Fluid Mechanics
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Lecture - 32
Sedimentation and separation

So, we are going to continue with the same theme that is.

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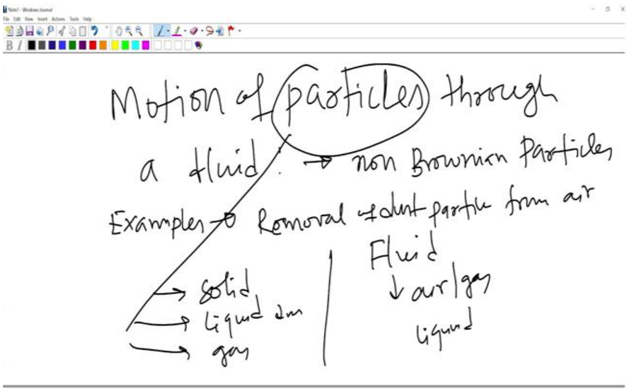

So we are going to continue the same theme that is the motion of particles through a fluid. So; however, we are going to look at the case of non-Brownian particles that is the particle that are much larger in size. So, it turns out that you know the motion of larger particles there is non-Brownian particles is kind of you come across this in a lot of processing operations any example. So, can you think of some examples where you come across motion of particles in a fluid?

So, when you concern operations such as say removal of say dust particles from air or you could also have a case where there is a flue gas which is coming from you know one of the industries right and if this industry could be carrying a particle ok. So, in general you are the particle that we are going to consider like other cases it could be a solid particle, it could be a liquid drop or a gas bubble and again your fluid can be either you know air or a liquid right or a gas or a liquid ok.

So, there are several operations in which you have particles moving in a fluid one example could be as I said the removal of dust particles you could also have cases where you have like say solid particles in a waste fluid and if you want to like say discharge the waste liquid, you may have cases where you know you would like to recover particles from liquid waste plus you also have cases where in a lot of industries which produce acid ok. You have acid mist is basically is fine droplets this is basically acid and before you exit a mist you know into atmosphere you would have to recover you know this droplets from the you know the waste ok.

So, there are a lot of cases where you do come across motion of either solid particles, liquid you know droplets or gas bubbles in the fluid and therefore, whatever we are going to do today is going to be of interest in that context of some of these operations now whenever. So, we said that you know when you think about I have some issue ok. So, now, so, when you had fine particles right we kind of saw that you know there the particles were moving by themselves of course, there was a thermal energy because of the presence of fluid ok, but in a sense they were moving by themselves.

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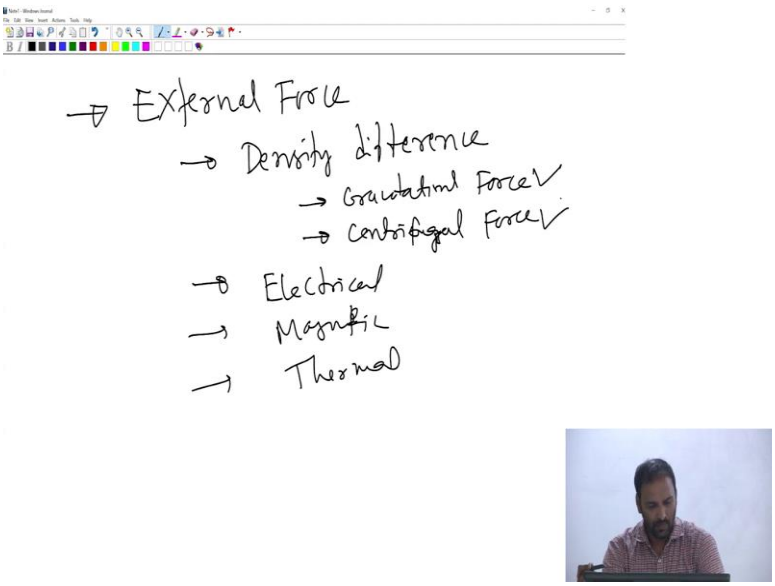

But whenever you look at larger particles you know what you need for the particles to move is it? External force right ok. And this external force can either come from density difference that is the density difference between the particle and the fluid ok. So, can you think of an external force where the density difference can be exploited to you know move the particle under consideration particle right. Can you think of an example? Can you think of an external force where I can exploit the density difference to effect the motion of particle in the fluid? Gravity right ok. So, your gravitational force is an example of a case where I can actually exploit the density difference to set a particle you know in the fluid to move ok.

