, n times pi D equivalent times L was equal to S 0 into L into 1 minus epsilon divided by phi s into D P into 6 right something like that that is your yeah can you work with 1 and 2 and let me know what is your D equivalent? Can you do it in your notebook tell me what is the d equivalent? Ok.

So, what you what you can do is, I have this equation that relates the this volume imbalance right or I can do is I can substitute for n from equation one I can substitute for n from equation 1. So, what I have done is. So, n I am going to write it as S 0 into L into 1 minus epsilon into 6 divided by phi s into D P multiply I have right yeah divided by pi into D equivalent into L right that is your n right multiplied by you have this term that is pi D equivalent square divided by 4 into l should be equal to S 0 into L into epsilon say it is not gets cancelled on both the sides right.

So, if you work it out it turns out that you know your this is D equivalent it turns out that you know your D equivalent is going to be two by third into phi s into D P into epsilon divided by 1 minus epsilon ok. You can work it out right therefore, your D equivalent is 2 by 3 into phi s into D P into epsilon divided by 1 minus epsilon.

$$D_{eq} = \frac{2}{3}\, \phi_s D_P \frac{\epsilon}{1-\epsilon}$$

So, you have been able to get hold of a expression for D equivalent which I can further use and then substitute in the Hagen Poiseuille the equation and calculate what is the pressure drop right. So, let us go back to that equation right.

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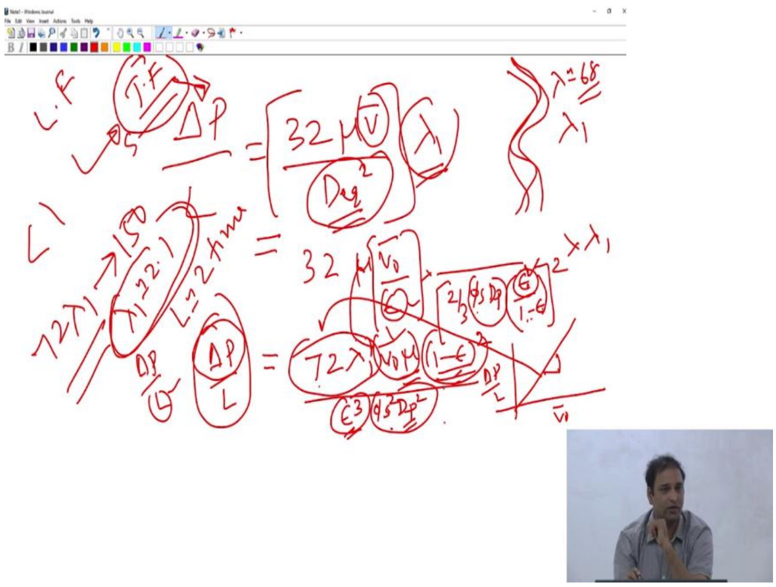

So, your delta P by L was equal to 32 mu V bar divided by D now D equivalent square right. Can you substitute for D equivalent square and then just ? So, what is done is then you know. So, we of course, we said that you know this is actually applicable only for straight conduits right or the straight pipes. So, because of the fact that you know your channels are not straight, you would have to introduce a factor called lambda you know lambda 1 which is what is called as a tortuosity factor ok.

So, therefore, I am going to multiply this everything by lambda 1; lambda 1 is a parameter which is called as a tortuosity factor this basically takes into account the fact that the pipes are not straight they are tortuous in nature ok. And there are we are going to people have done experiments to determine what this lambda is we will talk a little bit about that in the you know in the next few minutes ok. As mu and of course, this is not going to be is V bar I would have to replace that by v 0 divided by epsilon right that is your the average velocity right and I would have to substitute for what is D equivalent ok. So, what you can do is that is 2 by 3 into phi s D P square into epsilon divided by 1 minus epsilon right that is your not D square right that is just a D P right. And of course, there is a lambda 1 that is the tortuosity factor ok. So, if you work it out what you actually get is something like this. You get 72 times lambda 1 times v 0 bar times mu into 1 minus epsilon square divided by epsilon cube into phi is into phi s square into D P square yeah that is what you are going to get ok.

