Fluid Mechanics
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Lecture - 37
Settling of Colloidal Aggregates

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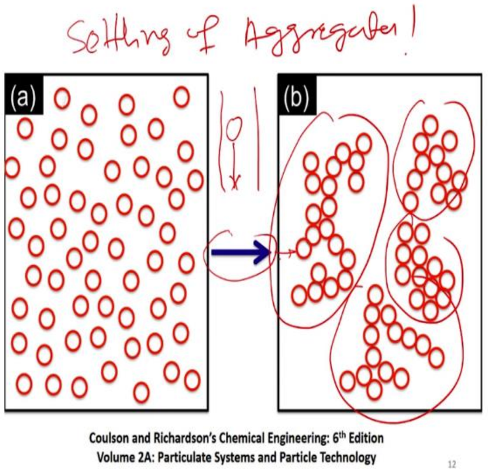

So, now that I have this case right. So now, I would like to look at the objectives of you know the reason why I have kind of given you a brief background of you know inducing aggregation is because I would like to look at settling of aggregates. So, far we were looking at you know a simple case of you know I have a spherical particle settling in a fluid a stagnant fluid, now I would like to look at settling of aggregates. So now, if you ask me a question is to how these aggregates form whatever background that I have given so far would help right ok.

So, you start with a dispersion stable dispersion add additives and you basically end up with this contradiction right. I would like to think about settling of aggregates. So, what kind of formulation do you think I should use do you think whatever we developed for the case of single particle settling, do you think it is going to work? Do you think that whatever settling principle that we kind of you know formulated for the case of you know single particles, do you think that is going to work with aggregates?

So, it turns out that you know it is not going work ok, but, but the reason why the settling of aggregates is much more complex; number 1 is that you know you may have aggregates of different sizes in the right, I have this nice dispersion and I go from you know this state to this state right you know I form aggregates, but aggregates could be of different sizes ok. And because these are particle clusters; these aggregates can also deform right you know they may change their size; they may change their shape. It depends on you know whether these aggregates are kind of you know settling in a quiescent liquid that is that is a stagnant liquid or is it in a flow you know flowing conditions everything becomes important ok.

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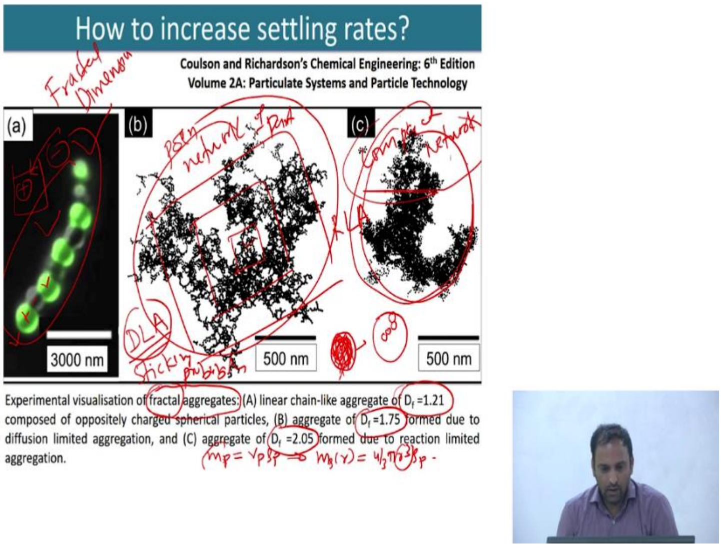

So, we are going to look at a simple and so, when it comes to the type of aggregates that are typically formed, people use a concept called there is a concept called fractal aggregates. So, what you are looking at is three pictures of typical aggregates that people kind of you know have looked at in the literature and there are some ways of characterizing these aggregate and this is basically done by using a concept called fractal dimension.

So, the reason why this fractal concept kind of is applicable here is because in a lot of cases the aggregates that that form they are kind of self similar ok. What I mean by that is if I look at you know a case b here, I look at aggregates in this small box or I go on increasing the box size right. It turns out you know if I kind of do analysis of the structure of the aggregates in terms of you know the number of particles in that box, in terms of the way the particles are arranged it will more or less look very similar ok.