Any other example where I can exploit the density difference for setting a particle to move?

What is that osmosis why is osmosis related to density difference osmosis is basically a typically you have some kind of a permeable membrane right where you know you can look at separation in terms of you know there is a. So, there could be a membrane which is permeable only to some species right. So, in that context you can use that, but can you think of centrifugal forces like we know centrifugal forces again one force that I can actually exploit the density difference any other external force that I we can think of again something we discussed the other day it could be electrical forces, it could be magnetic or it could be thermal forces right.

So, essentially so, you can set a particle into motion by using any of the external p. So, we going to see some examples where we are going to look at the motion of a particle under the influence of gravitational force and centrifugal force in this class ok. Now so, we will think of a simple example ok. So, let us think about simple example.

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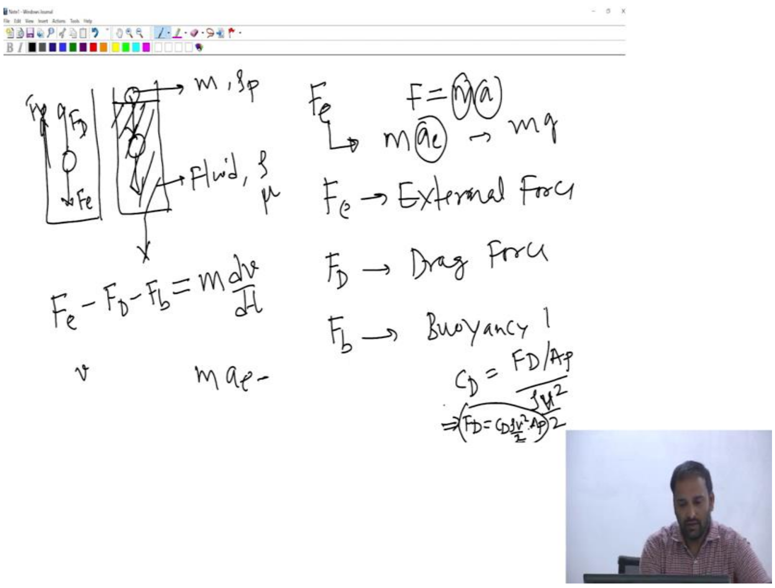

So, I have a like see a column and this column is say is filled with fluid and let us say that the densities of fluid is ρ, the viscosity is mu and I have a particle to begin with you know may be sitting right on at the top of the column I am holding it and let us say that the mass of the particle is m and ρ P is the density of the particle ok.

And I just let the particle move down under the influence of some external force. So, let us say then the external force is acting on the particle is F e ok. So, whenever a particle is set into motion say that you know there is an external force that is acting on the particle say that is a F e ok. Now we have kind of discussed in the class that whenever there is an external force which sets the particle into motion, there is always going to be a drag force right whenever there is a relative motion between the particle in the fluid, a drag force comes into picture therefore, there is going to be also drag force acting on the particle ok.

Now, when the particle enters the fluid, it is going to displaced its going to displace some fluid as well right therefore, there is also going to be buoyancy as well right there is going to be F b, this is your external force this is your drag force, this is your right buoyancy or buoyant right now what you can do is you can think about writing a simple force balance ok. So, we can use Newton’s second law that is your F is equal to m a ok.

So, can we think about the direction of each of these forces? So, if say that you know the external forces say acting down. So, if you have now that you know you consider particle in the fluid. So, you have maybe let us make it simpler. So, I have a particle in a fluid this is your F e which is say acting downward what could be the direction of F D which is the drag force is going to be acting up right that is your F D because we know that the drag force opposes the motion of the particle and typically it acts in the direction opposite to the external force and of course, its direction is you know its going to be parallel to the external force as well and buoyancy. So, because if this the fluid is being displaced up right therefore your buoyancy is going to be acting in the upward.

