Fluid Mechanics

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Lecture - 33
Settling velocity - Stoke's regime and Newton's regime

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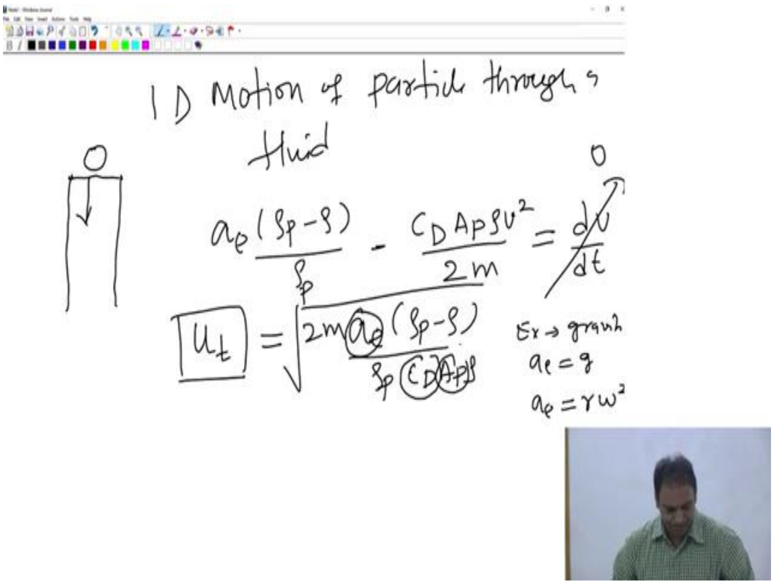

So in the last class we were talking about 1 Dimensional Motion of particle through a fluid, right ok. So, what we did was we kind of came up with a governing equation that basically describes you know when a particle is moving in a fluid, but that is in 1 dimension. Such a case I mean as we were discussing it could be a simple you know, if you take a case where if you have a column of liquid say that the column the liquid in the column is stagnant and if you have a particle that is moving under the influence of an external force.

So, we basically set up a working equation, right. And that working equation would look something like this right a times e into ρ p minus ρ divided by ρ p right is it ρ p or ρ p right minus C D into A P into ρ v square by 2 times m was equal to dv by dt right that is the working equation right everybody knows how to get this right.

$$a_e\left[\frac{\rho_P-\rho}{\rho_P}\right]-\left(\frac{C_D\rho v^2}{2m}\right)A_P=\frac{dv}{dt}$$

Now, as I was saying this is, this equation is a general equation which is valid for particle of any shape, right and for any fluid particle combinations, right. Now if you want to simplify this further before we do that I said that you know, as soon as a the particle has dropped into the column what will happen is, you will start with the particle being addressed now once you drop it, it is going to accelerate right and then this acceleration is going to die down soon.

Because of the fact that there is a drag that is acting on the particle and ultimately the particle will start moving with a constant velocity, right ok. So, at some point in time your acceleration is only 0, ok, that is because the particle is going to reach a constant velocity. And you can get that constant velocity as I mentioned the previous class it is denoted by u t or the terminal velocity people also call it as terminal settling velocity and that we obtained it as some a times e into ρ p minus ρ divided by ρ p into 2 times m right divided by C D A P into ρ under square root ok, right that is what we had developed.

$$u_t = \sqrt{\frac{2a_e(\rho_P - \rho)m}{A_P \rho_P C_D \rho}}$$

$$a_e = g\ (Gravitational\ force)$$

$$a_e = r\omega^2\ (Centrifugal\ force)$$

Now, if you want to simplify this further, I was mentioning that you know you would have to. So, of course, we talked about two different cases right; one I can say that a e is the acceleration because of the external force it could be working with any external force that you wish, ok. But if you take a simple case of external force being gravity we said you a e is going to be g which is the acceleration due to gravity. And of course, if you take your acceleration if your external forces like say centrifugal force, ok. We are going to replace that a e with r omega square right.

So, now if you want to simplify this further we said that you know we should know what is the C D, right if you look at the equation here right ρ p is a you know the property the particle ρ is a fluid A P is. So, there are certain parameters which depend on the fluid and there are certain parameter that depend on the fluid that you are considering, right. Now if you want to simplify this further we said that you would have to go back and look up things like this, ok.

