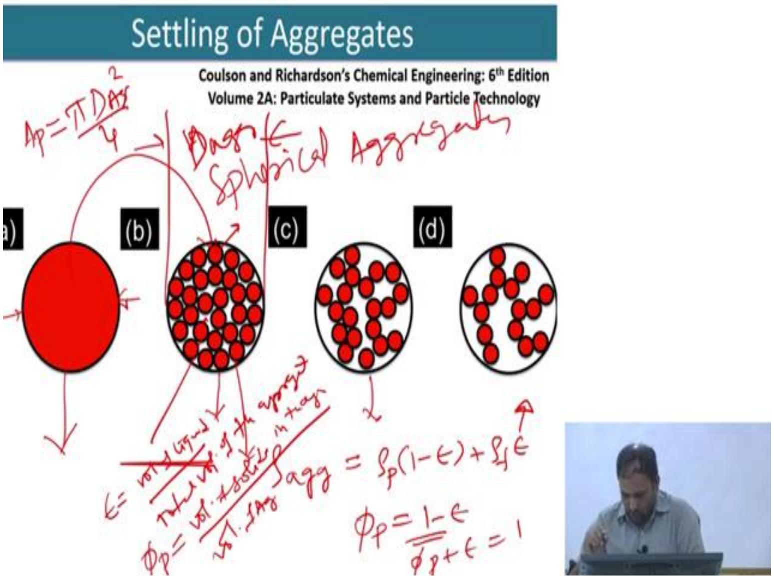
Fluid Mechanics
Prof. Madivala G. Basavaraj
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 39
Settling of colloidal aggregates - free settling

(Refer Slide Time: 00:12)

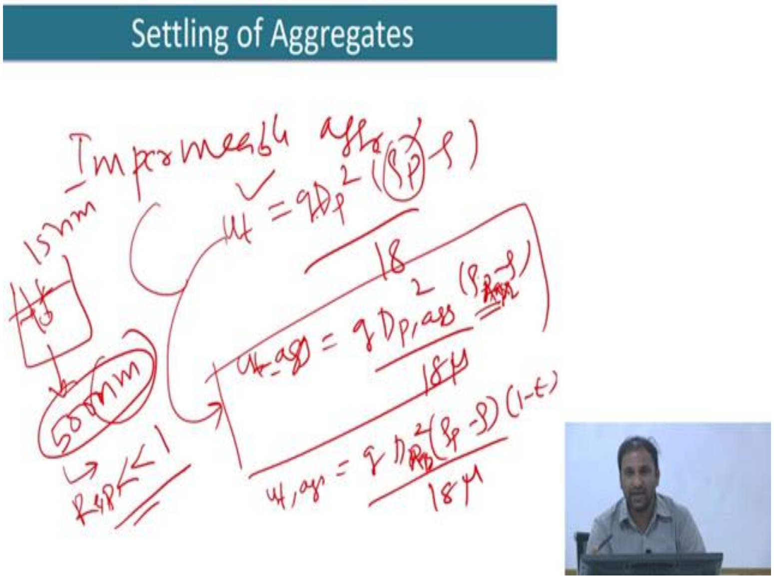


Of course we are not going to do that we are going to do a very simple case of we are going to assume that we have looked at cases where you know I have a particle that was settling right. Now, we are going to take a case, I am going to say that you know now I am going to assume the aggregates to be still spherical. And now this spherical aggregate is it has particles and we said that you know the properties of the aggregate is typically in between the properties of the fluid and the particle right. In that sense you know we basically defined a quantity called rho aggregate which basically goes as rho of the particle into 1 minus epsilon plus rho of the fluid into epsilon right epsilon is something called as a liquid fraction right.

If I have these particle that are solid you know here and there are some vacant spaces right. So, epsilon is basically the volume of liquid in the aggregate divided by the total volume of the aggregate ok, that is your epsilon that is a volume fraction of the liquid in the aggregate.

Similarly, I can also define a solid fraction right, so phi p if I say solid fraction this is a volume of the solids you know or the particles in the aggregate divided by the volume of the aggregate right. I can define that and they are related by the fact that you know your epsilon is 1 minus epsilon phi, because your phi p plus epsilon should be 1 right the total fraction of the solid in the you know aggregate plus the total fraction of the liquid in the aggregate should be 1 right.