Somebody has a question? So, its going to be acting up right. So, therefore, my force balance basically becomes F e minus F D minus F buoyancy. Now that is mass times acceleration if the particle is moving with a velocity v if you have a particle in the fluid and if it is moving with a velocity v your acceleration is d v by d t ok. Now in general your F e which is the external force, I can write this as some mass time some acceleration a e that a e would depend on the type of external force that you are considering if you are going to consider gravity its going to be m times g right and if you are going to consider the centrifugal force it could be something else right.

$$F_e - F_D - F_b = m\frac{dv}{dt}$$

$$F = ma_e$$

Therefore, I could write m times a e that is your external force minus F D how do we get an expression for F D? We will go back to the definition of the drag coefficient ok. So, we had said that your CD is F D divided A P divided by ρ u square by 2 now ρ v square by 2 now because you know v is a velocity which was the particle is moving therefore, I can write F D as CD times ρ v square by 2 times a p right that is your right that is your the drag force. Therefore what I can do is I can substitute for these in the expression and I can write sorry with that ok.

$$F_D = C_D\frac{\rho v^2}{2}A_P$$

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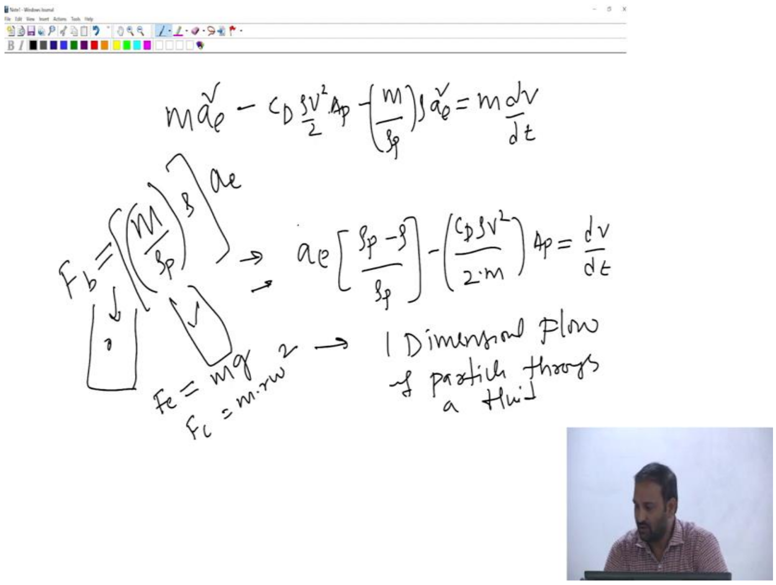

M times a e minus CD times ρ v square by 2 times A P minus. So, you have a buoyancy right F b. So, this you actually obtain from the Archimedes principle right if m is the mass of the particle that you are you know that you are considering, m divided by ρ P is going to give me the volume of the water that the particle is going to displace what is that ρ P is a density of the particle right times ρ which is the density of the fluid that is going to give me what is the mass of the fluid that is displaced times the external force external you know a e which is the acceleration because the external force is what is going to be your f b. Therefore, I can write this as m divided by ρ P into ρ times a e should be equal to m into d v by d t right.

So, I can do a little bit of rearrangement:

$$ma_e - C_D \frac{\rho v^2}{2} A_P - \left(\frac{m}{\rho_P}\right) \rho a_e = m \frac{dv}{dt}$$

$$a_e \left[\frac{\rho_P - \rho}{\rho_P}\right] - \left(\frac{C_D \rho v^2}{2m}\right) A_P = \frac{dv}{dt}$$

Now this particular equation it is the governing equation for something called as a 1 dimensional flow of particle through a fluid and the reason why is 1 dimensional is because you are basically considering a particle which is moving in a direction where the direction of motion of the particle is parallel to the direction of the external force and the buoyancy ok.

But; however, you may have cases where that need not be the case you could have a case where let us say a simple case of instead of you know have having a particle settled in a column of fluid like this if I tilt the column itself ok. And if the particle is still settling you know under say the action of gravity for example, you know you are the direction of motion will not be parallel to the direction of the external force and the buoyancy ok. In such cases you know your particle will not move in the a 1 dimensional fashion it could be moving in a 2 dimensional fashion or 3 dimensional fashion that depends on the kind of you know the problem that you are trying to work with.

At least in this case we are considering a case where the particle is like say in the case of gravity is dropping down you are then the external force is basically acting external force as well as the buoyancy is basically acting parallel the rest direction may be different, but they are acting parallel to the motion of the particle. Now so, now, it turns out that if you look at this expression right.

