Fluid Mechanics
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Lecture - 40
Settling in multiple particles system

So, we briefly discussed a multi particle system yesterday.

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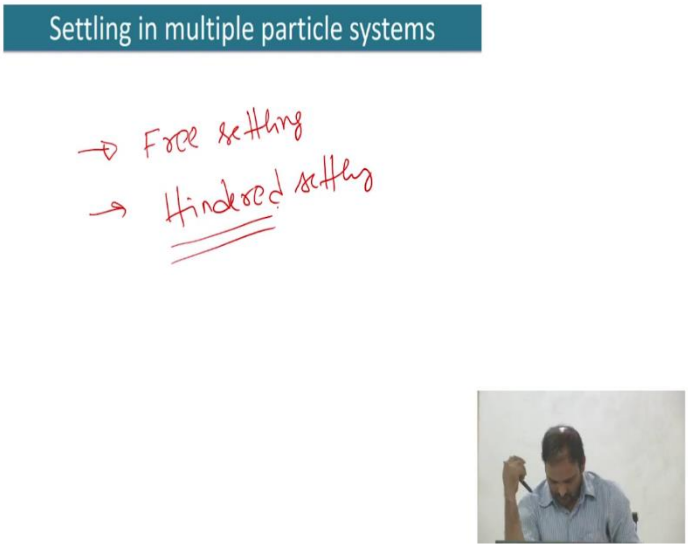

Settling in multiple particle systems

→ Free settling
→ Hindered settling

So, we defined something called as you know free settling ok, something called as hindered settling right. And we said that you know hindered settling is going to occur if you have cases where the particles are close enough or if there is a kind of a coupling of the way the particle settles with the wall of the container or with other particles in the system ok, then that is when we call what is called as a hindered settling, where the motion of the particle is kind of influenced by the other particles in the system plus the wall of the container right.

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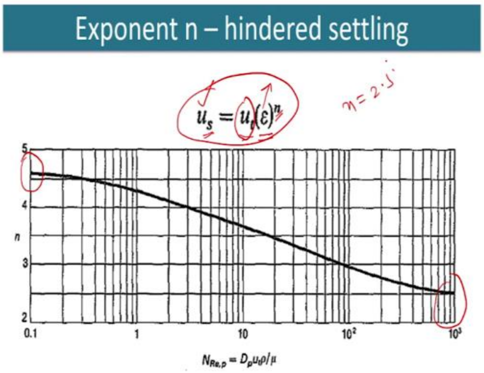
Exponent n – hindered settling

$u_s = u_t(\varepsilon)^n$

$n = 2.5$

And we said that you know, so basically people have kind of proposed you know some empirical equations of this sort that is the settling velocity under you know hindered settling condition goes as you know this is a free settling velocity multiplied by some term, which is which depends on epsilon, which is the volume fraction of the liquid that you have in the you know slurry or the suspension. And n is an exponent that takes the different values depending upon what kind what kind of fluid particle system are you working with ok.

And we said that you know if you take you know Stoke’s law regime, then you know n is going to be of the order of 4.6. And if you take the Newton’s settling regime, your n is going to be of the order of 2.5 right; that is what we said. And then we just want to set this problem up.

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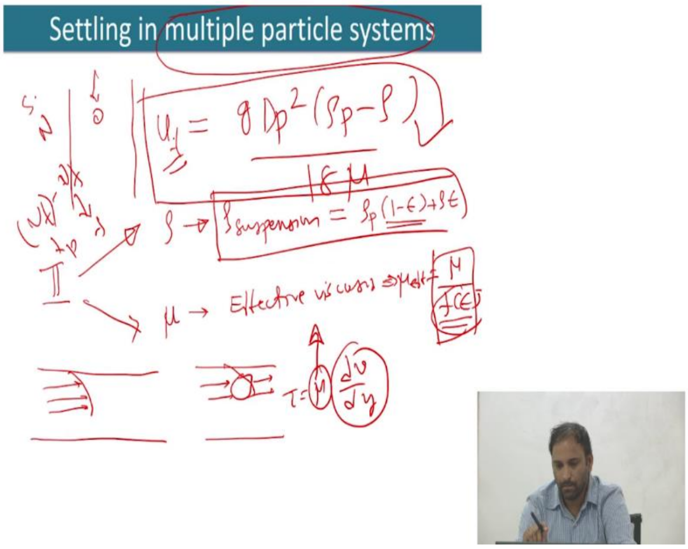
Settling in multiple particle systems

$$u_t = \frac{g D_p^2(\rho_p-\rho)}{18\mu}$$

$$\rho \to \rho_{suspension} = \rho_p(1-\epsilon)+\rho\epsilon$$

$\mu \to$ Effective viscosity $\mu_{eff} = \frac{\mu}{f(\epsilon)}$

$$\tau = \mu \frac{dv}{dy}$$

And I said that you know if you really want to look at you know the settling velocity in the multiple in the multiple particle system, I mentioned there any you can actually start with you know assume a particular settling regime either Stoke’s or Newton’s ok. And then you basically write down whatever is the working equation for settling for each of these cases, so we just took g D p square into rho p minus rho divided by 18 mu. You take that and you modify it to suit the multi particle system right; that is what we mentioned.

And in that connection we said you are going to make two modifications ok, modification one is that your rho which is the density of the fluid is now you would have to worry about replacing that with the density of the suspension or the slurry right. And we said that basically goes as rho of particle into 1 minus epsilon where epsilon, you know 1 minus epsilon basically gives you the fraction of the solid that you are in the system plus rho of the fluid multiplied by epsilon right that is one modification.

And the second modification that we said is you should replace your mu with some sort of effective viscosity ok. And we said this mu effective ok; mu effective you know is going to be something like mu divided by f of epsilon ok. And we said that you know this function f of epsilon is typically less than 1, that is because we have seen examples where we know that you know the viscosity of a fluid with the particles present is higher than the viscosity of the need fluid itself right.

And I was trying to answer a question yesterday where I am said that if you have like say a liquid flowing through a container right through a pipe for example, and if you say that you know there is some sort of you know there is a you know you can think about them as streamlines for example right. Now, what will happen is if you have a similar case with the particles what happens you know the presence of the particle it basically obstruct the fluid flow right.