(Refer Slide Time: 01:58)



Now, and we said that we are going to take a case of impermeable aggregate; we said we are going to take a case of a impermeable aggregate. And for that so, basically there is no exchange of fluid between the aggregate and the fluid right you know in terms of. So, what that makes is that you know I can still assume if I as; so what it basically means is that if I go back to this case right, if I assume that the aggregates are impermeable ok. So, I do not have to worry about you know this fractal dimensions and stuff like that because of the fact that you know your A P is a projected area right ok.

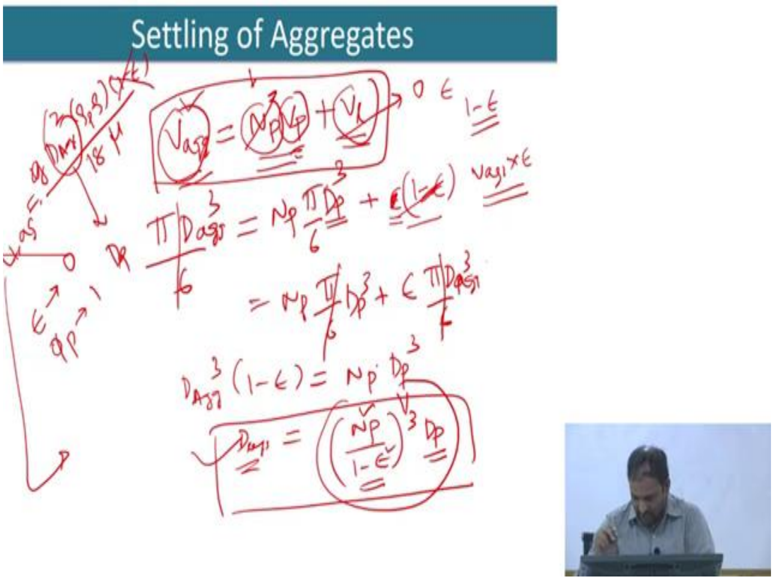
Now, if I instead of a solid sphere settling I am going to assume that there is an aggregate that settling and if I have some way of measuring you know the diameter of the aggregate and the projected area that you know it would still be pi D aggregate square divided by 4 right by pi D square by 4 right.

So, therefore, what I can do is I can actually whatever expression that we developed for you know some of the settling regime that we talked about right g D p square rho p minus rho divided by 18 mu this was for an isolated particle that was settling I can actually use the exact same formulation that is ut aggregate. Now, is going to be g D aggregate square into rho p minus rho divided by 18 mu again this is for the case of isolated aggregates right.

And I gave an example as to you know why I could still go ahead and use the Stokes settling expression because if I take like say 15 nanometer particle ok, I said if you make even an aggregate which is say 500 nanometer in diameter. If I do that I said you know the fact that you know the particles, the aggregate that you are getting is still in the you know in the nanometer size range your Reynolds number if you calculate right R e P would still be much much less than 1 therefore, I could still go ahead and use this you know. So, this is where we had stopped yesterday right.

Of course what I can do of course this is not rho p right, this is rho aggregate right, this is rho of it is not rho p anymore right. So, you this is a density of the particle in this case this is going to be rho of the aggregate right and we said because I know that you know rho of the aggregate can be expressed in terms of the density of the fluid and the particle you can write this as ut aggregate is equal to g D p square into rho p minus rho into 1 minus epsilon right divided by 18 mu that is where we had stopped yesterday right ok.

(Refer Slide Time: 05:08)



So, now I want to simplify this further again here this is aggregate right this is D of aggregate. So, if I want to simplify this further what I can do is I can actually take. So, your volume of the aggregate is the number of particles in the aggregate multiplied by the volume of the particle plus the volume of the liquid right is that is it ok. So, all I am doing is you have a you know aggregate of certain volume and that should be equal to the number of particles that you have in the aggregate multiplied by volume of the aggregate this basically gives you the total volume of the solids in the aggregate and this V l is the volume of the liquid that you have in there right.