$$\frac{\Delta P}{L} = \left[\frac{32\mu\bar{V}}{D_{eq}^2}\right]\lambda_1 = 32\mu\left[\frac{\bar{V_0}}{\epsilon}\right]\frac{\lambda_1}{\left[\frac{2}{3}\ \phi_s D_P\left(\frac{\epsilon}{1-\epsilon}\right)\right]^2} = \frac{72\lambda_1\bar{V_0}\mu(1-\epsilon)^2}{\epsilon^3\phi_s^2 D_P^2}$$

All that you have done is you basically simplified it further right you substitute it for what is D equivalent in terms of 2 by 3 phi s D P into epsilon divided by 1 minus epsilon square because you have 1 epsilon that epsilon squared here and there is 1 epsilon here. So, therefore, you have this epsilon cube with the denominator. And of course, you have phi s square into D P square and mu and v 0 bar that comes from here right and one minus epsilon square that comes because of the fact when you have 1 minus epsilon in the denominator right and of course, if you work it out. So, that is going to be your 72 times lambda 1 ok.

And what people have done is they have done experiments in which they have they have a way that is delta P by L right. If you take columns of you know in which you maintain different delta P is you know and then you vary your delta P by L or if you take columns where you know keep your delta P constant. And if you vary L right you vary this when people have done a lot of experiments by using packed beds of different materials and they have kind of found that you know this 72 times the lambda 1 which is a pre factor, it apparently it is about 150 which leads to lambda 1 of the order of 2.1 and this 2.1.

So, one way to think about this what is called the tortuosity factor one way to think about this would be that the fact that you know the channels are not straight of course, the fluid is going to spend more time in the packed bed because you know it has to pass through a greater length of the tube ok.

And the fact that you know this is slightly greater than 1, it tells you that you know the of course, they are a tortuous and you can think about you know this being the typical time that the fluid spends in the column would be roughly of about 2 times you know what it would spend if the channels are straight ok. This is roughly is what you can see and of course, this depends on the kind of flow conditions ok.

So, so, far what we have done is we have assumed Hagen Poiseuille equation which is valid only for laminar flow conditions ok. So, now, similarly what we are going to do is, we are going to develop a similar expression for the case where you have a turbulent flow condition ok. Can we do that any doubts with this? So, this tortuosity factor there is no way of directly measuring what it is ok. The only thing that you can do is you formulate the problem in such a way that you know you are under this laminar flow conditions, that is no your Reynolds number is less than one for the flow through packed bed problems right.

And then what you do is you carry out experiments if you have a way of measuring delta P and if you have a way of varying say v 0 bar if I know what is epsilon and you know your D P and everything. Therefore, what you do is, if you were to plot how this delta P by L varies as a function of v 0 bar what you would do is, you will get a straight line that is passing through the origin right and from this slope they basically back calculate what this lambda 1 is that is how it is done ok.

And this is based on a set of experiments done on columns of different you know packing material you know different length columns and stuff like that and this tells you something about lambda tells you something about how torturous the channels are ok; it turns out that you know the lambda that you get for this case is 2.1. And however, if you go to the turbulent condition the lambda that you get is much higher ok. So, case of the order of something like 68 or something like that ok. We looked at a number a little later therefore, this lambda 1 correction is necessary because of the fact that you know the flow is not through straight channels any questions?

So, it is almost time up we will stop here. So, I will take a few questions if you have some ok. We will continue with you know doing something very similar for the cases under which your flow through you know the tubes are under laminar flow conditions any questions you have a question both of you yeah yes any questions yeah no. You can tell me you know let us discuss that now I mean if you have any question that you have you should you should just raise and then figure out we do you know understand that no is the concept clear of the of the capillary model or the channel model. It is a simple concept that you know you are basically assuming that you know the flow through packed bed indeed can be thought in terms of flow through channels of course, these channels are not straight they are going to be different you know streams you know joining from different you know locations of the packed bed ok

However for the sake of simplicity we assume that there are straight channels we said there are n channels of exactly the same length as the length of the packed bed. And we did very simple volume balance and the surface area balance that gave us a way of estimating what is the equivalent diameter or you can think about this as a equivalent diameter of the conduits through which the fluid is flowing and then we substitute that in the Hagen Poiseuille the equation. And then came up with the expression for what is the pressure drop through the packed bed it turns out to be one fifty times mu 0 v 0 bar into mu one minus epsilon square divided by epsilon cube into phi s square into D P square that is the that is where we stop here today we will continue with the tomorrow.