So, I can simplify this further right if I consider the external force F e to be a gravity I can write this F e to be you know your m times g right are or if you consider external force to be some kind of a centrifugal force its going to be m times r times omega square right therefore, I can substitute for these I can simplify this further ok. One interesting thing to note from this equation is that if you look at like say let us write this equation for the case of gravity ok. So, that is ok.

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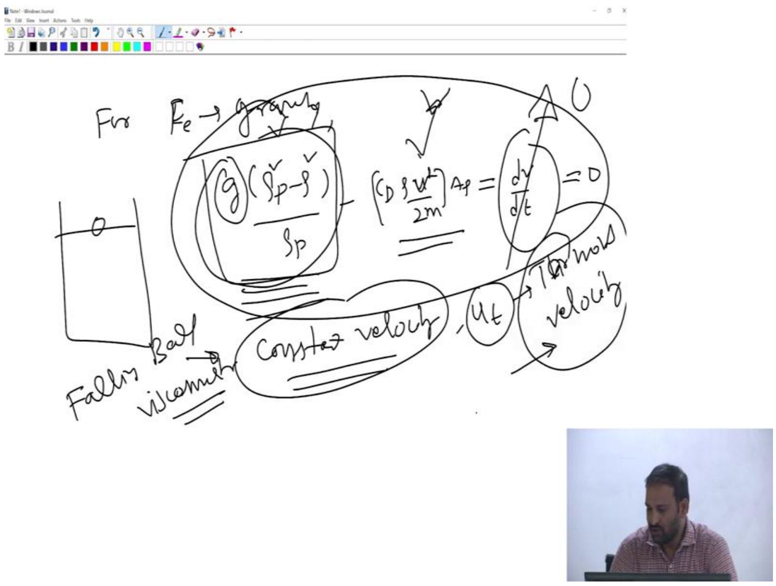

So

$$\frac{g}{\rho_P}(\rho_P-\rho)-\left(\frac{C_D\rho v^2}{2m}\right)A_P=\frac{dv}{dt}$$

So, if you look at let us say gravitation you know the case of gravitational you know settling of particles are you know the flow motion of particle that and the gravity this is your constant right you know your once you fix a fluid particle system and once you fix the external force to be gravity in this case your there is a constant term ok.

That means if I go back to this example right. So, I had a particle. So, the moment I drop the particle ok. So, your this term is fixed right; however, if you look at this term right because of the fact that the particle is going to accelerate right the particle is starting from rest the moment I drop the particle is going to accelerate because of which this is this term is going to go on increasing right because you know if this term goes as v square if the particle starts to go to higher velocity therefore, this term is going to go up you know go on increasing.

However this cannot increase forever right at some point you're going to reach a case where this term would be equivalent to this term that. So, under these conditions the particle is said to start moving in a fluid with a constant velocity and because its going to move with the constant velocity your d v by d t is going to be 0 and this constant velocity is the maximum velocity that the particle can achieve under a given fluid particle combination ok. So, therefore, I can equate this to 0 and get an expression for that constant velocity which is what is called as a which is typically divided by u t which is what is called as a terminal velocity ok.

That means under the action of all these forces the particle that initially accelerates through the fluid it is going to reach a constant velocity at some you know point during its flow and that constant velocity is what is called as a terminal velocity we kind of looked at the application of this terminal velocity concept when you looked at the measurement of viscosity of the fluid right. We said that there is something called as a we if we looked at you know we looked at different cases of measurement of viscosity right there is some case where this is called as a falling ball viscometer right.

Do you remember that there is something called is a falling ball viscometer in which what is what you do is you make a particle you know flow through a fluid make it fall through a fluid and then you basically exploit such you know 1 dimensional equation of motion to really get an expression for that terminal velocity and that terminal velocity as we going to see it is going to be related to the viscosity of the fluid in which the particle is moving right. So, people kind of use this concept to get the viscosity of the fluid in commercially available instruments for measuring viscosity ok.

So, let us do that. So, let us going to let us equate this to 0 and then get an expression for sorry ok.

