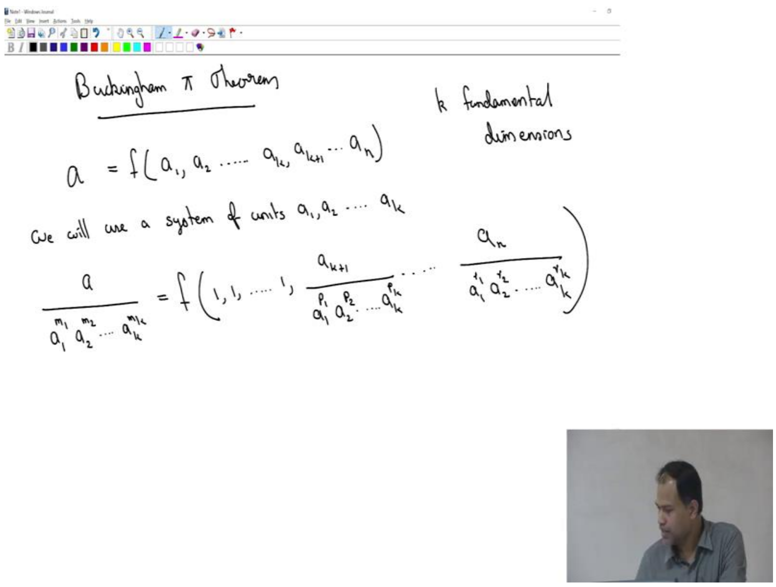
Fluid Mechanics

Prof. Sumesh P. Thampi
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 27
Non-dimensional analysis - 5-Concept of similarity

(Refer Slide Time: 00:19)



So, to remind you what we were looking at the; so, we said we have some physical process let us say which contains a dependent variable a which is a function of several independent variables

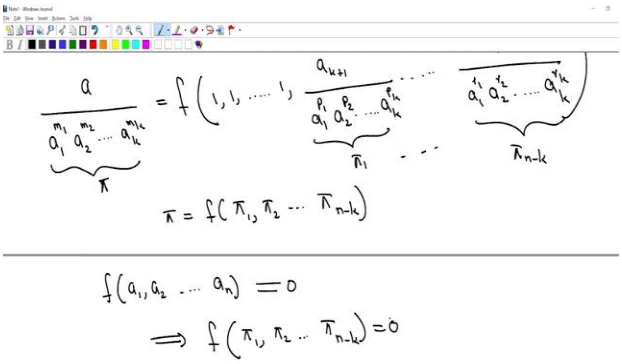
$$a = f(a_1, a_2, \dots, a_k, a_{k+1}, \dots, a_n)$$

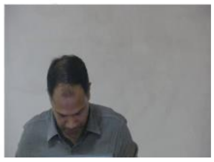
$$\frac{a}{a_1^{m_1} a_2^{m_2} \dots a_k^{m_k}} = f\left(1,1, \dots 1, \frac{a_k}{a_1^{P_1} a_2^{P_2} \dots a_k^{P_k}} \dots \frac{a_n}{a_1^{r_1} a_2^{r_2} \dots a_k^{r_k}}\right)$$

So, this is where we ended up yesterday. So, what did we have we have generally had a as a function of several independent variable and we selected a system of units which is given by a few of them. So, we had k fundamental units.

So, we have taken k variable and we expressed all other variables in terms of that k variables and that is all the proof is because each of these quantities now you have ended up are non dimensional numbers ok.

(Refer Slide Time: 03:03)





$$\pi = f(\pi_1, \pi_2, \dots, \pi_{n-k})$$

$$f(a_1, a_2, \dots, a_3) = 0 \rightarrow f(\pi_1, \pi_2, \dots, \pi_{n-k}) = 0$$

So, if you have n variables and k dimensions you will find out n minus k independent way in the n minus k dimensionless groups. Then I want to tell you something else and that is actually a story; it is a story that typically goes with Buckingham pi theorem actually dimension analysis. The story that we may see its almost true I think over a time it has got lots of spices into it, but still I think its overall a good story to see. So, let us see ok.