$$V_{agg} = N_P V_P + V_l$$

So, now what I can do is this is pi D aggregate cube by 6 right and that is number of particles and volume of the particle is say pi D particle right. So, that is the you know the dimension of the particle that make the aggregate right plus the volume of the liquid I could write this as the volume fraction ok, I could write this as because I know can I express this V l in terms of V aggregate, I can do that right. So, what I can do is I can basically write this as if I say that 1 minus epsilon would 1 minus. So, epsilon is the liquid fraction right 1 minus epsilon is the solid fraction right. So, therefore, if v agg multiplied by epsilon would give me what is the volume of the liquid right, yes now yeah. So, therefore what I can do is I can write this as pi by 6 into D p cube plus epsilon times pi D p cube by 6 into sorry D aggregate cube into that is it right that is the let me just check yeah is it ok.

$$\frac{\pi D_{agg}^3}{6} = N_P \frac{\pi}{6} D_P^3 + \epsilon(1-\epsilon) = N_P \frac{\pi}{6} D_P^3 + \epsilon \frac{\pi}{6} D_{agg}^3$$

So, therefore, what I can do is; so this gets cancelled right your pi by 6 gets cancelled everywhere. So, therefore, I could write this as I can D aggregate power 3 into 1 minus epsilon it is going to be N p into D p cube therefore, I could write this as D aggregate you know is equal to N p to the N p by 1 minus epsilon to the power of one-third into D p is it ok.

$$D_{agg}^3(1-\epsilon) = N_P D_P^3$$

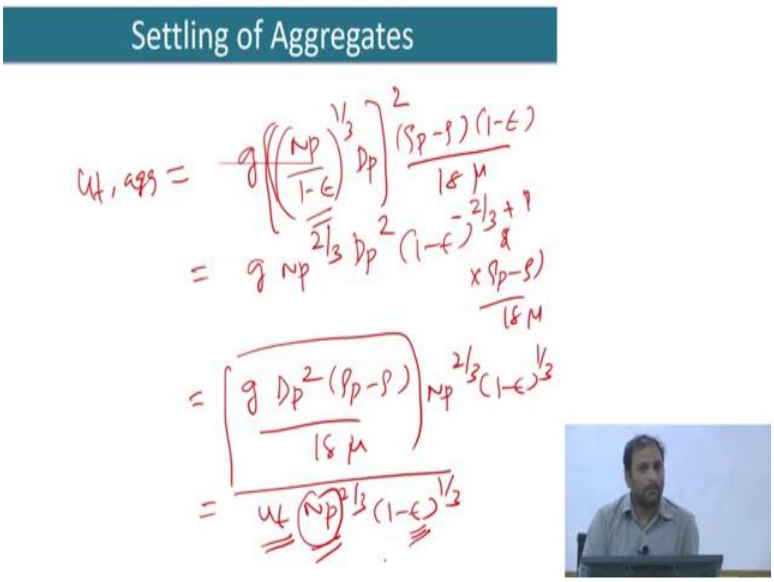
$$D_{agg} = \left(\frac{N_P}{1-\epsilon}\right)^{1/3} D_P$$

So, therefore, I am able to express the diameter of the aggregate in terms of the number of particles that you have in the aggregate times you know the volume fraction of; so your epsilon which is a void people also call it as you know fraction of the liquid, people have called it as void and things like that and D p is that is a diameter of the particle right. Let us think about the limiting case say right we just said that ut aggregate goes as you know g D aggregate square into rho p minus rho into 1 minus epsilon divided by you know 18 mu right

Now, if I take the limiting case of you know your what can you say about limiting case of epsilon going to 0 or your phi p going to 1, in that case if I take if I look at this expression your V l is 0 right and your number of particle is going to be 1 right. So, what I am trying to say is that this expression that we have it kind of you know becomes your single particle setting velocity right because your epsilon has 0 therefore, this term goes right number of particle is 1 here therefore, your D aggregate is basically same as D you know particle right.

So, essentially I mean you know whatever formalism that we wrote up here you know that is no extending whatever that we had for single particles to aggregate. It does seem you know it does make sense because I am able to recover that limiting you know case of single particle settling ok. Now what I can do is I can actually substitute for D aggregate in terms of you know this quantity right into the settling velocity expression.

