Fluid Mechanica
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Lecture - 30
Motion of a particle in a Fluid

We will talk a little bit about motion of a particle in a fluid. So, when we look at this title right, motion of a particle in a fluid, do you come across cases where you see motion of particles in a fluid? I mean I am talking a particle, but you know in a general sense it could be a object right, it could be any object it could be any object. Can you think of examples where you come across motion of objects in the fluid?

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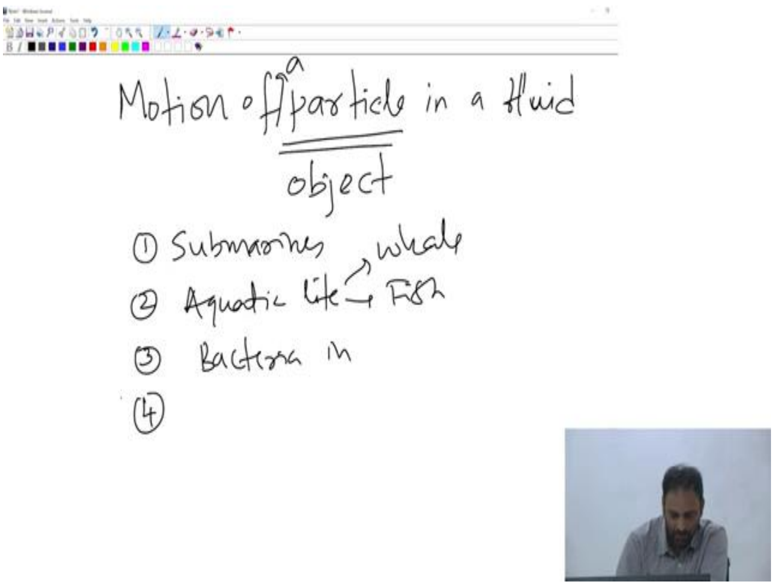

Yes right, some examples; I am sure all of you know right some examples of, it is a submarine right so, if you have a submarine in you know you know in like say sea for example, right it is moving underwater right. A lot of aquatic life ok, it could be whale right, it could be fish right or it could be any other thing ok. You can you also come across motion of bacteria in a fluid right, plus if somebody is interest in swimming right you can think about swimming of people in a fluid right. So, there are a lot of instances where motion of an object or a particle comes into picture ok. I am going to do is I am going to ask you to do a simple calculation ok.

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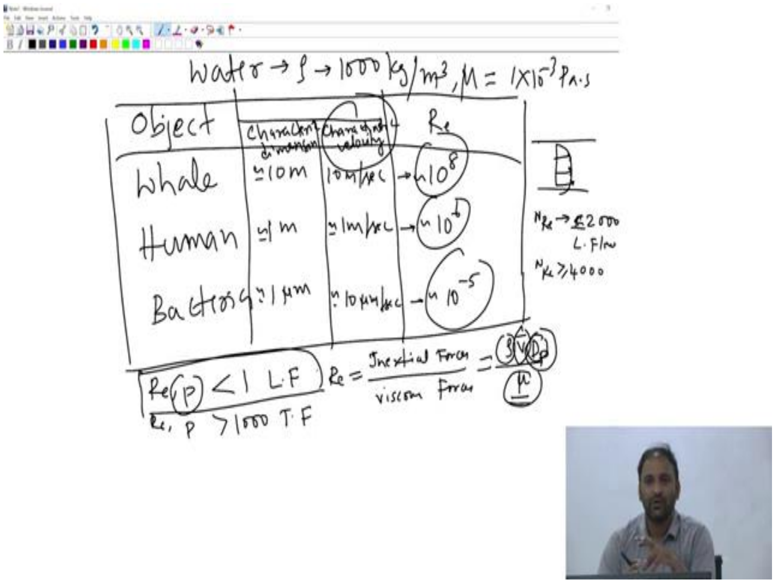

Let us think about your; I am going to have a table ok; I am going to say this is going to be your object ok. Let us take three cases, maybe four cases; a whale and let us think about a human or a man and let us take a bacteria ok. They are all swimming in water and the properties of water like say is ambient temperature so, your ρ is going to be 1000 kg per meter cube is the density of water and your viscosity, what is the value of viscosity of water 25 degree centigrade? It is 1 into 10 power minus 3 Pascal second right; that is your viscosity ok.