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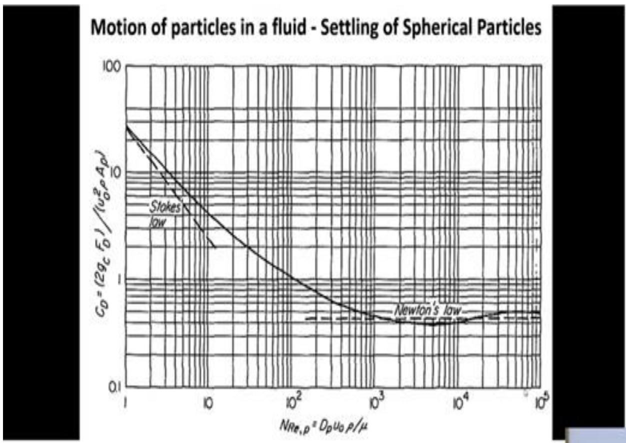

This is a an example where you know, what is plotted is a the drag coefficient C D on the y axis versus Reynolds number on the x axis, right. If you look at this plot there are kind of two limiting cases, ok; one is in the low Reynolds number regime, ok. When the Reynolds number is less than 1, your C D versus Reynolds number can be represented by an empirical equation which is something like C D is equal to 24 divided by the Reynolds number, ok. I can simplify the expression that we just wrote for the Stokes law settling regime.

And if you look at the high Reynolds number regime when your Reynolds number is more than 1000 up to about 200000, your drag coefficient more or less remains constant, right typically about 0.44. Therefore, what I can do is, I can actually substitute for these things in the working equation. So, what I am going to do is simplifying this, ok.

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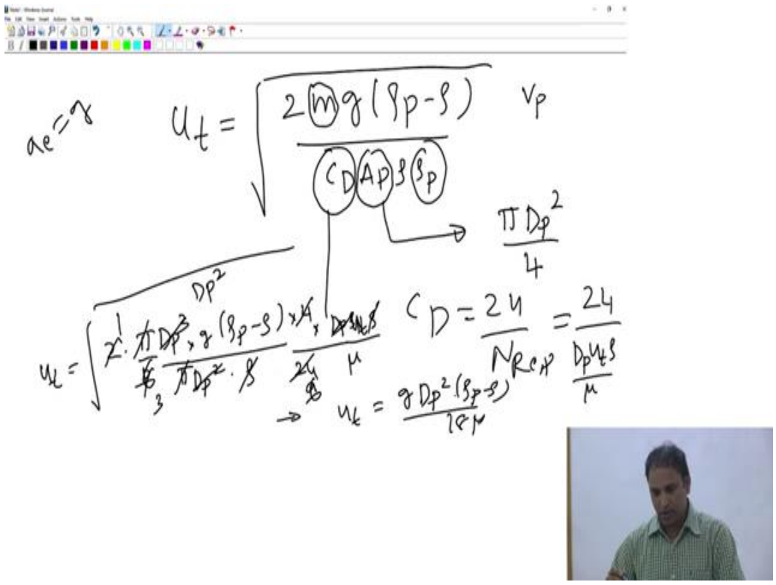

So, let us; so therefore, your u t was if I take a e to be g right. So, it was 2 times m times g into ρ p minus ρ divided by C D A P into ρ under square root, right that was the did I miss something, I have ρ p as well, right. So, now, what I can do is you have m here right and you have ρ p here I can replace that with V P which is the volume of the particle right and I also have C D which is sorry A P which is as I said it is a projected area which is pi D p square by 4 and C D as we have been saying this is C D is 24 divided by particle Reynolds number or Rep or N Rep.

Therefore that is 24 divided by D p U t ρ divided by mu right where mu is the it is cos to the fluid. So, if you put in this, we can do that quickly. So, therefore, your u t is going to be 2 times instead of m divided by ρ p, I am going to put it as phi D p cube by 6 right that is your volume, right multiply by half g ρ p minus ρ divided by I have C d here. So, I am going to write it as, ok.

Let us do for A P first, A p is going to be I am going to have 4 in the numerator it is pi D p squared that is for A P, right. And I also have ρ here. Now for Reynolds number it is going to be I am going to multiply by mu divided by 24 times D p u t into ρ, right that is your mistakes. So, your C D is 24 divided by Reynolds number right. So, that is going to be D p by ρ mu that is that is, right. So, therefore, I can cancel these two here and I can cancel, I think I have done a mistake, right. This is going to be D p here in to u t here in to ρ right. So, then you have D p square here and there is D p power 4. So, this going to be

D p square will come in the numerator. So, you have 4 here and there is going to be so, 6, right.