In that sense, you know, your liquid is going to be stopped. So, now, the fact that you know the there is a slow down slowing down of the liquid. So, your average velocity gradient basically reduces ok. And if you have this you know mu tau is you know mu times d v by d y, the fact that you know the average velocity gradient goes down when you have the particle you for compensating that you know you viscosity has to go up ok. This one way of thinking why does the viscosity of the fluid increases if you have particles in the system ok. So, therefore, your f of epsilon is a function which is less than 1 that some you know you would have to come up with a way of working around it right.

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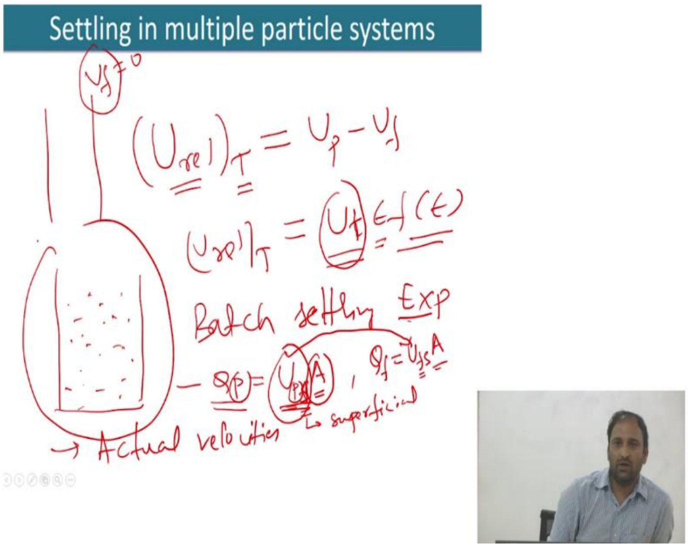

Settling in multiple particle systems

And I also mentioned that I mean in the case of multi particle system, your relative velocity right is typically related by that is the relative terminal velocity that is because of the fact that you know you have in the case of single particle system, your fluid was stationary you know you are U of the fluid is essentially 0 right, only thing that was settling was the only thing the that was moving was a particle right. But if you have a multi particle system as though as the fluid is displaced, the fluid is going to go up ok. There is going to be U f component as well and this U f is actually U of particle minus U of the fluid right; that is how it is. And then you know and we just derived that and it basically goes as U t which is the terminal settling velocity under the free setting conditions multiplied by epsilon into f of epsilon right.

If you go back to this expression substitute for you know rho in terms of rho suspension right, and then replace mu with mu effective, so that is what you will end up ok. Your U relative terminal is going to be settling velocity under free settling conditions multiplied by epsilon times f of epsilon ok.

Now, so before we go on you know go further I just want to introduce a few terms ok. Typically, when people do multiple particle settling right typically people carry out certain type of experiment which what are called as a batch settling experiments ok. In batch settling what basically happens is you know you have a container and that container is basically filled with particles ok. And basically you watch this container with the fluid particle system as a function of time that is what is typically done ok.

Now, if we if you define couple of quantities something called as a Q P something called as QP, where QP is the volumetric flow rate of the solid particles in the system ok. The particles are coming down or settling right, and there are some velocity associated with the way the particles move in the fluid right ok. Based on the velocity and if there is an area that is available for them to settle down right. So, your QP is what is called a volumetric flow rate right, and that would depend on the some velocity right. And P stands for the particles right.

I am going to use a subscript s sorry subscript s ok. And this s is what is called a superficial, it is basically stands for superficial times A, where A is the cross sectional area that is available for the particle to settle right. So, if you have a container, if you have a cylindrical container, your cross sectional area is going to be you know your whatever circle area right now. So, if you define ok, if you define the velocity that is associated with the particles by considering A, which is the which is the entire cross section area of the container ok, then this velocity that you are you have it here; it is what is called as a superficial particle velocity ok.

$$Q_P = U_{P_s} A$$

Similarly, if I define you know a volumetric flow rate for the fluid that is Q f ok, and if I say that the velocity with which the fluid is basically you know moving is again f for fluid and s for superficial. Again if I use the entire cross sectional area of the container that is when these velocities are what are called as a superficial velocities.

$$Q_f = U_{f_s} A$$

But however, we know that you know out of this entire cross section, certain area is occupied by the particle and certain area is occupied by the fluid right. Therefore, if I actually if I actually consider the actual area that is available for the you know respective fluid and the particle ok.

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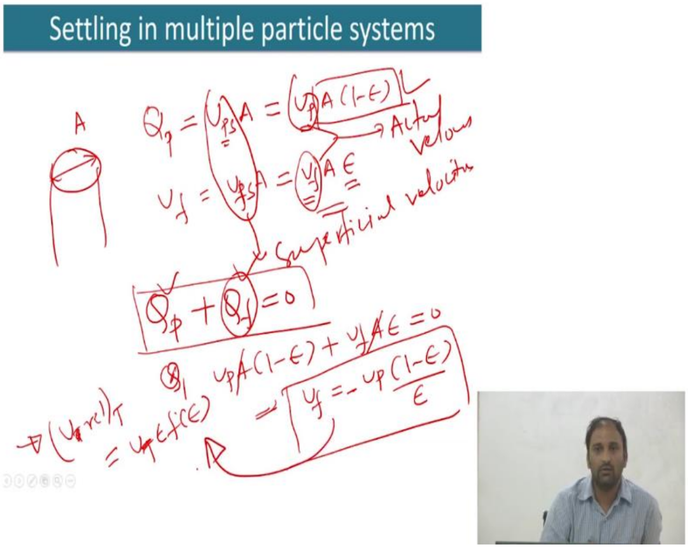

Settling in multiple particle systems

Then you define what is called as actual velocities ok, which basically is if you say that your Q p ok, is U particle superficial times A ok. A is equal to U of just the particle times what is the area that is available for the particle to settle? It is going to be what is that?

So, if you have a container right ok, say that you know that the cross sectional area is A right. And if I say that the cross if I want to have an idea about what is the cross sectional area that is available for the particles to flow, it is A into 1 minus epsilon right. Because if I if I say that you know the 1 minus epsilon gives you the solid fraction right, the only thing that I am assuming here is you know volume fraction is typically ,it is a 3D quantity right you know it is basically for the entire fluid particle system. If I assume that the area fraction is equal to the you know in a 2D sense ok. What is the fractional area that is available for the for the particle settle is this right. Is that or do you have some?