(Refer Slide Time: 09:53)



Therefore your ut aggregate would go as g into instead of D p; D aggregate square I am basically going to put up N p divided by 1 minus epsilon right to the power of one-third right that is what I had into D p whole square right. And of course, there was rho p minus rho there was 1 minus epsilon divided by 18 mu right that is what you had. So, therefore, this goes as g into N p to the power of two-third right and I have D p square right and there is 1 minus epsilon and that is to the power of two-third right minus two-third plus 1 because that is in the denominator right and there is a one-third power here and there is two here therefore, if I take it up it becomes minus 2 by 3 plus 1 ok,

And of course, you have and that multiplied by your rho p minus rho divided by 18 mu therefore, if you sum simplify this it becomes g D p square into rho p minus rho divided by 18 mu into N p to the power of two-third into 1 minus epsilon to the power of one-third ok. And therefore, that is a single particle settling velocity that is your ut itself times N p power two-third into 1 minus epsilon to the power one-third ok.

$$u_{t,agg} = g\left[\left(\frac{N_P}{1-\epsilon}\right)^{\frac{1}{3}} D_P\right]^2 \frac{(\rho_P - \rho)(1-\epsilon)}{18\mu}$$

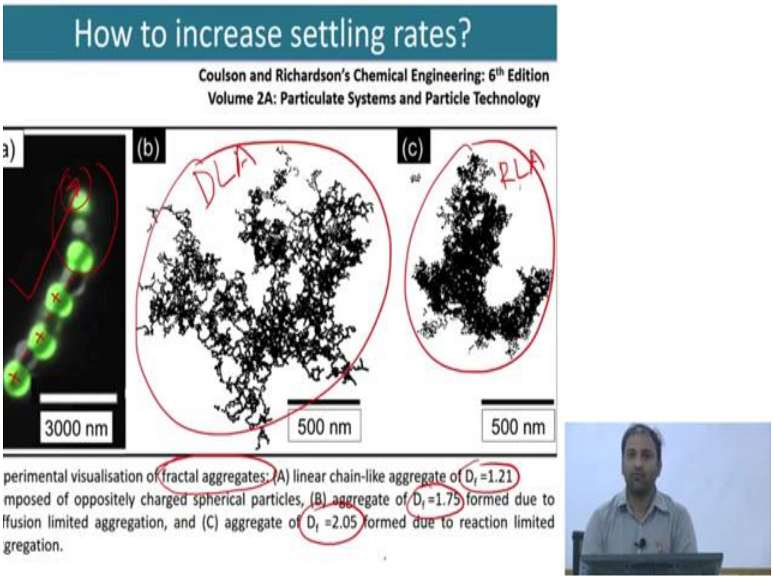
$$= gN_P^{\frac{2}{3}} D_P^2 (1-\epsilon)^{\frac{1}{3}} \frac{(\rho_P - \rho)}{18\mu}$$

$$= \frac{gD_p^2(\rho_P - \rho)}{18\mu} N_P^{\frac{2}{3}} (1-\epsilon)^{\frac{1}{3}}$$

$$= u_t N_P^{\frac{2}{3}} (1-\epsilon)^{\frac{1}{3}}$$

So, what you are able to do is I mean of course, this is a very simple formulation, you are able to express the settling velocity of the aggregate in terms of the settling velocity of the individual particles. And you know and this expression does make sense because you know that as a number of particle becomes larger and larger of course, your settling velocity has to go up right that is what it basically captures and it also turns out that it basically also depends on the porosity of the aggregate ok. If you have some way of measuring you know the number of particles in that aggregate and if you have some way of measuring what is the porosity of the aggregate you can basically go back and look at this equation you know have a way of calculating your settling velocities ok.

(Refer Slide Time: 12:12)