Therefore there is 1 here 3 here ok. So, therefore, basically if you work it out and of course, your ρ and ρ is going to get canceled. Therefore, if you work it out it turns out the your u t is going to be g into D p square into ρ p minus ρ divided by 18 mu because 3 times 6 is 18 and you have mu in the you know denominator and your numerator is going to be g D p square into ρ p minus ρ you can work it work this out, ok. This is the working equation if you have like say a particle which is settling in the Stokes regime ok.

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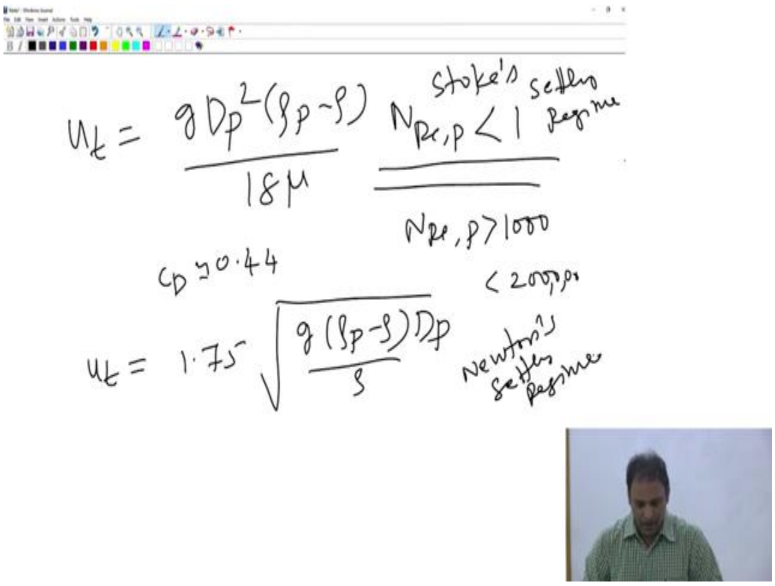

So, Stokes regime is basically if you have your particle Reynolds number is less than 1. Your u t is going to be g into D p square into ρ p minus ρ divided by 18 mu, ok.

$$u_t = \frac{g D_P^2 (\rho_P - \rho)}{18\mu} \; ; N_{Re,P} < 1$$

Now, similarly if you work out for Reynolds number, that is greater than 1000, but less than 200,000, ok. In that case your C D is going to be constant and that is 0.44, ok. If you substitute for C D in the expression for u t and then if you simplify it further what you would do is you would get u t as 1.75 into square root of g into ρ p minus ρ into D p divided by ρ, ok. You can work this out at home ok.

$$u_t = 1.75\sqrt{\frac{g(\rho_P - \rho)D_P}{\rho}}; 1000 < N_{Re,P} < 200{,}000$$

Therefore, these are the two limiting cases, for the settling of particles in a fluid, ok. That you know expressions for the terminal velocities which are applicable for the limiting cases one for when the Reynolds number is less than 1 is what is called as a Stokes settling regime. Other for cases where the Reynolds number is in between 1000 and 200,000 is something called as a Newton's settling regime, ok. Now so, of course, these are applicable for a spherical particle, right, because that is what we considered in this case pi D p square by 4 being substituted for A p which is the projection area which is only true for spherical particle right. So, therefore, if you have other objects of other sizes, so, it turns out that just.

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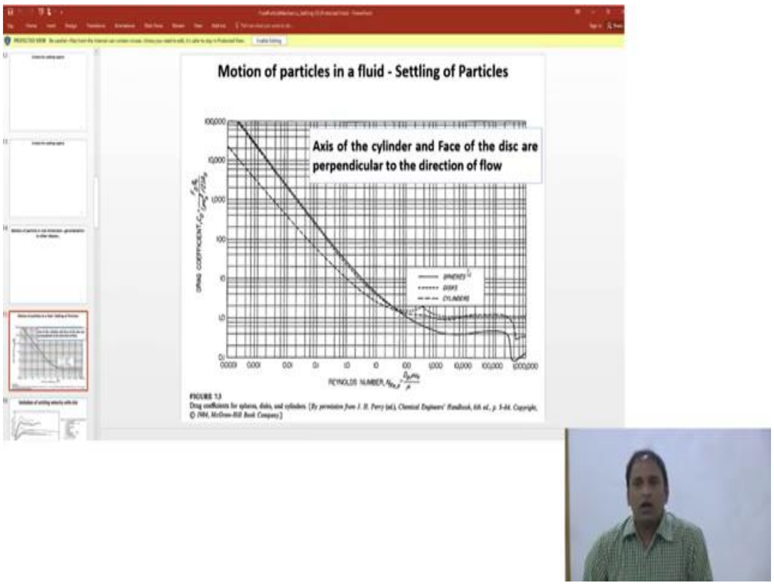

