Fluid and Particle Mechanics
Prof. Sumesh P. Thampi
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 03
Newton's law of viscosity

So, when you talk about problems in fluid mechanics, one of the property that comes up often is the viscosity of the fluid.

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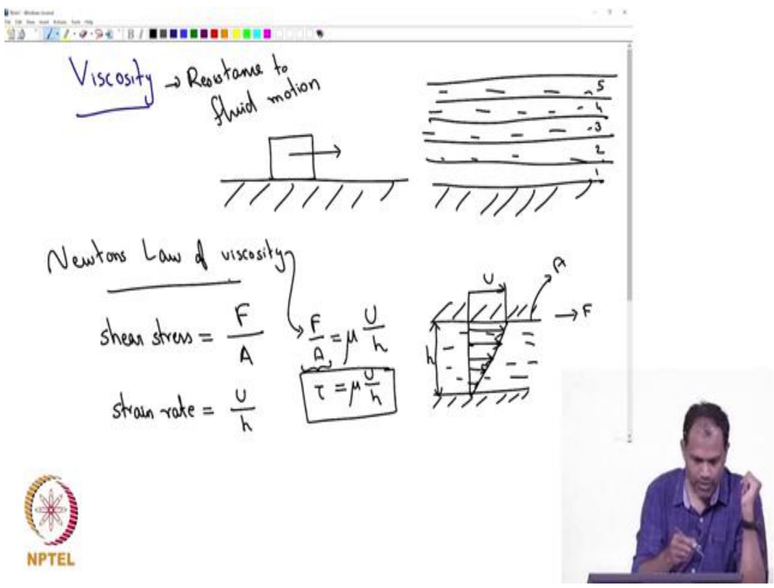

So, let us just briefly look at what is viscosity? So, viscosity actually refers to the resistance of the fluid when it is trying to flow. So, the concept is very simple, let us say you have some solid object; let us say you have a solid object and that is placed on a another solid substance. And let us say you are trying to move this box in a particular direction and what happens is it because the and the object on which it is kept is not moving, its going to x you know its going to resist the motion of the this the solid object that you have placed. In other words this box is going to be experiencing a frictional force that is generated between the box and the underlying substrate.

Similarly, if you have some wall let us say and then there is fluid on top of it, if the fluid on top of this wall is trying to move the wall is going to offer some resistance it's going to generate some frictional resistance and viscosity exactly characterizes that resistance. Now, the point is that you can imagine that the fluid that is above the wall is constituted by various layers of the fluid, let us say the way I have drawn this lines ok. So, this is a layer 1, this is layer 2, layer 3, layer 4, layer 5 and so, on.

So, the wall is going to give some resistance to the layer, the layer that we we have numbered as 1 the layer 1 will offer a resistance to layer 2, the layer 2 will resist offer a resistance to layer 3 and it keeps going ok. So, the viscosity of the fluid acts between any 2 layers of fluid that you are going to see or you will have it ok. So, the viscosity is nothing, but the resistance that comes about when 2 fluid elements are moving with 2 different velocities. If the fluid elements are moving with the same velocity then viscosity has no role to play.

So, viscosity is nothing, but really the viscous or the resistance for the flow for the fluid motion ok. So, that is a resistance to fluid motion or a measure of resistance to fluid motion. So, how does one calculate this ok? So, that is where Newton's law of viscosity comes into picture, Newton's law of viscosity it says that you take fluid between 2 plates. So, you have a plate and another plate and then there is fluid in between as shown by the dashed lines and let us say you apply a force F on the top plate ok.

So, what will happen is that the top plate will start moving, which means the fluid just below it will start moving and in the fluid below it would start moving and finally, the entire fluid would be moving except the fluid that is at the bottom of the plate. So, if you look at the velocity profile meaning that the velocity at various locations it would look something like this.

So, the top plate would be moving with a velocity almost with the velocity as that of the top plate, then the next layer would move with a slower velocity, the next layer will be even slower and that keeps going till the last layer is not moving. So, this is how the velocity profile is going to look like. Let us say to get that you have applied a force F, then you can define a shear stress as the force that you have applied per unit area where A is the area of this plate ok.

$$Shear\ stress = \frac{F}{A}$$

So, A is the area, so force per unit area tells you the shear stress. The strain rate is defined as let us say this velocity at the top is u and the plates are separated by a distance h, then:

$$Strain\ rate = \frac{U}{h}$$

And from Newton’s law of viscosity:

$$Shear\ stress, \tau = \frac{F}{A} = \frac{\mu U}{h}$$

$$\mu \rightarrow dynamic\ viscosity$$

So, that is what Newton’s law of viscosity, so that is how viscosity is typically measured also.