So, in so, therefore, the fractal kind of concept are typically used in characterizing aggregates, what you are looking at is a case where there are three aggregates which are of very different morphology right. In one case, it looks more like a linear right aggregate; in the second case, it looks like a more like an open network of particles and in this case is a more compact network right. What people do is, we know that if you takes say mass of the particle goes as volume of the particle into a density right.

Now what I can do is I can say mass of the particle which is a function of radius basically goes as 4 by 3 pi r cube into rho p right. Think about a solid particle right think about a solid particle and if I were to start from the center if I go radially outward I can say that the mass of the particle basically goes as radius to the power of 3 right.

$$m_p = V_p \rho_p = m_p(r) = \frac{4}{3}\,\pi r^3 \rho_p$$

Now, that is the case if I have a particle which is completely or an aggregate which is completely filled with particle right or that is actually valid for a solid particle right; there is no spaces right.

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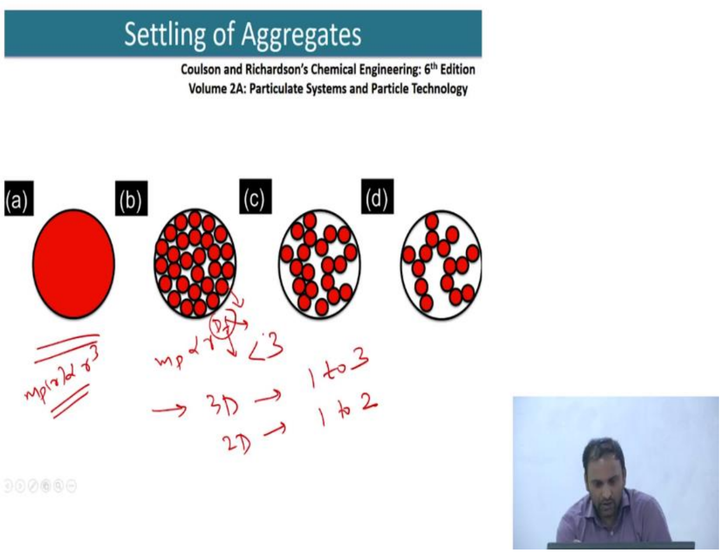

Now I go to a case where I have maybe I have a picture here right that you know your mass of the particle going as r power 3 is only valid for a completely solid aggregate right. Now the fact that you know I have a lot of vacant space here; there is a fluid that is also in the aggregate right. Therefore, your mp will go for go as r power some dimension Df where this Df is less than 3 right and this Df is what is called the fractal dimension and typical values of D, Df is that if you go for an aggregate in a three dimension, the value can go anywhere from 1 to 3; 3 is a maximum. And if you are working with two dimension, it can go from one to about two because the maximum that you can have is you know power is 2 right

If you look at the picture here, in A what you see is an aggregate of factor dimension 1.21. In B, you have a fractal aggregate which is a fractal dimension 1.75. In the case C, you have a an aggregate which has a fractal dimension of 2.05. And what kind of aggregates form and whether the fractal dimension is 1.75 or 2.05 it depends on again the kind of aggregation that happens in the system ok.

A is a case where what was done is you have a container; you have say a positive charged particle. Now what you do is you add a negatively charged particle to it ok. When that happens, what will happen is the moment there is a contact between a positive negative particle there is an there is a cluster formation maybe a two particle cluster. Now when the next one comes; it can either I mean you know of course, you know it can go out other location, but it can either go here or here right. In that sense there is a formation of a linear aggregate and that is what leads to a very low fractal dimension of 1.25 and what you see in the case B is what is called as a diffusion limited aggregation ok.

What does this mean is there a no again you have a dispersion of particles. The moment a particle comes into contact with another particle, they just aggregate ok. So, in case of diffusion limited aggregation people talk about what is called as a sticking probability and sticking probability is very very high in the case of diffusion limited aggregation. Because the moment there is an encounter between a particle and other particle or a particle in the cluster, the particle comes and attaches that is it ok.

However in the case of case C which is what is called as a reaction limited aggregation what will happen is, a particle joins in aggregate. It can also it will continue to rotate until it basically finds a more stable configuration and that leads to the formation of more kind of a compact structure ok. The reason why I am trying to say all of this is because understanding, aggregation sorry settling of aggregates is very complicated because you know you should know about the kind of clusters that form; whether the clusters are linear like this, whether they are more open structures like here or whether its a more compact structure ok. So, therefore, you would had to worry about the morphology of the aggregates, when you are trying to deal with settling of aggregates ok.