So, if you want to generalize for you know particles of any shape, what you should do is you should have some correlations like this ok. What you are seeing in the plot is again C D versus Reynolds number, but if you look it up there are different plots, ok. There is one plot for cylinders sorry for a spherical particle is a continuous line, right. The continuous line is for the spherical particles, which we just saw right the two limiting cases of C D being 0.44 or C D being 24 by Reynolds number and the other two are for particles of different shapes, ok. One is for disks other one is for cylinders and these have been generated by taking particles of particular dimension and particular shape and they have been held in some particular condition.

For example in this case axis of the cylinder and face of the disc are perpendicular to the flow direction ok. If you have a liquid that is flowing, the axis is in a direction perpendicular to the flow that is like this, ok. Therefore, when you want to work out the equation for settling velocities for different shaped particles you would have to worry about the orientation of the particle as well, ok. Whether the particle is oriented in the direction or in the direction perpendicular to the flow.

When the particle is oriented perpendicular is something was a bluff body is that is a terminology people use, in the literature. So, therefore, I would have to take care of you know such issues when you want to simplify these equations further for calculating settling velocities for different shape objects. Now, if you are given a problem that, I say that a problem is given to you and you want and they want you to calculate what is the settling velocity of the particles okay.

Now, how would you go about doing this say that, you know there is an problem that is given I can say that there is a particle of 100 micrometer diameter. It is settling in a liquid column, I say that the column is filled with water. I will give the density of the liquid you know water and then also viscosity of fluid. And if you would ask to calculate what is the settling velocity of the particle how would you go about doing the calculations.

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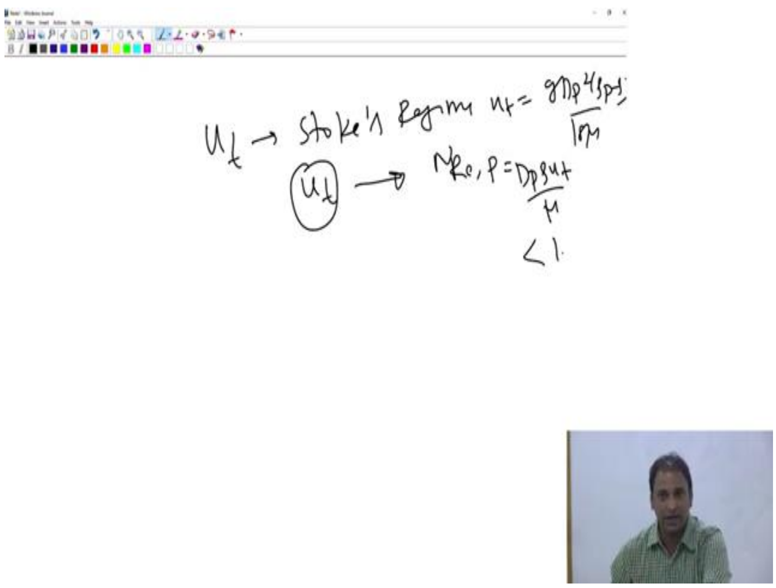

So, one crude way of doing this would be you calculate your u t for the Stokes regime, get what is your u t and then from that you basically calculate what is the Reynolds number after that I have done the settling velocity calculations. You assume that you know u t is say g D p square into ρ p minus ρ divided by 18 mu, you calculate the settling velocity then you get u t then you back calculate what is your Reynolds number, ok. If it is D u t by mu, right if the Reynolds number falls less than 1, then I say that you know whatever I have assumed that the particle is settling in the Stokes regime is correct ok.

Therefore, my assumption is right, ok. But that need not be the case all the time, right. Now you can end up with a Reynolds number which could be more than 1, it could be more than 1000. So, therefore, you kind of come up with a kind of a dilemma, as to you know whether I should be using the Stokes regime or the settling regime or the Newton's regime ok. So, therefore, what you can do is you can come up with a simple non dimensional number, ok. Which would, which can help us in terms of identifying the settling regime without going into this guesswork?

I said that the guess work you assume that you know the particle is settling in the Stokes regime. You calculate u t back calculate your Reynolds number see whether it is less than 1 or you know look at the typical range or typical number that you get out of the calculation. And then if that is not the case then you go back and then assume the Newton's regime. Then again calculate u t, calculate again Reynolds number right. So, instead of doing that what you can do is you can rearrange the expression that you have for your settling velocities, right.