Now, similarly, for the fluid ok, it is U p sorry U fluid superficial times A and that should be equal to U of f into A times epsilon right, because epsilon is the liquid fraction right. So, therefore, in any fluid particle system, if you are basically considering the entire cross sectional area that is available in the calculation of your velocities that is what is called as a superficial velocity.

$$Q_P = U_{P_s} A = U_P (1 - \epsilon)$$

$$U_f = U_{f_s} A \epsilon$$

But if you only consider the respective area that is available for either the particle or the fluid into consideration for getting the velocity of the fluid or the particle, you know these are what are called as actual velocities. In this case, these are the actual velocities. And these two terms are what are called as a super. Is that or any, any doubts?

Yeah, it is just a convention that people use that that is about it ok, whether you because we know that you know the way to go from the volumetric flow rate to the velocities, you know you would have to divide that by the area right cross sectional area that is available for the flow ok. If I take the entire cross sectional area that is available both for the particles in the fluid put together that is what is called as a superficial velocity. However, if I separate out the fractional area that is available for the particle and the fluid ok; if I use only respective area available for each of these species, then is what is called as a actual velocity that is the you know convention ok.

Now, so now, when you do a batch settling experiment ok, because the way the as I said right way the batch settling experiments are done is you basically take a container right fill in your particles, and the fluid right and you just watch as the settling occurs right. So, in such a case, if you I can write that because of the fact that I am not adding any fluid or the particle into the system externally, I can say that you know your Q p plus Q f should be equal to 0, because you know in the in a sense there is no net flow ok.

Therefore, you are you know the volumetric flow rate of the particles plus the volumetric flow rate because of the fluid that should be equal to 0 right, because there is no net flow as such now I am not adding anything to the system or I am not taking anything out ok.

So, now therefore, I can write Q p. So, now, I can define Q p in terms of you know your actual velocities right. Q p I can write it as U p times A times 1 minus epsilon plus U f times A times epsilon should be equal to 0 ok. Therefore, I can just cancel this throughout. Therefore, your U f becomes minus of U p into 1 minus epsilon divided by epsilon right, is that ok?

$$Q_P + Q_f = 0$$

$$U_P A(1-\epsilon) + U_f A\epsilon = 0$$

$$U_f = -U_P\frac{(1-\epsilon)}{\epsilon}$$

Now, with this what I can do is I am going to use this, and then go back to the equation that we developed ok, which was U T U relativity right, U relative T was U T times epsilon times f of epsilon right we had this expression right. So, I am going to substitute this back here ok.

$$(U_{rel})_T = U_T\epsilon f(\epsilon)$$

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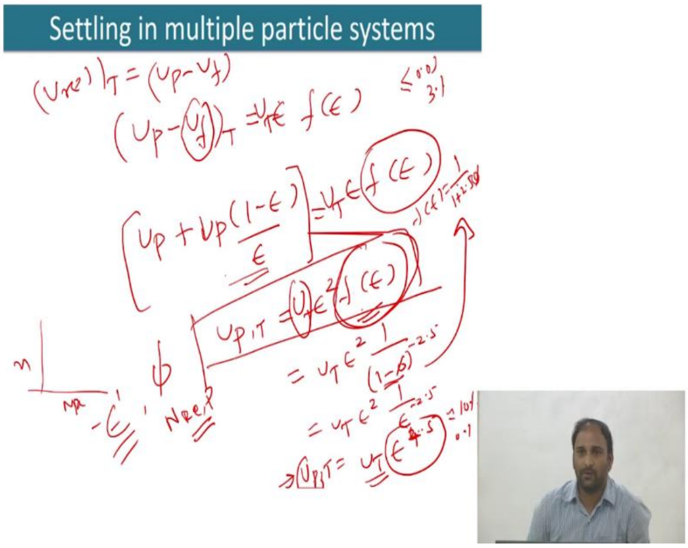

Settling in multiple particle systems

$(U_{rel})_T = (U_p - U_f)$ ≤ 0.01 3.1

$(U_p - U_f)_T = U_T\epsilon f(\epsilon)$

$\left[U_p + U_p\frac{(1-\epsilon)}{\epsilon}\right] = U_T\epsilon f(\epsilon)$

$U_{p,T} = U_T\epsilon^2 f(\epsilon)$

$= U_T\epsilon^2\frac{1}{(1-\epsilon)^{-2.5}}$

$= U_T\epsilon^2\frac{1}{\epsilon^{-2.5}}$

$\rightarrow U_{p,T} = U_T\epsilon^{4.5}$

So, what are we going to do? What I am going to do is I am going to write it as U p minus U f right that is my I said U relative T is going to be your U p minus U f right that is what I said because you know that because both moving here should be equal to epsilon square into f of epsilon right. Now, I know that if you go back this expression, so I can I can substitute for U f in terms of U p right, is that. So, I go here. So, this is U p plus U p into 1 minus epsilon divided by epsilon is equal to equal to, I am sorry about that that is going to be, this is not correct now, just epsilon right that should be equal to your epsilon into f of epsilon right. So, if I sum if I simplify this, I basically get U p terminal ok, I will say terminal is going to be equal to epsilon square into f of epsilon right that is what you get right. Because you know there is an f I can take U p out, so epsilon gets cancelled, one or epsilon is what you have here basically that is what I end up with. Is it ok?

$$(U_{rel})_T = U_P - U_f$$

$$\left(U_P - U_f\right)_T = \epsilon^2 f(\epsilon)$$

$$\left[U_P + U_P \frac{(1-\epsilon)}{\epsilon}\right] = U_T \epsilon f(\epsilon)$$

$$U_{P,T} = U_T \epsilon^2 f(\epsilon)$$

What is that?

Student: On the right hand side.

On the right hand side, no, no, because I see U f I am basically replacing U f in terms of U p into 1 minus epsilon divide by epsilon right ok, that is what I have done here.

Oh, yeah sorry there is U T here, I am sorry with that right that is your you know free settling velocity right or the settling velocity under free settling conditions right. Now, if I want to, now I still do not know what is f of epsilon right, because you know if I want to go back and you know think about you know that the plot that we saw right you know you had n which was a function of Reynolds's number right. So, if I want to get that n I still need to find out what is [vocalized- noise] what is this f of epsilon right.