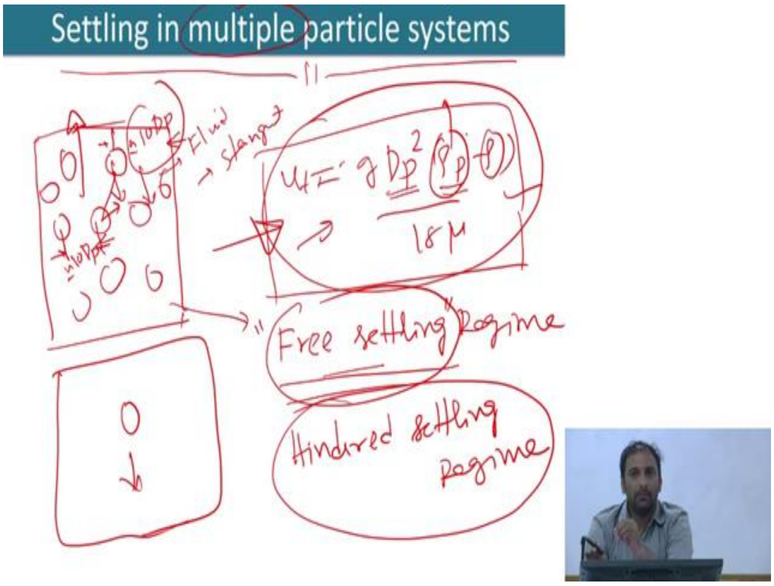


But in terms of you know making particles of you know the aggregates of this kind and then doing settling is very hard because first of all making aggregates of controlled size is very difficult and even if you are able to make it can I make aggregates of that look exactly identical right because somebody was asking a question yesterday, if you have aggregates of different sizes then you know it again is more complex right. First of all working with aggregate you know a single aggregate itself is tough because you know the your a p is different, porosity is all of that right, but now if I have a collection of such aggregates of different morphologies you know.

So, it becomes very hard to you know kind of solve this problem, but there is a lot of interest in looking at settling of aggregates because this is you know very much important in the case of application right. I said that you know in the case of you know water purification or you know you look at any other lot of colloidal processing when people do you add radius to make aggregates and then you know the way they behave in a fluid is important in the context of applications ok.

So, that is about I just want to give you a kind of a heads up as to how does one think about you know if you have instead of a nice spherical looking particle you know, if you have complicated objects like this how does one look at aggregation that was the intention ok.

(Refer Slide Time: 13:28)



Now, we are going to look at any questions; any questions so far. So, we will look at start looking at a multi particle systems ok. So, I am again going to draw the classic picture, so I have a column or maybe like say I have a container and say that I have a 1 particle that is in a fluid say that you know your fluid is stagnant right that is again I am going to write the same formula right.

Now, say that I am going to start working with multi particle ok, I am going to put another one more, but another one more, but another one right. If I go on increasing the concentration of the particles in the system can I again use a similar formulism that we had for you know single particle settling ok, any thoughts.

Do you think that whatever formalism that we have had for the single particle settling can be used for a multi particle system. Yes, now it turns out that it can still use it, but under certain conditions ok, if you are working under very dilute condition if the distance between the particles in the fluid if it is approximately 10 times the particle diameter and if the distance between the wall and the particle, if it is again approximately 10 times the particle diameter. It turns out that you can assume the particles would still be kind of you know flowing in a fluid in a very similar way as you know a single particle you know flowing ok.

And this is something called as a free settling regime ok; something called as a free settling regime ok. Even though there are multiple particles in the system the behavior of each particle or one particle does not have any influence on how the other particles would settle or the presence of the boundaries of the container does not affect, does not influence in any way how each of the particles would you know would kind of behave in the in the presence of an external field and that is what is called as a free settling regime right.

Now; however, if and as I said right a typical you look at any system right I talked about you know a viscosity measurements right. When people talk about viscosity measurements I said that there are something called as a rheometers and I said that you know there are different geometries right I talked about in a parallel plate geometry, cone on plate and you know co ed geometries right. When people and in all these measurements, what people do is people typically maintain a gap right there is a gap between you know two like say place for example, and one kind of comes up with a question is you know what is the gap that I should maintain ok.