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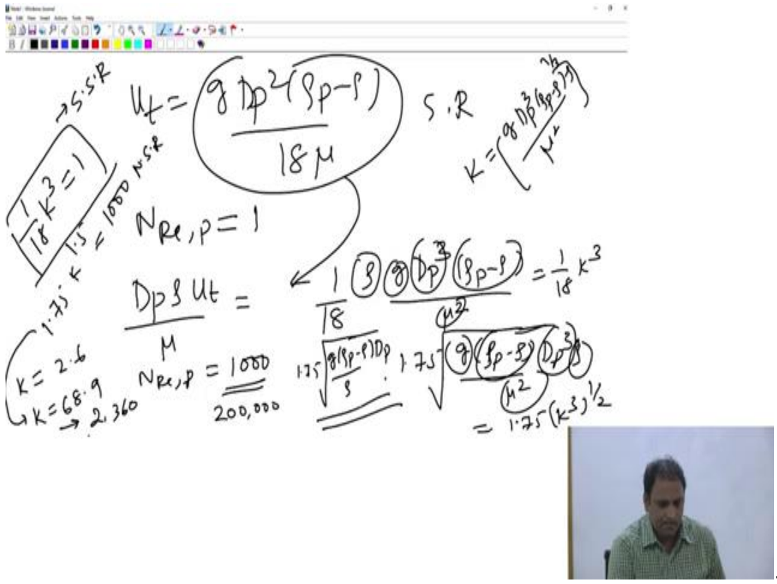

I know that u t is g D p square into ρ p minus ρ divided by 18 mu for the Stokes regime. I know that you know my Reynolds number, for the upper limit of the Reynolds number for Stokes regime is 1, right anything less than 1 is a. So, now, what I can do is I can substitute D p into ρ into u t divided by mu I can substitute for.

U t from the Stokes regime, so what do you get sorry, that is going to be your g D p square into ρ p minus ρ divided by 18 mu, right. So, what I can write this as. So, this is going to be I am going to write it as cube and I am going to write this as 1 over 18, right.

$$\frac{D_P \rho u_t}{\mu} = \frac{1}{18} \frac{\rho g D_P^3 (\rho_P - \rho)}{\mu}$$

Now let us look at the exact similar formulation for So, we know that your Reynolds number should be more than 1000 for the Newton settling regime. So, similarly I can write the Reynolds number as D p into ρ by mu into 1.75 right, square root of g into ρ p minus ρ into D p divided by ρ, right that is your expression further. So, now, what I can do is I can take the D p inside, right. So, I can write it as D p cube, right. I am going to take is that and then I have ρ here, again I am going to take ρ square inside I am going to write as ρ square, now of course, there is going to be mu square as well right, ok.

So, now I am going to cancel a few things, right. So, that is going to be there is one mu goes, right and what I has to have, but I forgotten that should be ok. So, did I do a mistake here, no right 18 there is going to be mu square here, right.

Right ok. So, look at the term that is in the there is g here right, there is D p cube here, there is ρ p minus ρ did I cancel one of the, what is the expression for the stoke settling, I made a mistake, right there is no. So, what we did is right, D p into ρ was in the numerator that is going be so, can you help me out. So, I have 1.75, let me just write it. So, your, it is 1.75 into square root of g into ρ p minus ρ into D p divided by ρ, right that is my Reynolds number.

So, here that is going to be ρ square here now that is going to be ρ here. So, therefore, it is seems. So, it is right. So, I think that is fine. So, I have a row here, right. I have a row here, g here write D p cube ρ p minus ρ mu square is it, ok. Therefore, I can actually define a factor K, ok. I can define a factor K as g D p cube into ρ p minus ρ into ρ here divided by mu square to the power of one third, right.

So, therefore, this is going to be 1 over 18 into k cube. And here it is going to be 1.75, again to k cube to the power of one and half is it.

So, therefore, from the just by rearranging the two expressions that; we had one for the Stoke settling regime another one for the Newton's settling regime. What we have been able to do is, we have been able to get 1 over 18 into k cube is equal to 1 is the case for Stokes settling regime ok. And 1.75 into k to the power of 1.5 is equal to 1000 for the Newton's settling regime, ok.

$$\frac{1}{18}\,K^3 = 1 \rightarrow Stokes\ settling\ regime$$

$$1.75\,K^{1.5} = 1000 \rightarrow Newton's\ settling\ regime$$

$$Where, K = \left[\frac{gD_P^3(\rho_P - \rho)\rho}{\mu^2}\right]^{1/3}$$

If you work it out in this case your k comes to be something like 2.6 ok. If I you know k cube is 18 therefore, k is going to be cube root of 18. So, that is going to be 2.6, ok. And similarly the k for the Newton's settling regime comes out to be something like 68.9, ok.