So, what people have done is there are a lot of theories available for what this f of epsilon is people have done a lot of theory as well as there are a lot of experiments available, wherein we know something about how the viscosity of the fluid changes a function of particle concentration ok. I am just going to put up some results from the literature.

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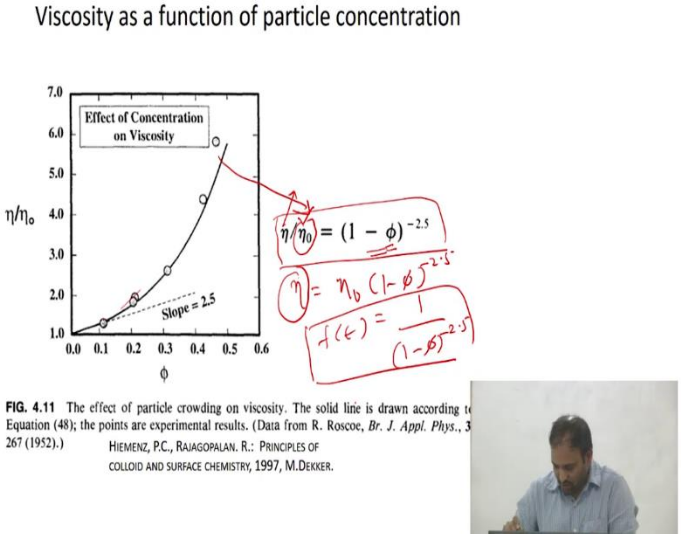

Viscosity as a function of particle concentration

$\eta/\eta_0 = (1 - \phi)^{-2.5}$

$\eta = \eta_0 (1-\phi)^{2.5}$

$f(\epsilon) = \frac{1}{(1-\phi)^{2.5}}$

FIG. 4.11 The effect of particle crowding on viscosity. The solid line is drawn according t Equation (48); the points are experimental results. (Data from R. Roscoe, *Br. J. Appl. Phys.*, 3 267 (1952).)

HIEMENZ, P.C., RAJAGOPALAN, R.: PRINCIPLES OF COLLOID AND SURFACE CHEMISTRY, 1997, M.DEKKER.

So, this is a plot a the line that you see is a line for this expression that you have at the you know on the slide, here eta is the viscosity of the particle filled system or is the viscosity of the slurry or the suspension. And eta naught is the viscosity of the pure fluid without any particles the suspending medium, and f is the volume fraction. And this line is actually the fit to this equation, and the data point that you see here is a experimental data right. And this is one of the theory that is developed by Einstein ok.

And, so therefore, what I can do is in terms of the effective viscosity function. Now if I write this as eta is going to be eta naught into 1 minus phi into right minus 2.5 ok. And we know that the we know that the viscosity of the particle fluid system is actually greater than you know is larger right. So, therefore, your f of epsilon the function that we want it should actually be 1 over 1 minus phi into minus 2.5 right; that is your f of epsilon ok.

So, what these relations which basically correlate the viscosity of the particle filled system to the viscosity of the neat system and the volume fraction. What they will give you is they will tell you, a way of actually getting this the f of f function which is what I need for me to go back and you know kind of complete my derivation of you know finding out, you know what the settling velocities are right. Let me just go back to right here.

So, what I can do is I can write this as U T into epsilon square right into 1 divided by 1 minus phi to the power of minus 2.5 right, I can write this as U T into epsilon square into 1 divided by 1 minus phi is your epsilon right that is the liquid fraction to the power of minus 2.5. So, therefore, your U T basically becomes two point 4.5 right, your U T is U p relative that is a the particle term in. So, this U p stands further the particles right that is the terminal velocity of the particle which is settling in a multi particle system where the hindered settling becomes important, basically goes as the free settling velocity multiplied by your epsilon to the power 4.5 and of course, there are some conditions under which this is applicable ok.

$$U_{P,T} = U_T \epsilon^2 \frac{1}{(1-\phi)^{-2.5}} = U_T \epsilon^2 \frac{1}{\epsilon^{-2.5}} = U_T \epsilon^{4.5}$$

Typically this is typically used where the concentration of your particles is about up to about 10 percentage or 0.1 ok. But of course, depending upon the epsilon that you are looking at you know the very liquid fraction or the phi that you are looking at depending upon the Reynolds's number of the particle you are working with ok. You will have different functional form for f of epsilon, you would have to use appropriate functional form and then substitute in an equation like this ok.

And as we said right this is typically this is only applicable if I am working with the particles of smaller size where the Reynolds's number comes in to the Stoke settling regime right because we started off with you know the Stoke's settling regime right. However, if you want to work with other you know settling regimes you would have to appropriately modify you know the expressions properly yeah ok. So, therefore, basically the idea of you know talking about this you know it kind of tells you something about how do people think about multi particle system, and then yeah go ahead, yeah.

That is because see these all these expression that we develop right you know the Newton's regimes the Stoke's regime everything, typically this is applicable for cases where you have a stagnant fluid ok, and say they do not you let a particle into motion either by you know inducing gravity all right. In that sense you know your fluid is still stationary ok, there is no net flow of the fluid as such ok. Though this is U T, it should actually be U T minus U f fluid ok. This is for the particle and this is for the fluid, but the fact that the fluid is stationary you know it basically turns out to be you know the terminus is the particle itself yeah. Epsilon should be? Yeah, I mean as I said right people have kind of developed like for example, so this is typically valid for about 10 percent.

But what are that dilute systems, yes, yes. It turns out that you know the f of the functional form that you have ok, people have different functional form. For example, if you go for concentrations which is typically up to about say zero point o three that is 3 percentage by volume ok. This f of epsilon is typically 1 divided by 1 plus 2.5 times 5 which is basically a special case of this right. If I take this and then you know if I expand it into you know higher order terms right if I neglect the higher order terms basically I recover this right. So, therefore, this is f of epsilon that you choose it depends on you know the kind of concentration that you are working with you know. So, in that sense you know so you would have to choose appropriate values of f of epsilon yeah ok. Any more questions? No?