So, I now let us think about the characteristic features of these objects ok. Let us say that, what do you think is a dimension of whale, characteristic dimension? What is the dimension of whale typical dimensions?

Student: 20.

20 meters right. It is 10’s of meters. Let us say we can approximate it to be something like say 10 meters with the order of meters ok. So, let us say its 10 meters what about humans? What is that?

Roughly again, you know let us say of the order of meter right, 1-meter right is of the order of meter. What about bacteria?

What is that? What do you think would be the size of the bacteria?

What is that? Micro?

Student: Micron.

Micron right, typical length scale could be of the order say 1 micrometer ok. There are there are lot of bacteria's, there are lot of viruses' and typical dimensions of these things are of the order of a micrometer. That is your characteristic dimension. What do you think about the characteristic velocity with which these things move? What do you think would be the speed if like say let us say bacteria, what do you think would be the speed ok? So, they typically swim with the velocity which is the order of say 10 power 10 micrometer per second ok. That is the speed with which you know they move in a fluid ok. What about humans?

What is that?

For me about depends on you know from person to person. So, if you if somebody is a great swimmer, you know they will do you know little faster right. So, typically again it is going to be something like you know 1 meter per second. That is the velocity ok, you can kind of you know swim the length of your body in a you know a second or something like that ok. What about a whale? Similar dimension so, it is going to be something like 10 meter per second ok. That is the typical ok. Now I want you to calculate, what is the Reynold's number that is associated. I want you to calculate what is the Reynold's number that is associated with the motion of these objects in water ok. That is the question.

So, how do we go about doing it? You can just do that, just plug in all these numbers. I would like you guys to tell me the Reynold's number that is associated with the motion of these three different objects, you know which have different length scales, which I you know have a different velocities with which they are moving. I would like you guys to calculate these numbers and tell me what is the Reynold's number that is associated with the motion of these fluids these particles. you do that quickly. All of you know right the definition of Reynold's number all of you know ok.

Reynold's number is it is defined as inertial forces divide by viscous forces right; that is what you learned and typically it is defined as ρ V bar some length scale by viscosity right. That is your definition of Reynold's number.

$$Re = \frac{\rho \bar{V} D}{\mu}$$

So, can you calculate what is the Reynold's number. So, mu is the viscosity of the fluid right so, that is given to you right. What about ρ? P is the density of the fluid that is also given to you. What do you say about D? D is your characteristic dimension right. When you talk about flow through a pipe typically D is going to be the diameter of the pipe right.

Now, because we are interested in looking at the motion of objects in the fluid ok, that D should actually be the diameter of the particle ok, because the relevant length scale in this case is the size of the object or the dimension of the object, which is swimming in a fluid. Therefore, your Reynold's number definition your your characteristic length scale is going to be D P which is the diameter of the particle ok. That is one modification that you would have to do for the Reynold's number formula that you learned.

What about V bar? Again, you know flow through pipe problem V bar is your average velocity with which the fluid is flowing through the pipe; however, in the case of fluid particle you know systems ok. If you have an object which is moving in a like say a stagnant fluid for example, then your V bar is going to be the characteristic velocity with which the particle is moving in the fluid ok. So, with this you should be able to get what is the Reynold's numbers right. Can you just can someone tell me what is the Reynold's number for case one?

Student: 10 power 8.

What is that 10 power?

Student: 8.

10 power 8. What about the next one?

Student: 10 power 6.

10 power 6.

10 power.

Minus?

Student: 5.

Minus 5, right? Now you also learnt a little bit about laminar flow and turbulent flow right in the class so far. So, now, there are certain range of Reynold's number for which you are going to consider the flow to be laminar flow. There is going to be a range of Reynold's number for which there is going to be a transition regime and of course, there is going to be a range of Reynold's number beyond which your flow is going to be turbulent flow right, and when you talk about flow through a pipe right, if you have like say fully developed flow you know your parabolic profile and all that what is your Reynold's number for laminar flow conditions?

What is that?

Approximately about 2000 right. Anything that is less than or equal to 2000 is going to be it depends on textbooks so, is 2100. So, it is about 2000 anything less than that its laminar flow. What about turbulent flow? That is your laminar flow, something about 4000 right. Anything about 4000 is about is turbulent flow. In the case of fluid particle systems so, these numbers get a little shifted ok. So, typically when you are you refer Reynold's number in terms of a Re with P at the as a you know subscript; that means, it is a particle Reynold's number.