Now, so what does? So, shear stress is nothing, but the force that you have applied right. So, it is a measure of force per unit area what the strain rate indicate? One another name for strain rate is that you know it’s changes in velocity per unit length right.

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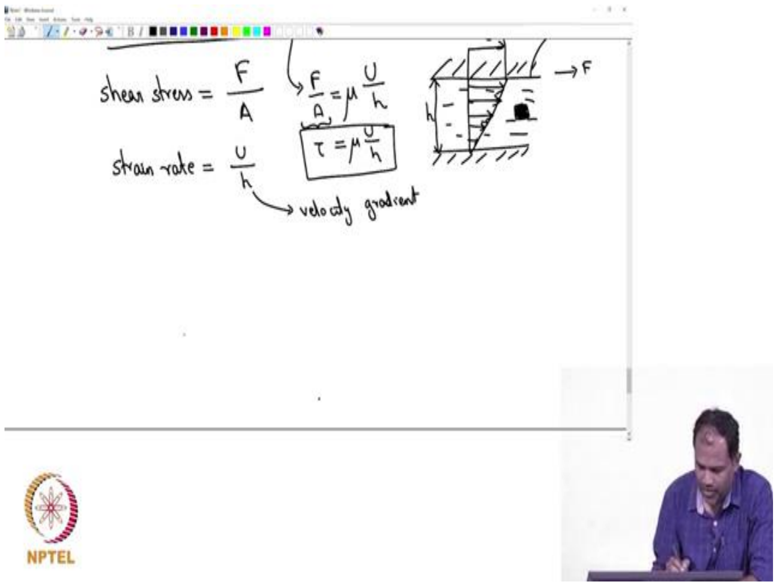

So, in that sense this is also a measure of velocity gradient. So, gradient typically refers to things how much it is changing in some direction ok, so this is basically change in the velocity. So, what does it mean? So, in order, to say that let us just take a fluid element, let us say a small fluid element that we take from here. So, the one which I have shaded is the fluid element that I have taken and I am going to make a slightly bigger drawing of that.

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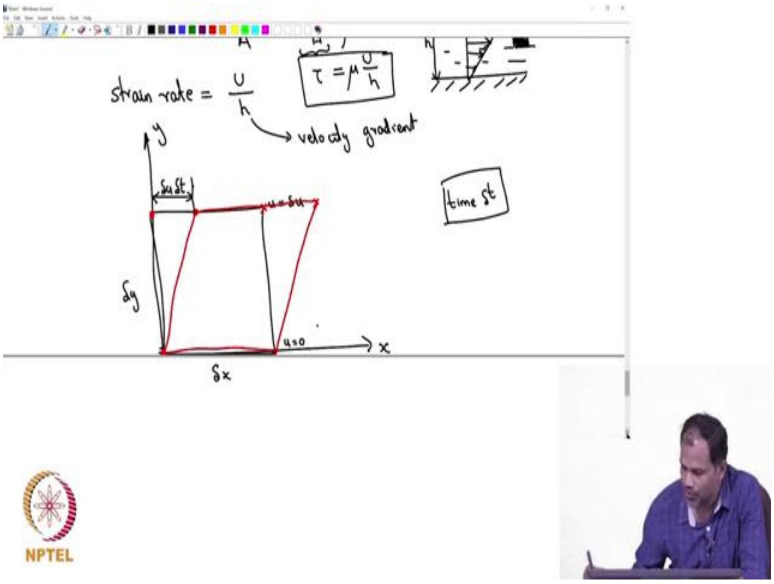

Let us say that is the fluid element that we have considered, let us call that axis x, let us call that axis y let us say this is small fluid element and elemental volume. So, this distance horizontally is δ x and δ y is the vertical or the height ok. Now, what we want to do is that we want to find out how what is the effect when there is a fluid, well actually what is the effect or what is what is going to happen to this fluid element.