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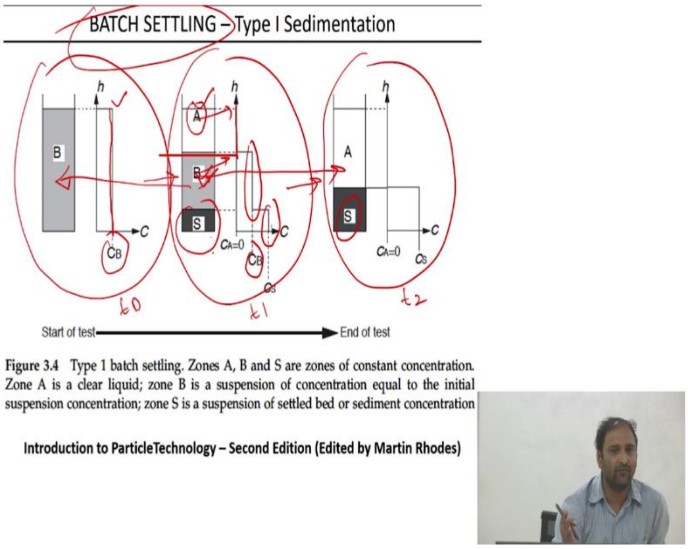

BATCH SETTLING – Type I Sedimentation

Figure 3.4 Type 1 batch settling. Zones A, B and S are zones of constant concentration. Zone A is a clear liquid; zone B is a suspension of concentration equal to the initial suspension concentration; zone S is a suspension of settled bed or sediment concentration

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So, so what I am going to do is. So, I am going to talk a little bit about in experimentally what people do is this ok. When people are given a slurry ok, and you want to find out you know their settling behavior. People do what is called as a batch sedimentation test ok; it is called a batch settling test or a batch sedimentation test. These experimental experiments are as simple as you take a container, fill your slurry, and just watch the container as a function of time ok.

So, what do you see are the images at three different instants of time ok, time t 1, t 2, t 3 if you want to call it as right, I say t 0 to begin with and some other time t 1, t 2. And what happens you know what you look at the case of picture corresponding to t 0, you have a slurry which is a uniform slurry, the concentration of the slurry you know as a function of height is exactly the same, that means, no matter from which pocket of the you know container I draw the solution, I measure their volume fraction is exactly same ok. So, uniform slurry and correspondingly, if you look at the concentration plot you know concentration everywhere throughout the height is c b, there is the concentration of the part that you start with and it is uniform throughout the height ok.

Now, with time what you would see is you get you know in the container, you will start seeing different zones that are formed ok. There could be a case you know where you have a zone with clear liquid that means there are no particles, in the system all the particles in that upper layer have come down ok. And you have a zone B, where the concentration of the particles in the zone you know in the second zone is exactly same as the concentration of the particle that you started off with ok. And then of course, at the bottom you have something called as S that is a sediment and where the particle concentration is going to be very high right.

So, therefore, if you are going to look at the particle the concentration is a function of height, the concentration is 0, there because there are no particles; it is a pure fluid ok. And then in the zone B, the concentration is going to be exactly same as you know your initial concentration as C B. And then of course, in the sediment, you are going to have a higher concentration right that is a concentration profile as a function of height ok. And then if you wait for long enough time what will happen is that you know, so you are you only your zone B completely disappears, you do not have B anymore ok, that means, all the particles have settled there, there they are all at the bottom and then you have a clear fluid ok.

This is a typical kind of a behavior that one sees you know when people do experiment like this. And what people do from this experiment is you know I can basically track down the interface right. If I look at the case you know the picture in the center, at some point in time, I start seeing an interface between A and B right. There is a clear interface ok. There is an interface that basically separates A and B right that forms at some instant in time right. And then of course, there is a there is an interface between B and S that could start forming at some instant in time or people doing such batch settling experiment is there you know you basically follow the interface position as a function of time ok.

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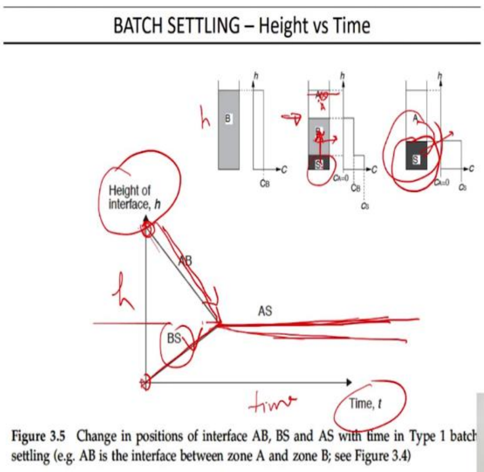

BATCH SETTLING – Height vs Time

Figure 3.5 Change in positions of interface AB, BS and AS with time in Type 1 batch settling (e.g. AB is the interface between zone A and zone B; see Figure 3.4)

And what you do is you can basically plot how the; this is the height of the interface as a function of time ok. Initially you only so in the beginning of your experiment, what you have is you basically have a clear interface right, I mean you have you do not have any clear fluid as such at some point in time ok, you will start seeing a interface between A and B formed is typically a start with some height. And as the time progresses this interface is going to descend right, it is going to come down. Therefore, you know you are going to see a decrease in the height of the AB interface as a function of time.

And BS interface there is an interface between you know, the zone where the concentration is same as initial concentration, and your solid that is not there initially there is no you know initially the height of that interface is 0 right. And as its as the you know interface forms this interface is going to move up right because of which I am going to see a increase in the height of this you know BS interface is a function of time. And the point where those two meet is when you know you have an interface between A and S ok. And it you could have a case where you know your A and S could be flat like this that is going to happen for cases where the solids that are present in the in the zone S, they are kind of they can support their own weight ok.

What, what may happen in some cases is that you know if there is a compaction that may happen even after the formation of this A and S interface what could happen is you could have, so of course, s we will have some fluid and the particle as well right. S is there is a lot of concentration of particles in the bottom there is also be some fluid ok. Now, imagine that you may have a case where the particles forms some kind of clusters in the in the in the bottom ok.

Now, with time you could you could still evolve. So, this A, A means you know in practice a make it may reduce slowly, but however if the particles that are present in this S can support its own weight typically you will see a very constant you know AS interface and that is going to remain as a function of time ok. So, that is the case where you work with some you know dispersion, and you see the formation of AB interface and AS interface and finally, you will have AS interface which does not evolve anymore with time right. Yeah.