If the particle Reynold's number is less than 1 that is your laminar flow condition. If your particle Reynold's number is greater than 1000's that is your turbulent flow condition and anything in between is going to be your transition regime ok.

$$Re_P < 1 \rightarrow Laminar\ flow$$

$$Re_P > 1000 \rightarrow Turbulent\ flow$$

Now, if you look at you know the flow through pipes and problems where you had you know fluids flowing through conduits, your transition from laminar to flow it is actually governed by you know your velocity of the fluid, viscosity, you know the parameters right. However, it turns out you know what you have is a case where you have everything, which is basically moving in the same fluid right, you have your fluid in which the objects are moving is fixed, that is your water ok, with the given density and viscosity; however, all that you change was the object right. You had a whale swimming you know a human and a bacteria and you can look at you know the Reynold’s numbers right they are markly different ok.

Therefore, the flow around objects are going to be very different than the flow or you know in pipes ok. And so, typically when your Reynold’s number is less than 1, you can think about it as a laminar flow conditions and depending upon the fluid object combination that you are going to look at ok, you are going to the object is going to experience very different kind of flow field around it ok.

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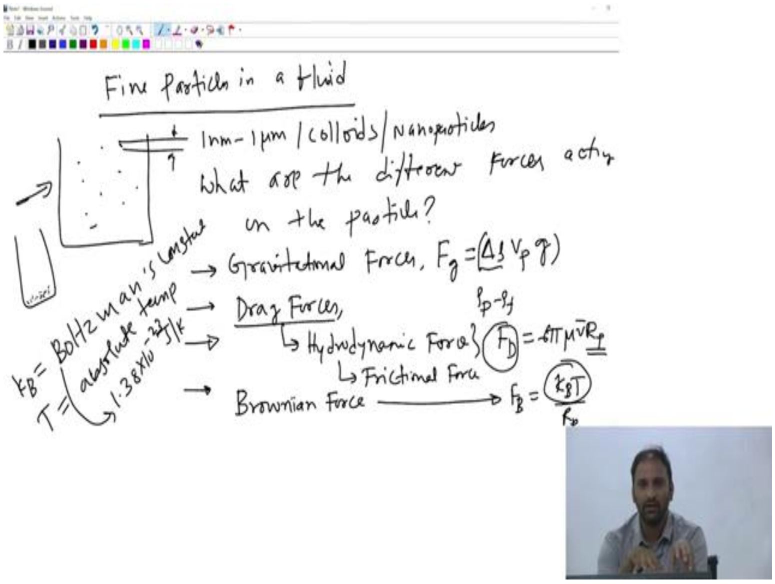

Now we will move a little bit into so, we are going to take a simple case of fine particles ok, in a fluid ok. What I mean by fine particle is, so, I have a container, I have particles ok, the dimension of these objects if I were to think about dimension of these objects is going to be in the range from about 1 nanometer to 1 micrometer ok. That is the dimension of the object. So, typically people call them as colloids or nanoparticles right.

If you remember the movie that I showed a couple of classes ago; so, there was a movie in which you had objects you know which were kind of moving around right and so, now, if you take that such tiny particles and if I put them in the fluid, I am going to ask a question as to what are the different forces acting on the particle?

I have tiny particles which are in a fluid and I would like to have an idea about what are the different forces that are acting on the particle. Can you name some forces? Of course, the first one is going to be; obviously, gravity right your gravitational force ok. So, if you call it as F g ok, that is going to go as delta ρ there is a density difference ok. Delta ρ is your ρ p minus ρ of fluid ok, where ρ P is a density of the particle and ρ F is the density of the fluid times volume of the particle right times the gravitational constant right your g acceleration due to gravity right. That is your gravitational force.

$$F_g = \Delta\rho V_p g$$

Any other force?

What is that?

Student: Drag force.

Somebody says drag force. Do you think that is going to be a drag force? So, we you have learnt that you know the drag force comes into picture whenever there is a relative motion between the particle in the fluid and especially when you take the tiny particles 1 nanometer or 1 micrometer particle say it turns out you know they were moving around and that moving around was because of what is called as a Brownian motion right.