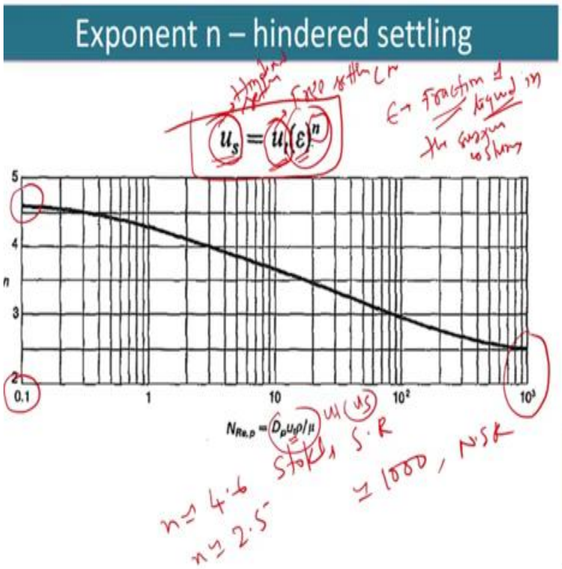
And even in such cases a typical rule of thumb is that you know you should ensure that the gap between the particles is at least about 10 times the dimensions of the object that you have in the fluid ok. That means, if I am working with like say 10 micrometer particle in a fluid and if I were to put them between two plates and if you wanted to do rheological measurements. The typical you know rule of thumb is that the distance between the you know plates has to be at least about 10 times the particle diameter ok.

In that case I would at least maintain a gap that is more than a 100 micrometer because the particle size that I am working with this 10 micrometer right ok. Therefore, but the similar kind of things also kind of works in the case of your free settling cases as long as the distance between the particles is you know larger than you know 10 times the particle diameter that is when you have this free settle ok.

Now, when you go on adding more and more particles into your system ok, what will happen is that now as the particles come down it is going to display some fluid right. So, I you know I do not have this case if you know quiescent or static fluid anymore ok, the liquid is going to go up right because as the particles come down it will display some fluid and that fluid is going to go up and your drag for single particle does not apply anymore because you know the fluid around the particle is also going to be in motion right.

So, therefore, then you would have to think about you know kind of modifying you know whatever formalism that we had for the single particle case in terms of you know using it for the multiple particle system ok. And so, when the motion of the particle is kind of influenced by the wall or the other particles in the system people use a term called hindered settling ok. When you have a large number of particles in your system ok, if one is coming down if there are other particles which kind of you know influence the drag that the particle you know experiences ok. So, in such cases you say that you know the settling is occurring under what is called the hindered settling conditions.

(Refer Slide Time: 19:44)



And what people have done is, people have come up with some very simple you know kind of empirical relationships what are you looking at is a case this is u s is something called as a hindered settling velocity that is the settling velocity in the multiple particle system ok. And it goes as the settling velocity of the single particle that is under this is under free settling condition, this is under hindered settling condition and that depends on epsilon which again is the fraction of liquid in the suspension or a slurry right ok.

$$u_s = u_t(\epsilon)^n$$

And this n is an exponent and that depends on the again the flow regime that people work with ok. If I take like say limiting case of you know there is very low Reynolds number typical value of n is about you know 4.6 that is you know for Stokes you know settling regime. And if you look at you know a typical case for you know Reynolds number that is above of the order of in a 1000, that is your Newton settling regime, it turns out you know the value of n is of the order of 2.5 ok

People have kind of done a lot of experiments and they have kind of come up with this empirical relationship, that the settling velocity under hinder settling condition basically goes as the free settling velocity multiplied by you know some function epsilon to the power of n where epsilon is the fraction of the liquid that you have in the system yeah.

What is that n that I calculate? Correct. So you are talking about here right ok. So, before I comment on that, what can you say about the settling velocity under hinder settling conditions, do you think it will be lower or larger than?

It is going to be lower right, yeah it is going to be lower therefore, you know I would still be under the; so if I were to say that you know I am under the. So, typically if you look it up let me just do a in one of the examples we have a case where we kind of calculate the free settling and hindered settling velocities of course, it is slowed down, but not by a large ok. So, so in that context you know I mean whether you take, so ut here or u s is not going to shift your you know Reynolds number by a large. Maybe we will comment on it and when we do that.

So, now, I just want to think a little bit about you know how do we get this ok. So, as I said this is a people have done a lot of experiments and they have kind of come up with this empirical relationship, it turns out know you can actually derive it ok. So, that is what we are going to do for the rest of the class.

Again starting point would again be assuming a certain settling regime you know I can work it out by assuming that to the particle settling in Newton’s regime or I can basically consider the Stokes regime. So, starting is going to be again this equation right. Now, I have this equation and I say that this is applicable for Stoke settling velocity, Stoke settling regime ok.