No, no, this is actually height as a function of time ok. What I am doing is imagine that I am taking a photograph of this right at different time ok. Say that you know the height of the container is say h for example, ok. All I do is I start noting so when say that you know at some instant in time, you know this is where the A, B interfaces ok. Now, what I do is, I take this point and I plot it here ok, that is my height of the BA interface right. So, you get the point.

So, all you are doing is the fact that you know, so the moment you start seeing a clear fluid in the in your experiment, at that point you have there is a creation of BA interface right or AB interface ok. All I do is I basically mark the position of that interface and I am basically following that as a function of time that is this line ok. And the moment I see the formation of a BS interface, I start noting down the position, and I basically follow the height of that as a function of time that is basically this line ok.

And the point where they meet is when the AS interface forms right ok, and that is your this line. As I said this could be remaining constant or this could evolve all depends on the kind of particle that you have in the system ok. If you have typically if you have a rigid particle ok, it remains constant. If you have like say fluid like particle you know the emulsions or droplets or something that or if you have particles that are squishy ok, people see a some slow evolution, but however that you know evolution could be very negligible in the case of hard particles yeah.

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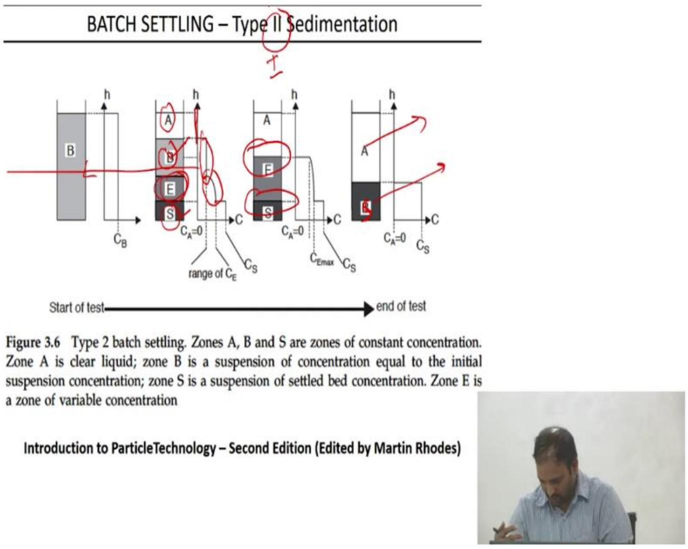

BATCH SETTLING – Type II Sedimentation

Figure 3.6 Type 2 batch settling. Zones A, B and S are zones of constant concentration. Zone A is clear liquid; zone B is a suspension of concentration equal to the initial suspension concentration; zone S is a suspension of settled bed concentration. Zone E is a zone of variable concentration

Introduction to ParticleTechnology – Second Edition (Edited by Martin Rhodes)

Now, so this is a another experiment where what people do is a similar to the previous case ok. You have zone A - the clear fluid. Zone B, where the concentration is same as you know the initial concentration. You of course, have the sediment at the bottom. There is a zone E, where the concentration fluctuations may happen ok.

What I mean by that is if I look at if I look at the concentration plot right, the concentration in the top zone is exactly you know 0, there is no particles there, and B is same as the initial concentration. And you have a zone E, where the concentration basically varies as a function of height ok; at the top the concentration is as same as you know what do you have in B; at the bottom the concentration is as same as what you have in S ok. However, across the height the concentration does change ok.

And you know again depending upon you know what you know so what will happen is your B could completely disappear, and then you know you only have an interface between A and E, and an interface between E and S. Again you follow that as a function of time ultimately you again end up with A and B in the, so A and sorry this is this should be S right. This should be S right that is a sediment at the bottom ok.

Now, whether the type 1 sedimentation or the type 2 ok, where you the formation you have the formation of this zone, where the concentration raised across the height, it typically depends on you know what kind of initial solid concentration that you are working with ok. Typically people have seen that if you are working with the a slurry where the concentrations are lower typically of the order of you know about 20 percentage or less ok, you see type 1 sedimentation ok. However, if you go for higher concentration you know initially that is when you see a type 2 sedimentation.

The reason why I wanted to talk about such kind of test is there you know you have a lab in the next semester where you want to look at settling of slurries. You have some experiments where you are going to look at containers which have slurries of different concentration. You are again going to look at you know monitoring the interface position as a function of time, and that will you know tell you that will help you in terms of obtaining the settling velocities ok. We will talk a little bit about that in the, you know in the in the in the next few minutes or so ok.

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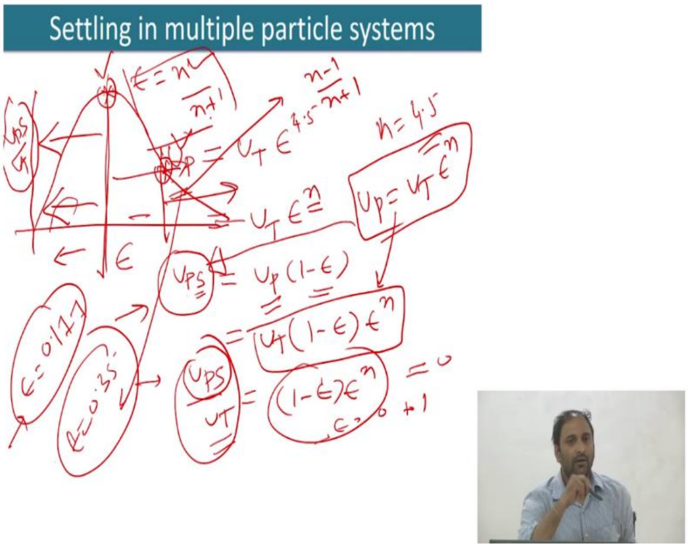

Settling in multiple particle systems

Now, any questions? So, now, let us look at, so we did this right, your U p you know goes as U T times epsilon to the power of 4.5 is what we did ok, but however in general it is typically epsilon to the power of n right, and I said and depends on what kind of settling that you are working with right. Now, from this what I can do is I can actually define U p s which is the which is what is called as a particle settling flux U p s is what is called as a particle settling flux, which is basically your U p times 1 minus epsilon right. I am basically going back to the superficial you know concept right. So, if you want to get the superficial settling velocity U p s should go as U p into 1 minus epsilon.