So, in the remaining class, I am just going to and I am going to use lot of assumptions ok.

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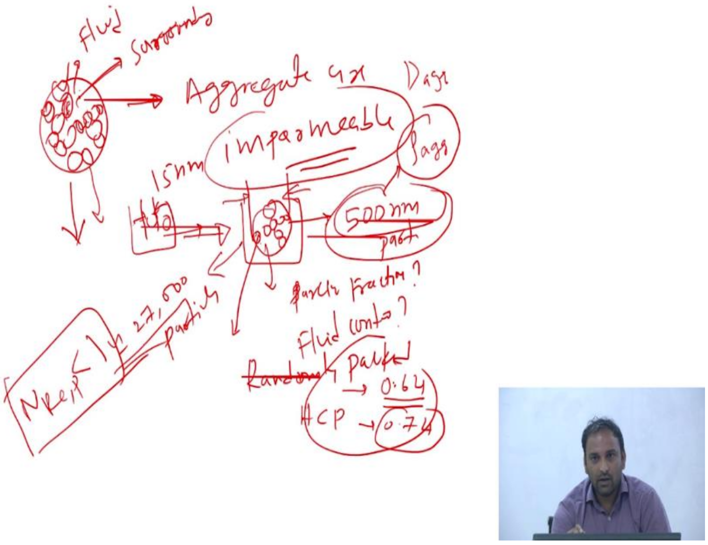

Let us say that if you want we have a an aggregate something like that ok. The particles are all connected. How would you like to look at the settling of this ok? I am going to assume that these aggregates are impermeable. What I mean by that is there is no exchange of the solvent between the aggregate and the surrounding ok. Because you know we know that these aggregates are its a composite you can think about aggregate as a composite particle right. There are particles of course, there is a fluid right. But now if I assume that there is no fluid exchange between the aggregate and the surrounding, they are what are called as impermeable aggregates.

And let us say that you know I have a simple case of a dispersion which contains say 50 nanometer particles ok. And say that I have made the particles to aggregate and I have formed a an aggregate which is like say some dimension, some example of say 500 nanometer particle. There is an aggregate of 500 nanometer you know dimension which I am able to form starting with a 15 dispersion that contains 50 nano meter particles and I have done that by adding additives; it could be salt or polymer or whatever ok.

Now, it turns out that you know even if I assume that this is a really densely packed aggregate say that what could be the maximum packing density. If I assume that you know, they are I have formed an aggregate and this is an aggregate a particle and say that, this is the most densed most dense spherical aggregate that you can think off ok. What do you think would be the packing fraction of particle in the aggregate?

Because I said that you know these aggregates are you know you have particles of course, there is a space where there is fluid as well right and I said that you know it is impermeable. Therefore, there is a particle plus there is also a fluid right. What could be the maximum fluid contained? It depends on the kind of packing that I am going to assume right.

If I say that you know the particles in the aggregate are randomly packed your packing fraction is going to be 0.64 right; that is a maximum packing density. If I say that you know somehow I have a way of making an aggregate where all the particles are kind of orderly arranged, some kind of a HCP kind of arrangement hexagonal close packing here it is going to be 0.74 right. Even if you take the most dense aggregate, it turns out that you know the typical number of particle that it may have is of the order of 27000 particle ok.

So, what I have done is I have taken a dispersion of 50 nanometer particles and then, I have made a aggregate by adding salt or you know polymer and if I assume that the density of the sorry the size of the aggregate that I have formed is 500 nanometer.

If I assume some packing density, I can roughly calculate what is the number of particles in the in that aggregate right by a simple geometric argument and it turns out that you know if I use a packing fraction of 0.74. This typically aggregate this aggregate will consists something about 27000 particle right. Now we know that any dimension which is sub micrometer size range, we know that if I were to calculate the Reynolds right your $N_{Re,P}$ is going to be definitely less than 1. Therefore, I can still assume the aggregate to be settling in the Stokes regime right.

So, right I can assume that right because you know I have made a fine; nanometer particle say you know I have a way of somehow getting what is the rho of the aggregate I have a an aggregate of like say size d aggregate and that is settling under the action of gravity and say there you know I have some way of getting what is a rho aggregate. I can actually if the fact that you know the your Reynolds numbers are less than 1. Whatever formulation that we are done for the settling of aggregates which is u t is g D p square into rho p minus rho divided by 18 mu. I can actually use exactly the same formulation I can write this for settling velocity of aggregate which is g into D.