If I substitute the lower limit of the Reynolds number for the Newton's regime and if I substitute to higher limit of the Reynolds number which is 200000 your k comes out to be something like 2,360. Therefore, the idea is this ok.

So, what you do is you calculate the k which is basically defined as g D p cube into ρ p minus ρ into ρ divided by mu to the power of one third you calculate that. If the value of the k that you obtain if it is less than 2.6 then you use the settling velocity for the Stokes regime and calculate your terminal velocities.

However, if the k factor that you are getting if it is between 68.9 and 2,360, that is the range of Reynolds number over which the settling occurs in the turbulent condition, that is the Newton settling regime, ok. In such case you would have to use the appropriate expression for the terminal velocity for the, you know particle in the Newton's regime ok. So, therefore, this formalism, in terms of calculating k, as a criteria for deciding whether I should go for a Stokes or the Newton's settling regime it is useful instead of doing a guesswork of assuming your particular settling regime and a back calculating your Reynolds number.

Any questions, do you have any questions with this? It is just a manipulation, right. It is just a rearranging of you know the expression for each of these you know u t's and then you know equating that to the limiting value of the Reynolds number for respective conditions, that is all we done yeah ok. So, now, we will talk a little bit about, before we talk about applications of these things.

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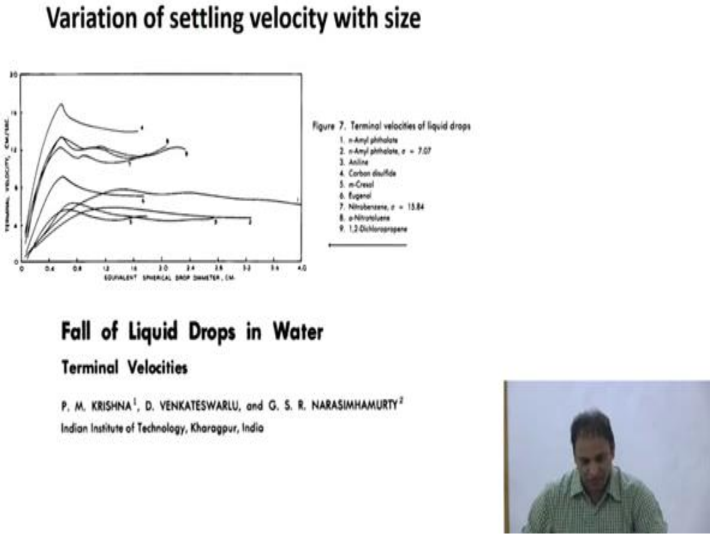

So, now what do you see is a plot, of a variation of settling velocity with size, on the y axis you have terminal velocity on the x axis you have equivalent spherical diameter. And there are different curves right 1 2 you know they are numbered from 1 to 9, ok. And they have taken different fluids you know for example, 3 is an aniline, 7 is nitrobenzene, ok. So, basically what we are looking at is, a case of fall of liquid drops in water, ok.

So, far when we looked at settling, we only talked about solid particles, right we talked about spherical particles rigid you know hard particles. Now, if you want to plot, terminal velocities is a function of size of the particle, ok. It would always be a monotonically increasing function right because in the case of Stokes settling your u t goes as D p square, right. And in the case of Newton's settling regime it goes as D p to the power of half right.

So, therefore, in both the cases as I increase the size my settling velocity should always go on increasing right; that means, it is a monotonically increasing function, ok. Now, if you look at this plot can you see some difference; the first difference which is very apparent is that it looks like the settling velocity increases, in analogy with what is known for smaller dimension for example, maybe in the range from 0.0 to 0.6 in that centimeter size range, the thermal velocity increases with size.

However, it attains a maxima and then it basically drops back, right ok. So, any thoughts is to why that could be the case. So, everybody understand the plot right, it is a plot of terminal velocities is a function of equivalent spherical diameter. So, why does equivalent spherical why is it not the diameter of the drop, why is it a equivalent spherical diameter? The obvious answer is going to be because the drop right, the drops can also change the shape.