Now, because I said that your settling velocities under hinder settling are going to be lower than the settling velocity under you know free settling conditions. If I calculate the Reynolds number it will turn out that you know your Reynolds number for the hinder settling condition would still be in the Stoke settling regime right. So, if I say that look I can take this expression and I can modify it for the case of hindered settling, what do you think should be the modification?

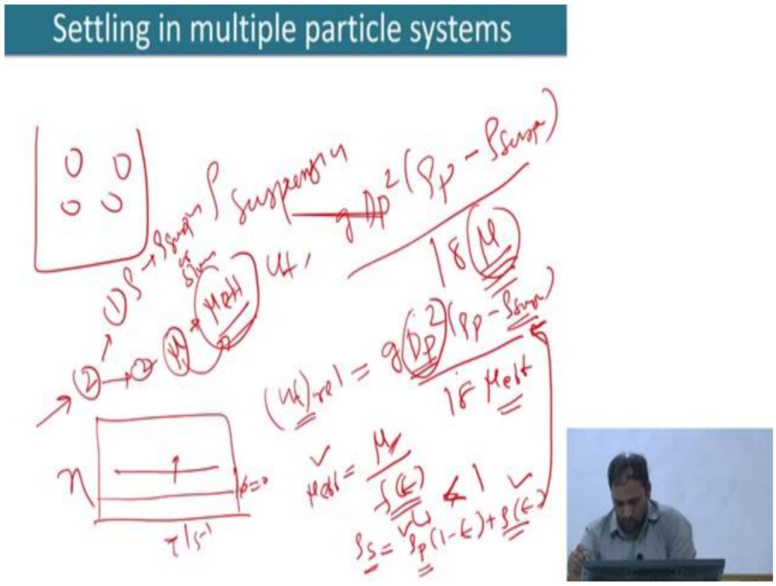
So, if I go back and look at you know the settling of the aggregates right I said that look I want to use the same expression, but I want to look at aggregates. I said that you know I would have to change D p into D aggregate that is what we did right and I said you change your rho p into rho aggregate that is a modification that we did right.

Now, if I want to again use a similar concept, but I would like to modify this for the settling in the hinder settling regime. What kind of modifications should I do for this expression? Yeah, one has to worry about these interactions, but you can still I mean I do not have to go for you know I mean; so, what he is trying to say that you know if I have a dispersion of particles say that case 1, case 2. In in case 1 you have like say neutral particles they are not charged and I take case 2 where I have exactly identical particles same number of particles in the fluid. In the case 2 there is a you know electrical double layer interaction, there is a repulsion between the particles. You know the question is the settling in this case and the second case are they exactly identical ok.

So, you can it turns out that you know I can club all of this into some effective parameters, I do not have to really worry about you know the inter particle interactions I do not have to worry about the charges and the Van der Waals force everything. I can still use a similar expression, but instead of using the properties of the fluid, I would have to use the properties of the suspending medium sorry let me just say what I mean ok.

All you have to do is now we said that your rho, I was going to take as rho of aggregate in the settling of aggregates right. Now, that you know I have a fluid now it is not rho p right your rho right your rho is the density of the fluid that does in which the particles are embedded right. I mean now what the particle effectively sees is not just the fluid, but the fluid plus the particles right. In that sense what I have to do is my now rho which is the rho of the fluid right or the rho of the suspension ok.

(Refer Slide Time: 26:31)



So, what I mean by that is instead of using rho which is the density of the fluid you would have to use the rho of the suspension ok. Therefore, one of the modification is your g D p square into rho p minus rho of the suspension divided by 18. Now, your mu right in the case of single particle settling mu is the viscosity is the fluid, but now I have a fluid plus the particles, I would have to worry about not mu of the just the fluid, but mu of the a particle filled system right. Therefore, this, the modification that I am suppose to be making is two modifications; one is you replace rho of the fluid with rho of the suspension or the slurry if you want to call it as ok. The other modification is that you replace your mu with what is called as a mu effective which is the viscosity of the; not the just the fluid, but the particle filled fluid right.

$$\rho \rightarrow \rho_{suspension}\ ; \mu \rightarrow \mu_{eff}$$

And we kind of had an example in the previous class where we looked at you know how does the viscosity changes the function of maybe like say shear stress or shear rate right as a as a concentration of particle right. So, we said we had a case where you know the particles the addition of the particle you know did increase the viscosity right, if you have phi is equal to 0 add a little bit of particles the viscosity goes up right.