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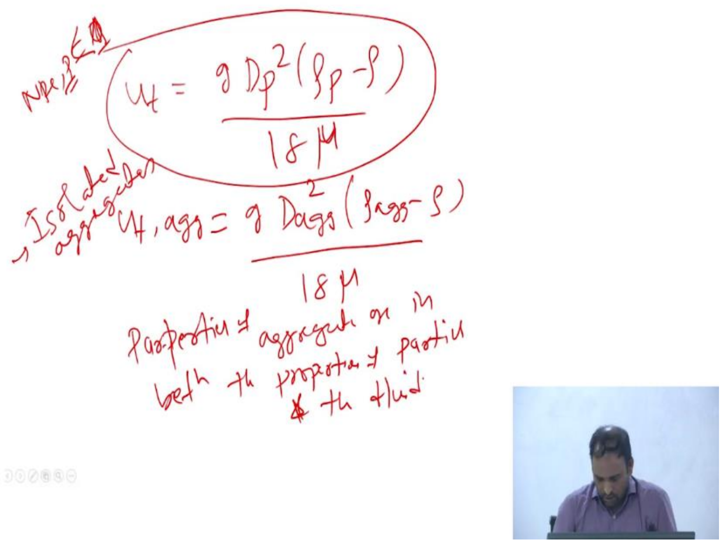

Now, aggregate dimension square into rho of aggregate minus rho divided by 18 mu I am of course, talking about isolated aggregate right settling you know in a fluid right is that ok? Any questions right?

$$u_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}$$

$$u_{t,agg} = \frac{gD_{agg}^2(\rho_{agg} - \rho)}{18\mu}$$

All that is being done is I am just saying that you know the aggregates that are formed of course, they contain a large number of particles because of the fact that the dimensions are you know still sub you know micrometer. I can still you know use the stokes formulation because you are now Reynolds number of the particle or the aggregate if you want to call it as would still be less than 1. I can actually use the same formulism that I use for settling velocity of the particle.

No you can calculate that right because I said that you know the if the particle dimensions again you know you would have to look at the particle dimension, it turns out to know if it is a 500 nanometer particle right and if I have the settling in water, if I were to take the density of water and the you know right. It turns out; if you calculate Reynolds number, it is the it will be definitely be less than one yeah ok. So, now, when you are working with aggregates, it turns out that you know the particle the properties of the aggregates are in between that of the. So, properties are in between the properties of particle and the fluid um.

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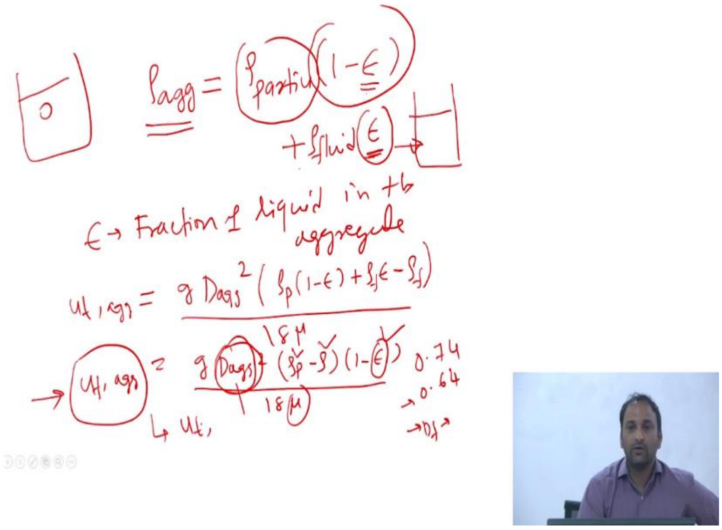

What I mean by that is your rho aggregate right which is a the density of the aggregate, I can write this as rho of particle times 1 minus epsilon I will I am going to define what is epsilon in a while plus rho of the fluid times epsilon where epsilon is what is called as a fraction of liquid in the aggregate ok. That is a fraction of liquid in the aggregate that is a that gives you the liquid fraction and 1 minus epsilon will give you the solid fraction ok.