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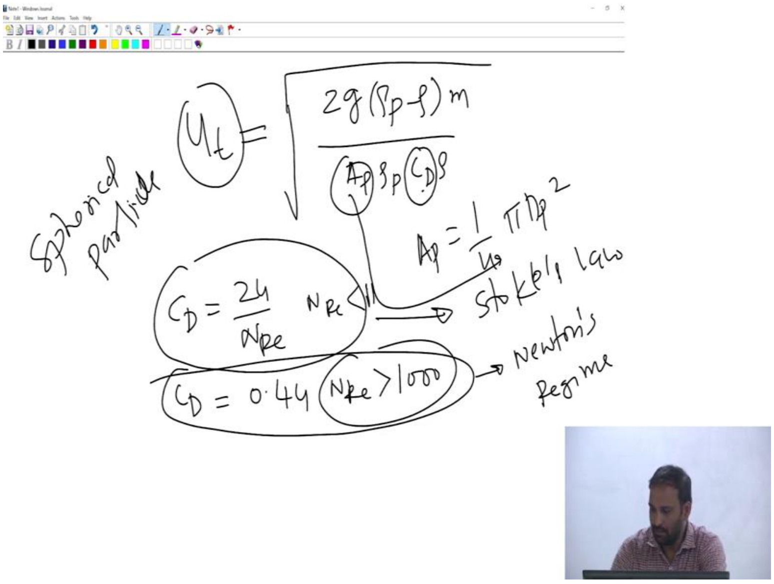

So,

$$u_t = \sqrt{\frac{2g(\rho_P - \rho)m}{A_P \rho_P C_D \rho}}$$

Now, the equation that we wrote right it is kind of a general equation that is valid for objects of any shape ok. We really did not make any assumption that the particles are spherical or cylindrical or whatever right. So, the 1 dimensional equation of motion that we wrote right let us go back this expression or this equation is valid for particles of any size you know or any shape right. So, now, what you can do is, you can now if your working with the particles of specific geometry I can simplify this expression further and I can actually get right.

So, now if I want to work this out like say if I am considering a spherical particle ok. We said that this A P which is the projected area, we said there you know what you should do is if you have a particle is moving in a fluid you should look at a plane perpendicular to the fluid and you should look at what area do you see ok. If its a spherical particle your A P is going to be 1 over 4 pi D P square right where D P square is the diameter of the particle I can substitute it for A P and what about CD ok? So, what people have done is, people have done a lot of experiment ok.

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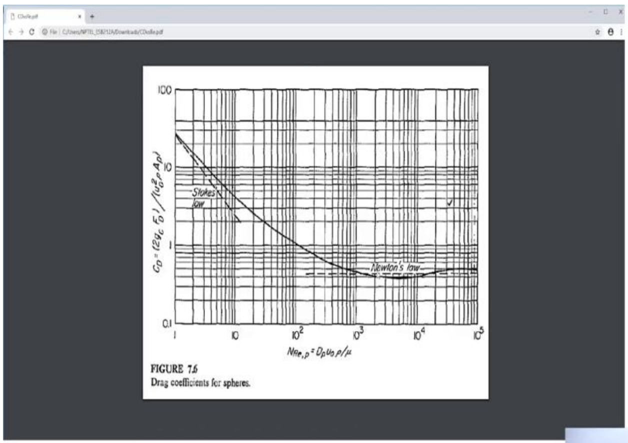

And kind of tabulated you know the results of you know the measurement of CD which is a drag coefficient as a function of Reynolds number this is the CD that we have been talking about ok.

So, that is your CD on the y axis your x axis is the Reynolds number ok. It turns out that if you look at this plot there are 2 limiting cases there is a case where if you look at the low Reynolds number case there is basically a line with a constant slope ok. This is what we kind of exploited in the some of the previous examples where we wrote the CD to be 24 by Reynolds number right that basically corresponds to that Stokes law regime its what is called as a Stokes law regime which is valid when the Reynolds number is less than 1 ok.

So, therefore, if I am working with conditions where the motion of the particle in the fluid occurs under the condition such that the Reynolds number is less than 1 for CD I can replace this as 24 by Reynolds number. However, if you look at the case where you know the Reynolds numbers are larger you know for example,10 power 3 in that region there is a dotted line, that dotted line basically corresponds to a case where your CD is more or less constant and typical value of CD in that case is basically.