And whenever there is a relative motion between the particle in the fluid, you know your drag force is going to come into picture. People call it as drag force there is also another term for it is called as a hydrodynamic force. You can also call it as hydrodynamic drag and that if you say it as F D, any thoughts what is a functional form for this? I am sure all of you have learned this at some point. It is 6 pi ok, mu which is the viscosity of the fluid some average velocity with which the particle is moving and your R or P or whatever right your size of the particle ok. That is your drag force.

$$F_D = 6\pi\mu\bar{v}R_p$$

Any other force? So, is gravity, is drag force, any other force that you think is important?

What is that?

Student: Friction.

Friction, drag they are the same know; when people also call this as frictional forces if you want to write another name ok. Other one what is called as a Brownian force and this

Brownian force it goes something like as F B; it goes as K B T that is your thermal energy, divided by R P ok. So, KB K B is something called as a Boltzmann constant right and T is your absolute temperature the value of K B T is something like 1.38 into 10 power minus 23 joules per Kelvin ok.

$$F_B = \frac{k_B T}{R_P}$$

$$k_B(Boltzmann\ constant) = 1.38\ X\ 10^{-23}\ J/K$$

$$T \rightarrow Absolute\ temperature$$

Let us think about again calculating these forces ok. I would like you guys to calculate what is your what is the FD ok.

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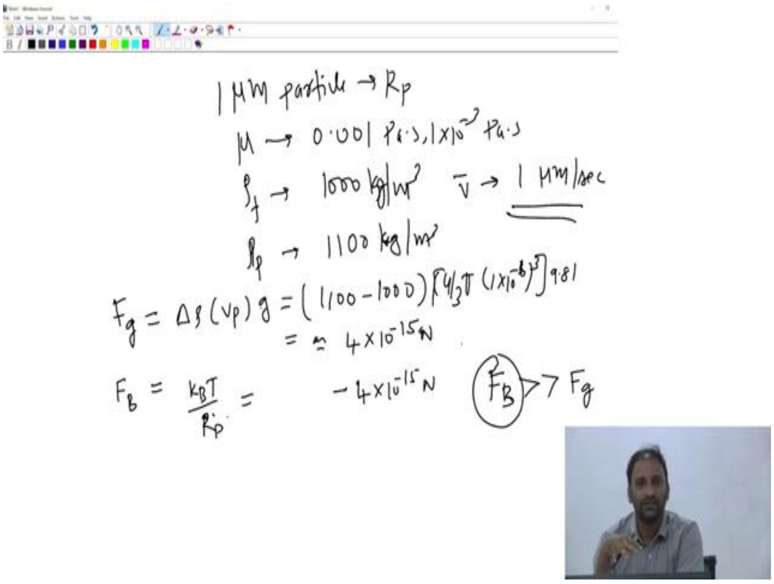

So, if you take a lets think about a simple case of 1 micrometer particle ok, like this is going to be your R P and again let us go back to your viscosity of the fluid is 0.001 Pascal second or 1 into 10 power minus 3 pascal second. Your density of the fluid is again 1000 kg per meter cube ok., let us take a particle the density of like say 1100 kg per meter cube and if you take a 1 micrometer particle typically it is known that you know it kind of moves around with the velocity which is the order of 1 micrometer per second ok.

That is a typical ok, if people have done these measurements, so what they have done is, they have taken a microscope, they have observed the velocity with which the particles move and that typically for a 1 micrometer radius particle it is of the order of 1 micrometer per second. If you put in all these numbers can we get an in get values for say F g and F B. You just quickly work it out.

What is that?

Volume is 4 by 3 pi r cube right. You know the you know the dimension of the particle you can actually calculate what is your volume of the particle. Yeah what are the numbers? Yeah somebody has numbers. What is F g? This should be of the roughly 4 into 10 power minus 15 Newtons. This also should be of the order of or if you have these numbers let us let us quickly check yeah. So, so the number that you will get would be your drag force is of is of the order of 4 into 10 power minus 15 Newtons and your FB would also be of the similar order ok.

$$F_g \sim 4\ X\ 10^{-15} N$$

$$F_B \sim 4\ X\ 10^{-15} N$$

Now, so what I can say from this calculation is that, if you have a particle of 1 micrometer radius and if it is immersed in a in water and if the density difference is about 100 kg per meter cube, it turns out that you know your F g and F B they are comparable right. Now what happens if I reduce the size of particles? If I have like say instead of you know 1 micrometer, if I take say 10 nanometer particle ok. Obviously, what you will see you know is you will see that you know your F B which is the a Brownian force is going to be much greater than F gravity right ok.