So, therefore, when one is dealing with drops in a fluid I would have to worry about shape changes ok; that means, depending upon. So, the reason why it would go to a maximum and then drop off because one naive answer could be; that in the initial for the smaller size regime the particle, the droplets will remain spherical ok. However, when you make the droplets bigger and bigger at some point they are going to flatten, right and then your drag is going to be more, more drag means it is going to settle slower, right. So, anyway we can think of we will argue a little bit of along these lines in a minute.

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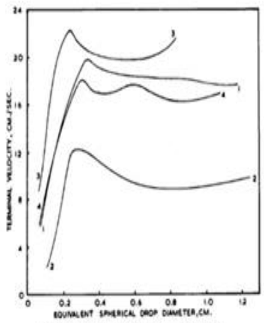

Figure 8. Terminal velocities of liquid drops

1. Carbon tetrachloride
2. Epichlorohydrin
3. 1,2-Dibromoethane
4. 1,1,2,2-Tetrachloroethane

This again, some another some data again for, so I do not know if you notice this, right. This is a paper from IIT Kharagpur which is published in 1950s, ok. So, people who are worrying about settling of you know droplets and you know other stuff you know then it is a right, because it is and again the settling of drops in a fluid is also relevant to one of the common thing that you guys see. Any example raindrops, right ok. When the rain falls right it is going to be these are going to be water droplets as they approach earth they are going to again flatten in the direction you know of the gravity, right. So, in depending upon the size of the droplets that are going to be generated, ok

If that size the droplets are very small, then you can assume the drops to be spherical in shape they would you know come like a hard sphere; however, if you have a, if there is a larger raindrop the moment it exceeds a particular size, if I were to go by this plot for like say 4 which is some carbon disulfide. There is peak right I can draw a vertical line and I can actually get what is the limiting size for which the droplet behaves like a sphere. But beyond that line you know where you see the drop in the terminal velocity that is when the shape changes would have occurs, ok.

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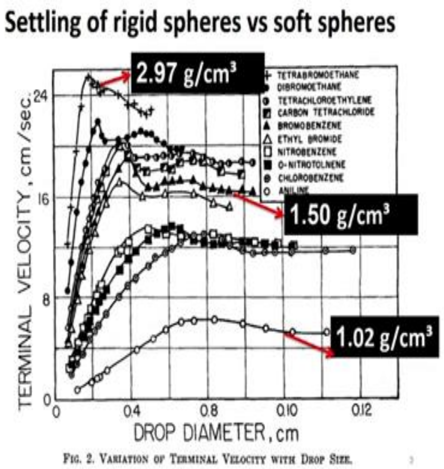

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Again there are more data this again from some other people, again the characteristic peak in the terminal velocity followed by a you know a decrease is evident here as well. Any thoughts as to why the other nice things to notice is you know, the peak where it the appearance of the peak right is seems to be shifting towards smaller sizes right. If I look at the bottom most plot which is for aniline, right or the top most plot which is for tetrabromoethane it turns out there you know the peak is kind of shifted right to the left side ok. Any thoughts why it could be? That is because there are different density, right. If you look at these fluids for example, aniline has a density of 1.02, which is very much close to that of water right.

However, if you look at tetrabromoethane the density is 2.97 much denser, ok. Again that is also evident from the slope, right. If the slope for at least in the initial regime where there is a linear increase, right the slope is larger for 2.97. Because the density difference is more we know that, they should be settling much faster right, compared to the case where the density difference is very little for the case of aniline, right.

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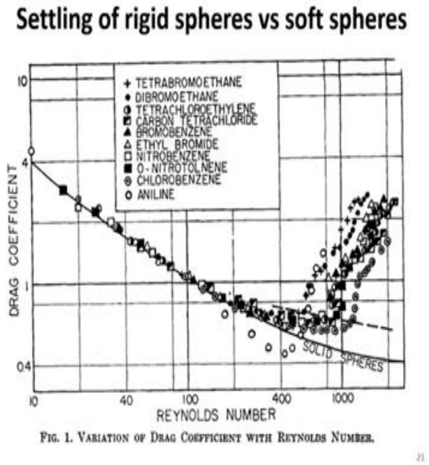

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This is a plot of again drag coefficient as a function of Reynolds number. It appears that for the low Reynolds number cases for example, Reynolds number from a 10 to about 400, roughly ok. All the data for the droplet seems to be in line with what is expected for solid spheres, but; however, if you go for larger Reynolds number; larger Reynolds number is because larger dimensions larger size of the droplets.