$$\rho_{agg} = \rho_{particle}(1 - \epsilon) + \rho_{fluid}\epsilon$$

Therefore, now you can say that you know the density of the aggregate is rho of particle times the particle fraction plus rho of fluid times you know fluid fraction right ok. Therefore if you think about density, density the aggregate would always be intermediate to the density of the particle that formed the aggregate and the fluid in which you know it is dispersed right. So, therefore, I can actually substitute for this I can substitute for this in the expression for you know the u t aggregate, I can simplify this further can you do that? u t aggregate is g D aggregate square into instead of rho of aggregates I can just write it as rho of particle into 1 minus epsilon plus rho fluid into epsilon minus rho of fluid right divided by into right. If you just you know simplify this further that is what you would get right yeah.

$$u_{t,agg} = \frac{gD_{agg}^2(\rho_P(1-\epsilon) + \rho_f\epsilon - \rho_f)}{18\mu} = \frac{gD_{agg}^2(\rho_P - \rho)(1-\epsilon)}{18\mu}$$

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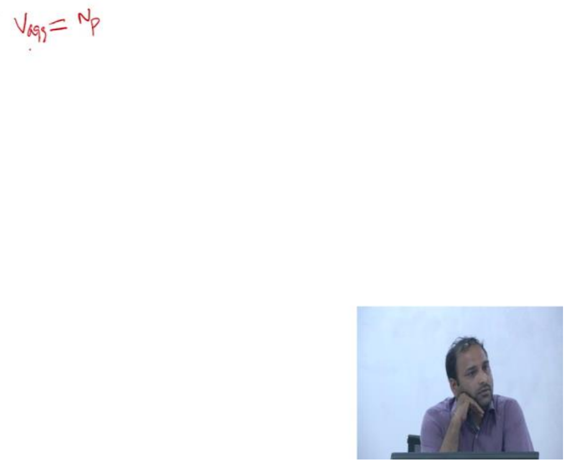

Now so what I can also do is I can define the. So, I can I can say that the volume of the aggregate the volume of the aggregate is equal to the number of particles I have in the aggregate or basically I do let us put it this way now I want to. So, let us say that you know I have of course, I do not have a way of calculating epsilon right which is a you know, but of course, if I can do some assumptions right. I know I can say that you know the maximum value of this is going to be 0.74. If I assume I ordered packing if I assume ran random packing is going to 0.64 and if I have a way of calculating fractal dimension, there will be some way of kind of you know writing this the structure of the aggregate in terms of the packing fraction right.

And rho p is the density of the particle if you know the parent particle that you are going to use right. I know the rho p, I know the properties of the fluid in which the particle settling right the only thing that I do not know is what is D aggregate that is the dimension of the aggregate if we have a way of measuring the aggregate dimension there are ways by which you can measure that ok. If I have aggregates in a solution whatever method that people use for particle size measurement people have people can use a similar methods for measuring what is the D aggregate therefore, if I have a way of calculating this or measuring this I can actually substitute this back into this equation and I can actually calculate what is a you know ut aggregation you know the there is a terminal velocity of the aggregate that was settling in a fluid right.

Ah What you can also do is, I can actually do further simplification and then try and express this ut aggregate in terms of the settling velocity of the particle itself and the way to do that would be yeah. Yeah.

No I am. So, see what I am trying to say is that look if I assume that it is an impermeable aggregate ok. I you know it has some characteristic size, it has some characteristic density I am assuming it as a spherical you know aggregate and that is settling in a fluid in such case; I mean this is going to work right. That is a projected area is still I am you know going to assume it has a 4 pi r square again as I said that this is simplification ok. The reason why it is a simplification is because if I take you know aggregates like this, you know it is more complicated right you know for example, the case 1 I have to worry about whether this linear aggregate that you have at what orientation is settling that becomes important right ok.

So, we have kind of simplified this problem and all that I am saying is that you know if you assume the aggregates to be a spherical ok. If I assume them to be impermeable the effective you know your ap which is the projected area is still going to be you know pi r square which is the pi times you know the radius of the aggregate square that is all right ok.

So, maybe I will stop here. So, so we kind of set up the problem now um. So, we have a way of thinking about if I assume that you know the particles are settling or the aggregates are settling in stokes regime, I can whatever formalism that we did for settling of solid particles, I can kind of rework that and then use a similar kind of formulation also for aggregates. We will try and continue with the rest of the class tomorrow yeah.

Thanks.