Therefore, I can write this as U T into 1 minus epsilon times epsilon to the power of n ok. And I can talk about the dimensional flux ok, which is U p s divide by U t would go as 1 minus epsilon into epsilon any, any, any issues that is right. So, I am basically going back to the superficial velocity concept ok.

$$\frac{U_{Ps}}{U_T} = (1-\epsilon)\epsilon^n$$

Now, it turns out that if I basically plot you know this U p s divided by U T as a function of epsilon ok, it turns out you know it basically goes through a maxima ok, and then it starts decreasing and then there is an inflection point here and then it again goes on decreasing further. You can you can do that ok.

What you do is you basically take this expression vary epsilon from 0 to maximum is going to be 1 right. If you do that ok, you basically get a plot like this. So, this the maxima and the inflection point, I can get it by basically differentiating and equating to 0, and then I can double derivative right. So, if I do that, this maxima seems to appear at a concentration where your epsilon goes as n minus 1 divided by n plus 1 sorry that is going to be n divided by ok. And this inflection point seems to occur at n minus 1 divided by n plus one ok.

So, so this plot is basically the dimensional settling flux where U p s is your superficial particle velocity right that has a velocity which is based on the entire cross sectional area divided by U T, which is the settling velocity under the free settling conditions right that is U p s by U T as a function of epsilon ok. If I do that the maxima appears to be at n divided by n plus 1, and your inflection point appears to be n minus 1 divided by n plus 1.

If I substitute for n for the case where we put n is equal to 4.5 right; if you do that it turns out on to this maxima corresponds to 0.177 ok. And this inflection point corresponds to 0.35 epsilon is 0.35 ok. So, the reason why I want to do this is because the type of settling that we talked about ok, whether you have a sedimentation type sedimentation 1 or sedimentation type 2 that basically depends on your initial concentration in the dispersion.

Therefore, so these numbers they basically define some kind of limits where people are going to see a very different kind of settling behavior depending upon the initial concentration that you have in the dispersion ok, yeah. Is it ok, any questions? So, all that we have done is we took this you know the general form of you know U p goes as U T power epsilon power n, so we took that ok. And from that we basically obtain what is the superficial settling velocity of the particles ok, which basically is U p times 1 minus epsilon right. And I substitute for U p from here which is U T times epsilon power n. I get this expression.

All I have done is I have done the first and the second derivatives equate that to 0, it gets me access to this at this point. From that I can basically get some values of epsilon that basically tells you something about the different settling behavior that we saw in type 1, type 2, ok, it basically corresponds to concentrations where the concentration is typically less than point 0.177, the type 1.

But whereas, at a higher concentration you would have you know a type 2 sedimentation ok. So, that is the reverse side because you know this is the higher concentration right your epsilon is smaller means your particle concentration is higher right. So, you will get type 2 sedimentation in this regime. And whereas, in this regime so basically from here to here you will have a type 1 sedimentation ok, whether you have a zone of variable concentration formed or not ok, it would depend on the concentration that you work with right ok.

So, now if you go back and take a look at these let me just go back to ok. So, if you look at if you look at typical sedimentation test right, you it so happens you know you will always come across cases where you have a sharp interface ok, where you have a sharp interface, and that sharp interface separates zones of different particle concentrations ok.

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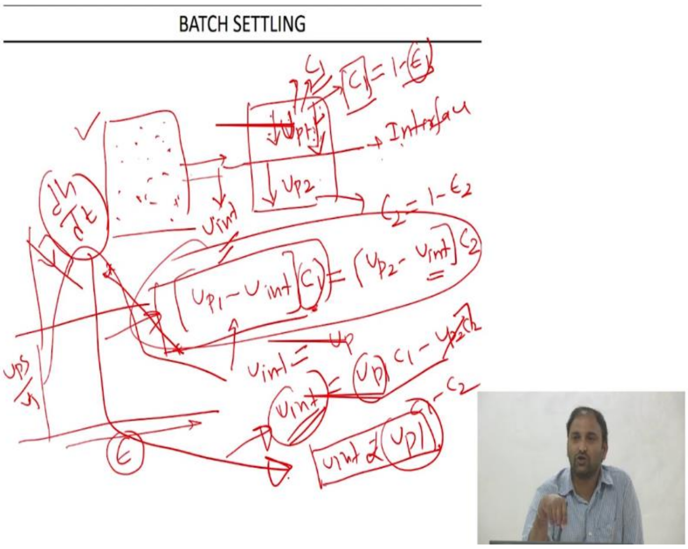
BATCH SETTLING

So, what I mean by that is let us take a simple case I say that you know I have a case where I start with you know a certain number of particles here ok. Say that at some point in time, you have a clear interface that is my interface ok. Now, you have particles of some concentration here; say that the concentration here is C 1 that basically is going to be 1 minus epsilon 1 right you. I am going to define concentration as in terms of your epsilon is your liquid fraction 1 minus epsilon 1 will give you the solid fraction. I am going to denote that as C1 in this case.

So, there is going to be some particles here say that the particle setting velocity is here is U p 1 say that. So, you have started with a dispersion of some concentration. And then as a as I observed the you know the contents ok, I see a case where there is a interface form that is a clear interface between your two regions, where in one region the concentration is C 1, the in the other region the concentration is C 2, which again is 1 minus epsilon 2 ok. And say that you know your interface is basically moving down with the velocity U int that is an interface velocity right.