So, therefore, you basically have 2 limiting cases this is CD is equal to 24 by Reynolds number that is for the case where you know your Reynolds number is less than 1 and you have another case where CD is 0.44 that is for the case where you know your Reynolds number is typically greater than 1000. Therefore, I can exploit you know the experimental data that is available on a on particles of different sizes that are being done and tabulated you know or all kind of which is available in the form of CD versus Reynolds number plot.

$$C_D = \frac{24}{Re}\ ; N_{Re} < 1$$

$$C_D = 0.44\ ; N_{Re} > 1000$$

I can substitute for these limiting cases and I can consider the of course, these are valid only for spherical particles the whatever plot that I showed its valid only for the motion of spherical particle in the fluid, if somebody is working with other types of particles or you know other geometry you know of particles you would have to think about using an appropriate CD versus Reynolds number plot ok.

Now that we have done that so, can we just replace let us think about the case one where Reynolds number your CD is 24 by Reynolds number which is what is called as a Stokes law regime can you substitute for A P and CD and get me an expression for the settling velocity or the terminal velocity ok. Let us do that for both for the Stokes law regime and also for the Newton's regime which is for cases where you know your Reynolds number is much greater than 1000, can you do that and let me know what the results are.

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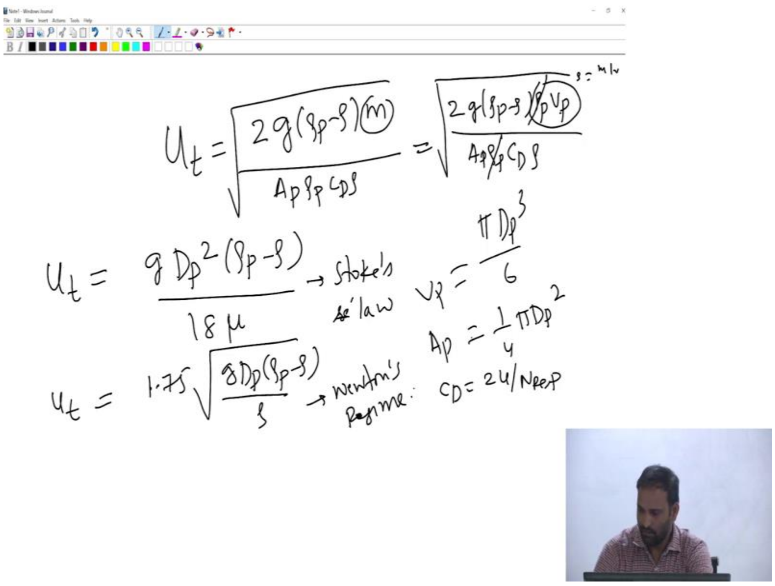

So, I can actually write m in terms of the density of the particle and the volume of the particle right I can do that therefore, this ρ P ρ P gets cancelled of course, I can express v P in terms of the particle dimension right if you know pi D P cube by 6 is your volume of the particle right and of course, I can substitute for A P to be 1 over 4 pi D P square right. So, if you substitute for all of that and of course, your CD is equal to 24 by Reynolds number right I say do all of that the expression I am looking for is something like this u t should go as g into D P square into ρ P minus ρ divided by 18 mu is what you would get for something called as a Stokes regime ok. And your u t should basically go as should go something like 1.75 into square root of g DP in to ρ P minus ρ divided by ρ that is for Newton's regime, which is basically you are simple substitution right ok. So, I should be able to get this right ok. Just work it out and let me know if you have issues with these things.

$$u_t = \sqrt{\frac{2g(\rho_P - \rho)m}{A_P \rho_P C_D \rho}} = \sqrt{\frac{2g(\rho_P - \rho)\rho_P V_P}{A_P \rho_P C_D \rho}} = \frac{g D_P^2(\rho_P - \rho)}{18\mu} = 1.75\sqrt{\frac{g D_P(\rho_P - \rho)}{\rho}}$$

Now it turns out that you know the concept, that we have kind of discussed today it's kind of useful in a lot of cases where people are interested to separate particles from a fluid ok.