In general the viscosity is known to increase with the addition of the particle therefore, your mu is should be replaced with mu effective where mu effective is the viscosity of the particle filled fluid system right. So, therefore, your ut now you are going to call it as ut relative, I will come back in a minute as to why we call it as ut relative is g D p square into rho p minus rho of suspension 18 mu effective.

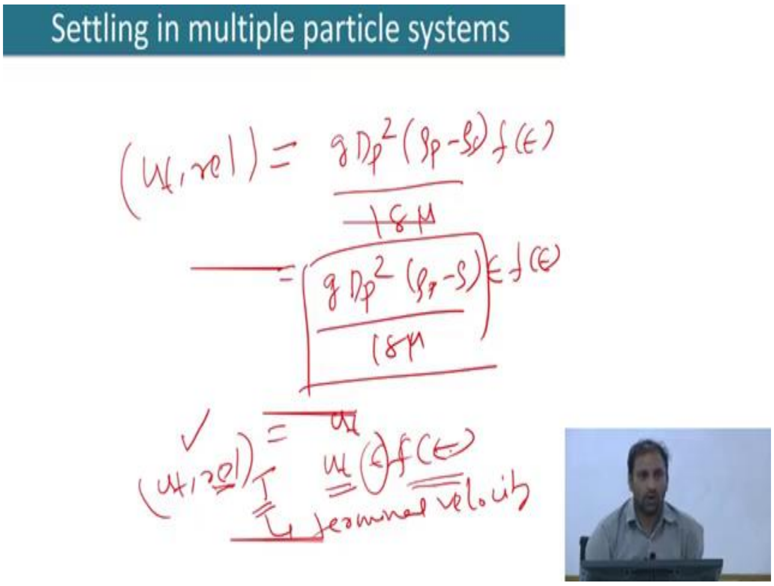
Right, because you know I do not have to worry about particle sizes right because you know I am assuming that you know you have lot of particles you know I am, you know when you have multiple particle system you know your D p is exactly same right. I do not unless I am making the particles cluster together; you know I am, unless I am making the particles larger by aggregation I would have to worry about you know changing this. But the fact that you know the particles remain as individual units and they continue to settle. So, I do not have to worry about this. So, I am going to keep it as the same and then I am going to change you know this and this right.

And as I said this mu effective it is typically written as mu times some function and this function f of epsilon is typically greater than, it should be less than 1 right because the mu effective is typically larger than the viscosity of the fluid itself right.

So, therefore, and your rho which is the rho of suspension I can write it as rho of suspension, I can write it as rho of the particle into again 1 minus epsilon plus rho of the fluid into epsilon right I can again similar formalism right. The density of the dispersion is basically you know some kind of a average of you know the kind of contribution that comes from the particle and the contribution that comes from the fluid right.

$$\rho_S = \rho_P(1-\epsilon) + \rho\epsilon$$

(Refer Slide Time: 30:34)



So, if I substitute these into this equation, it turns out that your ut relative would go as g D p square into rho p minus rho ok, I had rho of suspension they divided by 18 your mu effective it becomes f of epsilon here. If I work it out it becomes g D p square into rho p minus rho into epsilon into f of epsilon divided by 18 mu that is your single particle settling velocity that is your ut right, that is your sorry that is your ut into epsilon times f of epsilon ok. I am going to put T here because that is going to be again terminal velocity ok.

$$(u_{t,rel})_T = \frac{gD_p^2(\rho_P - \rho_s)}{18\mu} f(\epsilon) = \frac{gD_p^2(\rho_P - \rho)}{18\mu} \epsilon f(\epsilon) = u_t \epsilon f(\epsilon)$$

Now we are going to so, base therefore, your relative terminal velocity under the hindered settling conditions when you have multiple particle of the system basically goes as ut which is the velocity, you know terminal velocity under free settling conditions times epsilon which is the fraction of the liquid that you have in the dispersion multiplied by some function f of epsilon. So, we will talk a little bit about what this function is you know it may be in the next class ok. I will just stop here; if you have any questions I will take them otherwise we will meet tomorrow at 1 o clock you know.