Now, if the particle settling velocity is U p 1 in the above region, and if the particle velocity is U p 2 in the below region it turns out that I can think about a simple mass balance ok. That mass balance typically reads something like this. U p 1 which is the particle settling velocity in the region where the concentration is c 1 minus U interface that will give me the relative velocity of the particles with respect to the interface multiplied by the concentration of the particle that you have in the above zone that should be equal to the U p 2 minus U interface which again is the relative velocity of the particles in the bottom zone as a function you know related to the interface velocity multiplied by C 2 ok.

$$[U_{P1} - U_{int}]C_1 = [U_{P2} - U_{int}]C_2$$

So, all that you are doing is for the condition is for you to get a clear interface ok, for you to get a perfectly sharp clear interface what is the concentration of the particles that go that that reach the interface from the above should be same as the concentrations of the particle that leave the interface from the two different zones ok. This is basically if you talk in terms of this is basically the relative velocity right times if you look at this concentration right, it is effectively a mass balance ok. Mass flux that is coming down should be same as the mass flux that is you know going down ok. Now, from this what I can do is I can actually get an expression for U int ok that basically goes as U p 1. So, U int goes as U p 1 C 1 minus U p 2 C 2 divided by C 1 minus C 2 ok.

$$U_{int} = \frac{U_{P1}C_1 - U_{P2}C_2}{C_1 - C_2}$$

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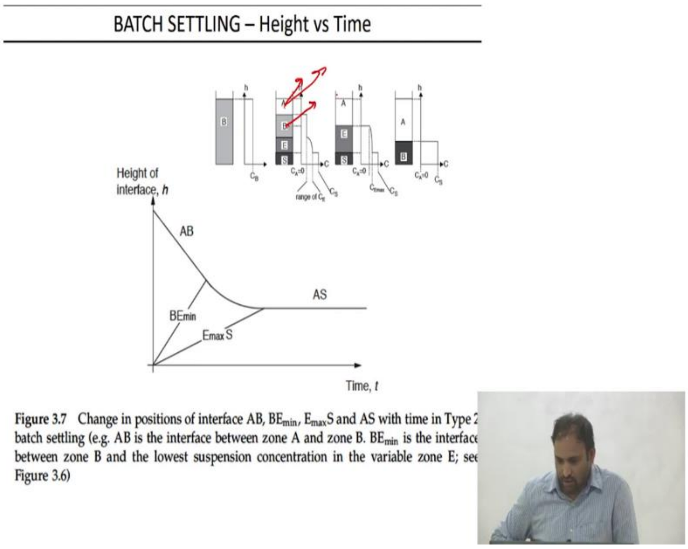

BATCH SETTLING – Height vs Time

Figure 3.7 Change in positions of interface AB, BE_{min}, $E_{max}S$ and AS with time in Type 2 batch settling (e.g. AB is the interface between zone A and zone B. BE_{min} is the interface between zone B and the lowest suspension concentration in the variable zone E; see Figure 3.6)

Now, if you go back to the cases where I had a region between the clear fluid and the zone of some concentration ok, I know that one of the concentration is 0 right. I know that the constant one of the concentration is 0 right. Therefore, therefore, your U int would be proportional to your U p 1, because if I have cases where I have a zone or an interface which separates a fluid with no particles, and a fluid with particles ok. In such a case my one of the concentration is 0. Therefore, if you just monitor U int that is the interface velocity I can actually directly calculate what is the particle setting velocity.

$$U_{int} \propto U_{P1}$$

So, so therefore, basically you have if you have cases where you know there is a clear sorry about that. So, so if you have cases where there is a interface between the clear fluid and zone where you have particles of some concentration ok, I can use this formalism that you know the settling flux or the mass balance if I do if I do it, if I get an expression for U int right, I can actually calculate what is the settling velocity just by monitoring the interface velocities right.

You know if you look at the case right I said that the interface was basically decreasing. All I do is I take this slope that d h by d t will give me the velocity with which the interface is basically falling that is exactly same as you know I can relate that to the velocity of the particles are basically settling ok. Is it ok? Yeah; so, yeah.

Yeah this is only when C 2 is 0 ok. But what people do is now I can I can actually write this in terms of, but of course, you know if you so when we did this U p s by U t right as a function of epsilon then when we did ok. And epsilon if I you know I just use epsilon here, but you know epsilon would be changing as a function of you know depending upon you know the your sedimentation that is a occurring right, your epsilon would be changing all the time right.

Therefore, what people do is you know we did that that the flux plot right, then you know you can actually use the I can take two points I can get a tangent. And from the slope there are ways of actually calculating what is your U interface and relating that to interface velocity ok. We will not going to the details of that. But the point that I want to make is that you know whenever in the experiment you are going to do in the next semester, you basically follow the interface between the clear fluid and the you know and the bottom fluid with some particle concentration. And because you are basically measuring U interface that will be a nice way to back calculate your settling velocities ok.

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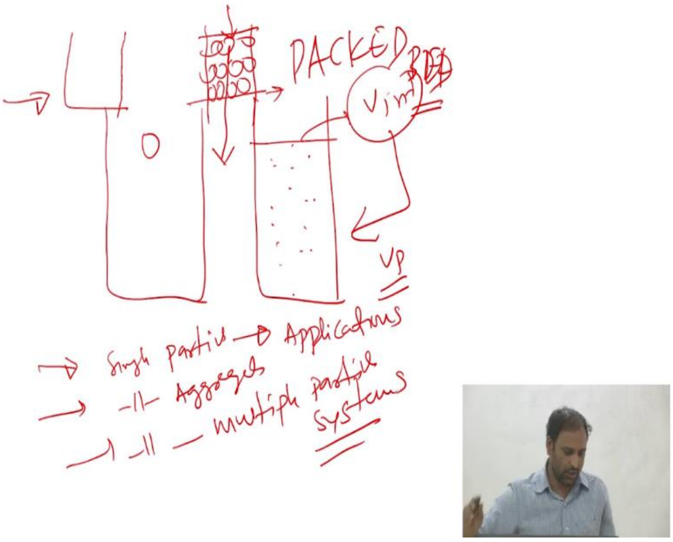

This is going to be useful because see if you are working with like say system where I can observe the particles and looking look at their you know position and find their velocities right. If I have a way of doing that you know my job is easy, but you know if you have cases where you have very fine particles, if the concentration is really large in such cases you know how do I calculate the particle setting velocities ok. So, the only way of doing that would be basically by monitoring this U interface and then from that you back calculate what is your the settling velocities ok.

Maybe I will stop here and the I think with this we are going to stop with you know the concept of so we have basically looked at three concepts settling of single particles ok, and implications of that in terms of applications right, and then we looked at settling of aggregates ok, and then we look at multi particle systems right that is what we have done so far ok.

So, what I am going to do in the next class is to look at a case where what you do is you basically have a like say a pipe for example, ok, and what we are going to do is I am going to have a support plate, and then I am going to fill this pipe with particles ok. And then you are going to look at how does you know the fluid flow occurs in cases where I have a container which is filled with particles, and we would like to look at you know the flow through what are called as packed beds right is what we are going to look at in the next class ok. So, we will do that on Friday, yeah.