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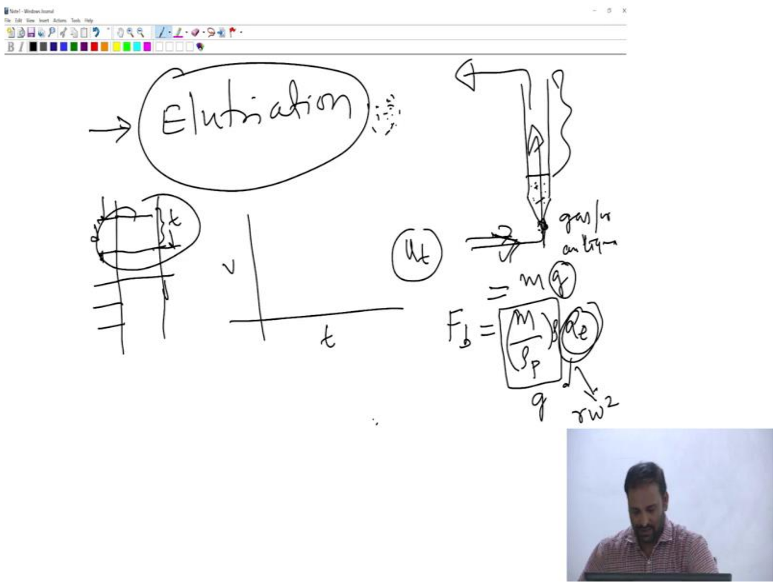

There is actually a process called elutriation say that you have a powder sample say that you know I have you know some sample are given to me which basically consists of powders powder and say that the powder has particles of different sizes ok. Now if the objective is to separate the particles that are present in the powder into different you know fractions larger size smaller size and stuff like that what is typically done is you take this into some kind of a container you fill the powder here now what you do is, you let a basically a gas or any you know liquid ok.

Now, what you can do is, what I can do is if I have a way of regulating if I have a way of regulating the flow rate of the fluid that I am basically you know using then depending upon; depending upon the velocity with which the gas or the liquid is moving up the column depending upon the velocity with which the fluid is moving up the column if the velocity of the fluid is larger than if the velocity of the fluid is larger than the terminal velocity of a particular particle that is there in the you know in the container. So, either the particle is going to be carried along with the fluid or will continue to remain in the column ok.

So, therefore, having an idea about the terminal velocity, its also called as a terminal settling velocity because when you are talking about motion of a particle under the influence of gravity is basically settling right. So, therefore, depending upon the if you have an idea about whether the particles are kind of smaller or larger, whether the Reynolds number is you know less than 1 or more than 1000 depending upon these conditions. So, you should be able to come up with some estimate of what is the settling velocity of the particle.

Now, once I have an idea about that basically by regulating the flow rate that is the velocity with which the fluid enters the column I can actually regulate that to separate particles in a different fraction. So, therefore, so, if I want a particle you know to remove particles of larger size of course, I would have to maintain a higher flow rate therefore, so, this is one of the cases where the settling principles are kind of used for separation of particles by this technique called elutriation ok. We will look at some applications of this in the class when you do some tutorial problems ok. So, any questions so far?

Correct.

So, typically when we said that you know when you start the particles you know say then you start with particle being addressed ok.

I see a lot of people talking. So, if you want to talk you can leave. So, say that you know you start with particle being on at rest let you know and you leave it and let it move under the influence of gravity. Now it turns out that you know that the typical time that it takes you know for it to attain that terminal velocity, it's usually very very small. In fact, you have an experiment in the next semester what are you going to do is, you basically have a column something like this what you do is, you have some markers on the column these markers are typically used to basically find out what is the distance that the particles travel in a given time ok. Now what I can do is, I can actually plot the velocity as a function of time ok.

F you do that you will see that you know. So, basically what you do is an experiment you know when the particle basically enters this you start a stopwatch when it leaves you basically stop right. I know the dimensions the distance that it travels and I know the time I can basically plot it right. It turns out that you know if you look at of course, when you do this terminal velocity measurement of course, you do not look at that initial region you know where the particle could still we are you know accelerating; however, it very quickly reaches the terminal velocity ok. So, what you one of the best way to do this would be I look at velocity at successive intervals, I look at you know at what instant you know the velocity between 2 successive regimes is constant that is how people find out the terminal velocity.

So, I agree that I know initially you know there could be some case you know where they would also need not be terminal velocity, but typically you kind of work with a really large column its not that you know. So, they basically the fluid that your using for separation it, typically expenses a reasonable amount of time in the residence time is typically larger. So, in that timeframe you know the particle does attain a terminal velocity and your separation is typically based on whether the velocity of the fluid is either larger or smaller than the terminal velocity of the given time ok. Any more questions no?