Therefore, you can see a significant change that you see because that is evident from the upturn of this C D versus Reynolds number plot, right. Again this is another indication of the fact that the droplets have changed their shape and one has to worry about such things you know when you look at.

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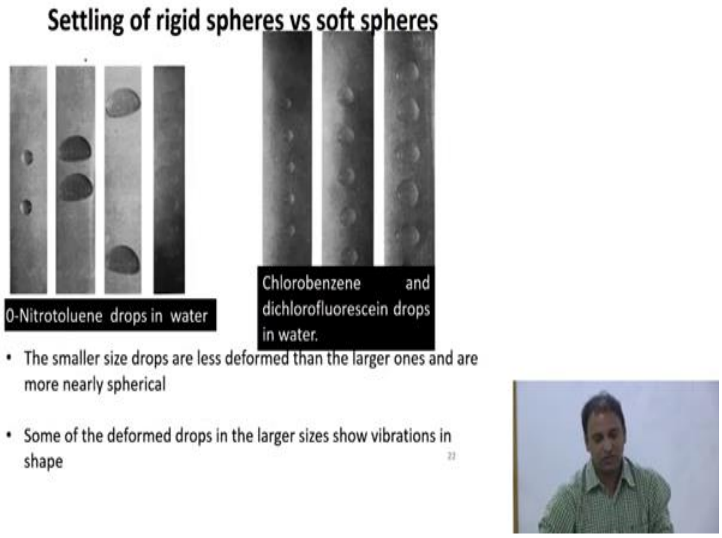

These are some images from again different literature the extreme the plot on the left. Again extreme left is that is where the droplet continues to remain in the spherical shape during it is settling ok; however, if we consider a larger droplet it turns out that you know that the drop start becoming elongated right.

And this elliptical the deformation of the droplet leads to droplets being in different shapes and of course, because of that your drag forces are going to be very different than what you would see for the spherical particle case. And that is therefore, you would you would expect that you know the particles would slow down because of the increase in the drag force ok.

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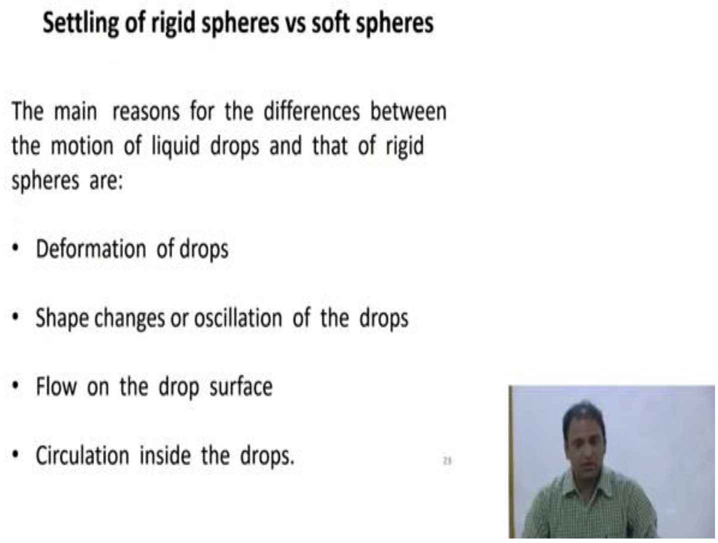

So, in general the main reasons for the difference between the motion of the liquid drops and that of the rigid spheres is because of the deformation of drops ok, which becomes evident when you go for larger sized droplets ok, number one. And this deformation can lead to either shape changes or shape oscillations ok. If you look at the movies ok, you know how these droplets settle.

Now you will see that you know the drops start going from one shape to the other ok, there is going to be drop shape fluctuations which you would have to worry about ok. And whenever you have a smooth or a you know a liquid like surface; the flow on the surface as well as flow within the droplets are going to be different ok. In the case of solid sphere spheres I do not have to worry about flow within the particle, right.

However, in the case of you know liquid droplets they are going to be some internal circulations which can also lead to a change in the way that droplet would behave in a fluid ok. So therefore, so when you are working with spherical particles; spherical particles and rigid particles of well-defined shape and dimension, things are kind of fairly well laid out ok; that means, you know there as C D versus Reynolds number plot you should know little bit about settling regime you know the correlation between C D and Reynolds number all of that is this kind of well laid out you know.

You can use those things to understand the settling behave behavior of solid particles; however, if you go to liquid drops as a bubbles, you will have to worry about some of these considerations.