Therefore, so whenever one is working with you know fine particle system it is a good idea to kind of think a little bit about different forces are acting and also do a simple calculation of the different force that will be acting on the particle that will give you some information about whether your gravitational force are really important in terms of making your dispersion or the fine particle dispersion unstable ok.

Whenever you know, when I drew this picture right so, now whether the particles continue to remain you know in the solution or they would go back and sit at the bottom, that would depend on the balance between the different forces that are acting on the particle right. Now there is a gravitational force which wants to pull the particles down and you have Brownian which wants to keep it in a suspended state right. Therefore, if you go for finer and finer particle sizes it so, happens you know your Brownian force overweighs the gravitational force that is one of the nice way of imparting nice stability for the particle dispersion that you are working with ok.

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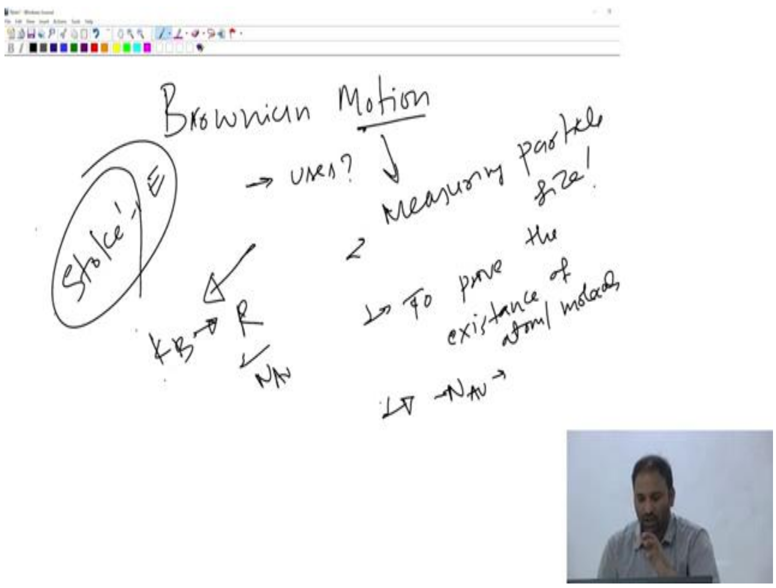

Now, that we know a little bit about fine particle dispersion and different forces now, I am going to ask a question as to so, I said that these particle fine particle they exhibit Brownian motion right. Do you think there is any practical use of this chaotic motion that you know the particle exhibit or the fine particles exhibit you know when you put them in a fluid? Do you think they are any applications of it or any uses? Anyone has read a little bit about Brownian motion? You read at some point for sure right in your plus two or you know maybe before that right.

So, it turns out that you know the concept of Brownian motion was really useful and also it is useful you can use this for actually measuring particle size ok, number one. Number 2, people have also used this Brownian motion to actually prove the existence of atoms and molecules ok. Before there was a debate about you know the whether the molecules are there atoms are there or not ok. So, people kind of use some nice simple experiment that they could do by monitoring the Brownian motion of particles you know in a fluid and they were able to kind of come up with a come up with the explanation for the fact you know the molecules do exist, plus have you heard of Avogadro number right?

So, we know that you know 1 mole of any substance has 6.023 into 10 power 23 number of atoms right or molecules right. Now, one of the proofs for the calculation of Avogadro number, it actually came from measuring the Brownian motion ok. What someone called John Perrin he is a French scientist; what he did is he used this Brownian motion concept and from that he was actually back he back calculate what is called as a KB which is the Boltzmann constant and there is a relationship between Boltzmann constant and R which is a universal gas constant and your Avogadro number. So, he used that to back calculate what is the Avogadro number. It turns out that you know the number that he got was very close to you know what is known now ok.

So, we will talk a little bit about these concepts in the next class ok. So, I am going to start with showing you again some more videos of Brownian motion ok. Yeah explain a little bit about why is it important you know and what kind of measurement that people did and then we are going to derive one very important equation; which is what is called as a Stoke's Einstein's equation; Stoke's Einstein's equation which is one of the fundamental equation that people use for measuring the particle size data ok. So, we will talk a little bit about that in the next class. I will stop here today yeah.