I just said that you know your you know your buoyancy right. So, if you this is basically based on the Archimedes principle right all you have to do is I have a fluid I have a particle that enters the fluid that is going to displace some volume of the fluid ok. I should calculate what is the mass of the fluid that it is displace and time acceleration right. So, your external.

Correct.

So, I only said you know in this case F b I wrote it as m by ρ P times ρ into ae right typically we you write it as m times g right. So, in this case g was the acceleration due to gravity I said there you know I am going to use a general term ae which is the acceleration for a given external force that is all. So, therefore, if I use gravity your ae is going to be g if I use you know centrifugal force is going to be r times omega square right that that was the only.

No it depends on the external force which you are considering for example, if I. So, when you use external like say when people use centrifugal forces right typically you know centrifugal courses forces are measured in terms of whether the centrifugal force is 1000 times g 500 times g and stuff like that right.

Now, when you do that you are you really. So, if the gravitational if the acceleration because gravity is very very little then you know you are only talking only about the dominant centrifugal force in that case in such a case your g has to come from the external force that you act that you are using though this weight concept is true only when you are talking about are the buoyant force coming from the weight right its only true only for the gravitational force.

But however, if you use other external force you would have to worry about the acceleration that comes from the external force correct right. So, now, no I mean it’s the same thing right your this is your mass that is displaced right m by ρ P times ρ that is the mass of the fluid that is displaced by the particle times your acceleration because m by ρ P is a volume times g is your mass times the acceleration right ok.

Any other questions? So, I would like you to guys to go back and work out these equations ok. Just write down this 1 dimensional equation put in appropriate terms I would like you guys to derive those two equations the Stokes law and the Newton’s law because that we are going to use them for some of the tutorial that you are going to do later.

But you know you see if I have like say let us think about let us think about something like that I have a like say a play that is rotating now they say I have put a fluid with some particle ok. Now if you look at you know the relative you know the I mean. So, what you are trying to say is true when you know when you are the centrifugal force that you are using there, it becomes kind of comparable to gravity ok; however, if you take cases where you know you are really working at a very large rotation speeds ok. So, I think the motion because of the gravity you can actually neglect.

But again now you know you would have to think about resolving it you have to find out you know whether the gravitational force you know the gravitation activation and the acceleration that is because the different you know are in the same direction are you not worried about resolving and then take appropriate components for your analysis ok. So, we so, you should also think a little bit about working out the equation that we developed also for cases of other particles ok. So, we looked at a simple case of spherical particles, but; however, you know you can also develop similar expressions if you are you know thinking about settling of let see cylinder for example ok

If I have a cylinder which is basically falling with this configuration you know under the influence of gravity, then you know your ap which is a projected area that is going to be the a of a rectangle right your length times the diameter. So, you should have to you know take a appropriate ap right everyone agrees right. So, if I have a cylinder which is rotating ok so, which is falling.

So, the area that is perpendicular to its flow it is going to be the rectangle right that is you know your length times diameter; however, if it is rotate you know if its basically following with this axis in the direction of the external force, your again consideration is going to be same as the sphere right pi r square it basically sees a circle right. So, should be able to develop these equations for particles of other arbitrary shapes and they should be able to work out the settling velocity expression.

Yes it will right because the plot that we showed that I showed for CD was in a Reynolds number, it was written there is only for a spherical particle case ok. However, people have done experiments on you know particle there are rough irregular you know again discs like particle cylinders and stuff like that you know.

Also you would have to worry about all of that as well. So, even if you say that you know you have to think about you know the area that is in contact with the fluid right. If you say that that is going to be if its 4 pi r square right its going to be 4 times larger if you want to do that way. I do not have an exact reason as to why it is done like that maybe I will just think about it maybe you know answer to the next class.

So, the definition of ap is the area that the fluid a plane that is perpendicular to the motion of the fluid right. So, if the liquid is flowing in a particular direction or a particle move right. So, you have to take an you know plane which is perpendicular in motion and the area that that plane sees is what it what that is how it is defined.

But I will just look up and let you know if the why it is in a particularly way ok. So, I will stop here today and then tomorrow we will try and look at tomorrow we are going to have a tutorial based on this whatever you discussed so far. So, is going to be in the in MSB 356.