So, now imagine this is the fluid element the shaded one is what we have considered and the velocity at the bottom of this layer is smaller than the velocity at the top of this element. So, to represent that let us say that the velocity at the bottom of the layer is u is equal to 0 and let us say u is equal to δ y is the velocity at the top or otherwise you could say that u is the velocity at the bottom and u plus δ sorry δ u u plus δ u is the velocity at the top ok. Or the whole point is that the to the top layer and the bottom layer are moving with different velocities.

$$Bottom\ layer\ velocity, u = 0$$

$$Top\ layer\ velocity, u = \delta u$$

And let us say you are looking at things after a time t. So, at time t is equal to 0 this square is the configuration and you want to find out what is the configuration after time t equal after time t. So, now, we have assumed that the bottom line having a velocity u is equal to 0, so; that means that if I look at this corner after time t it’s going to be there. If I look at this corner that corner is again going to be at the same place after time t. But if I look at that this top corner that is going to move, that is going to move with a velocity u and it will cover a distance u times t in time t.

So, this the point that I have marked as the red dot would be somewhere else after some time ok. So, after some time this the point that is marked as the red dot would have moved to a new point. Similarly let us say the point that I have marked as the cross would move to a different point because the top layer is moving. In other words the shape would look like that. So, the square does not remain the square anymore, the square would have changed to a deformed shape.

So, this is what is going to happen in a small time. So, let us say the time that we are talking about is really a small time δ t ok. What is the distance that the top layer would have moved? So, since it has got to a velocity of u and then we are looking at a time of δ t. So, this would have moved a distance δ u times δ t velocity times the time gives you the distance that it has moved. So, this is the configuration or this is what happens to the fluid volume. Let us calculate what is this angle let us call it δ θ.

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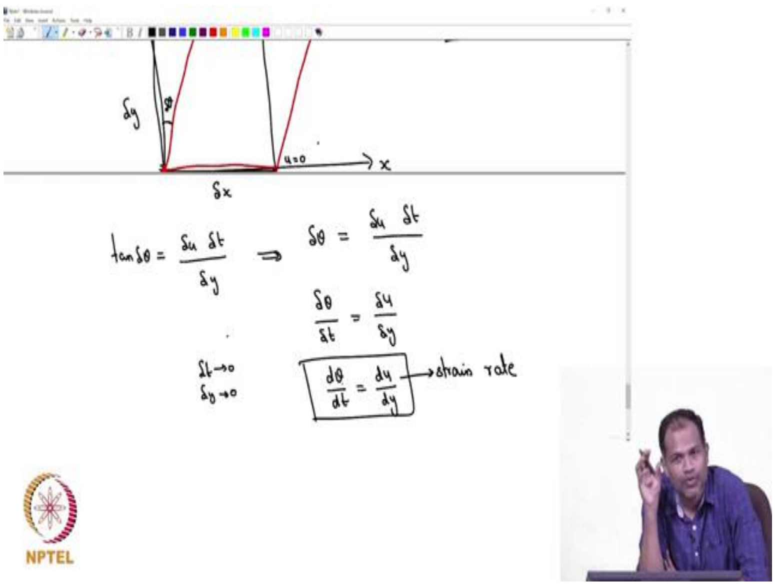

So, I can calculate tan δθ as:

$$\tan \delta\theta = \frac{\delta u \, \delta t}{\delta y}$$

And I am looking at δ θ being really small in which case tan δθ can be approximated as δθ therefore:

$$\delta\theta = \frac{\delta u \, \delta t}{\delta y} \rightarrow \frac{\delta\theta}{\delta t} = \frac{\delta u}{\delta y}$$

But I will, I am interested in trying to find out when δt→ 0 δy→0. So, we are looking at an infinitesimally small element which has got a differential velocity and in a small time ok. So, each of this differences can be approximated as derivatives and therefore:

$$For\ \delta t \rightarrow 0, \delta y \rightarrow 0; \frac{d\theta}{dt} = \frac{du}{dy}$$

So, remember for the Newton's law of viscosity, we defined τ or the shear stress is proportional to viscosity times the strain rate. The strain rate which is du by dy, so this is the strain rate right strain rate.

So, the physical meaning of strain rate is that it tells you what is the change in the angle in unit time. In other words the change in angle, so if d θ by dt was 0 that would mean that the shape does not change, the shape remains as a square which would happen if there was no difference in the velocity. So, d θ by dt is telling you how fast the square element is getting deformed ok.

So, shear strain rate is nothing, but the rate of deformation and Newton's law of viscosity simply says that the force that you are applying is proportional to the rate of deformation and the coefficient that connects the force with the rate of deformation is nothing, but viscosity. So, that is what Newton's law of viscosity is that also tells you how you define your shear stress, what is the physical meaning of strain rate.