(Refer Slide Time: 04:35)

Trinity test – the code name of the first detonation of a nuclear weapon

By US at 5.29 am on 16th July 1945 in New Mexico

Part of Manhattan Project

(Source : Wikipedia)

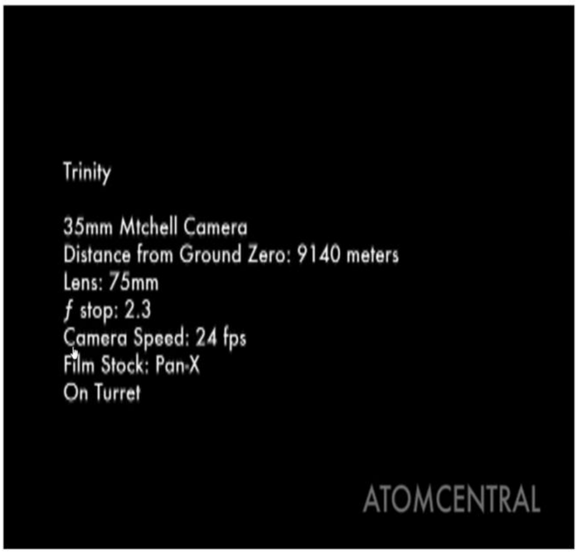
So, it is a here you go ok. So, trinity test actually it is a code name it was code name of the first nuclear bomb that was tested. So, now, I mean you theory act and everything is very common, but long time back that is 1945. When actually this test was held this was held in right now, it is in Mexico in a place this is in fact, this all things are taken from Wikipedia you can read more about there its says that it was in on 16th of July at 5.29 AM this happened. Actually it was held or it was suppose to happen at 4 o clock because of rain then you known it was moved to 5.29.

Now, all this details are precisely given and that actually shows how important this test was. We will see why this test important was and right hand side that you are seeing is just you know about that you will see if you go there at the moment it is a protected side.

(Refer Slide Time: 05:39)

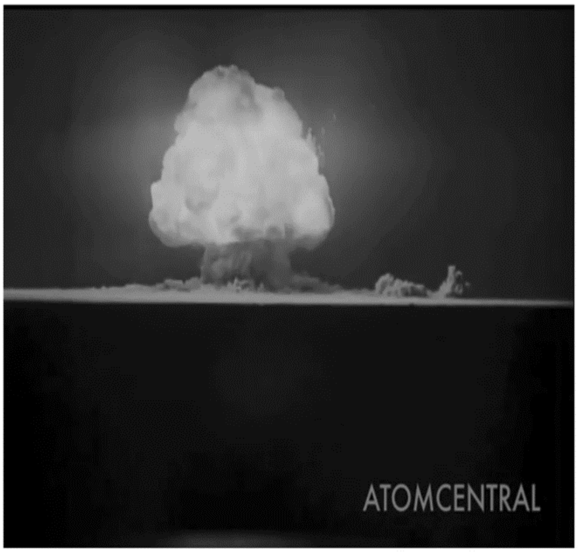
Let us see video of what this test look like.

(Refer Slide Time: 05:59)



So, this is shot from a distance of almost 10 kilometre away from where that test was actually happening; 10 kilometres is quite far you can imagine. Did you see that explosion? You saw something right.

(Refer Slide Time: 06:13)



.

So, there I think its better now. So, imagine you are seeing it 10 kilometre away from where you are sitting and is that huge the huge fire ball that rises. It is a near to the ground here, you will see the top portion.

(Refer Slide Time: 08:09)

What is the big deal about the test?

- The world had never seen an atomic bomb before
- People doubted about the effectiveness of bombs
- People wondered about the amount of energy that will be released
- Apparently, some scientists in the laboratory didn't even think that the bomb would work

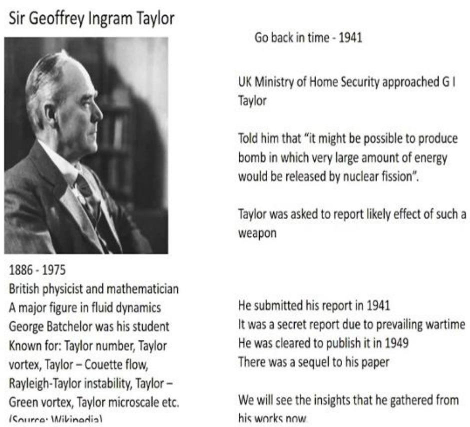
What happened?
The heat vaporized the steel tower
Desert sand was melted to a green glass (which is still slightly radioactive)
Windows 100s of miles away broke

When they found that it worked,

- *Little Boy*, the uranium bomb was dropped on August 6 1945 in Hiroshima
- *Fat Man*, the plutonium bomb was dropped on August 9 1945 in Nagasaki

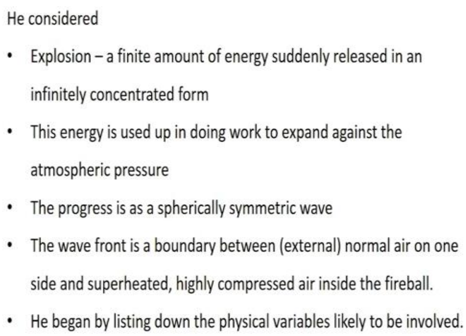
So, what is the big deal about the test? So, the point is that before this test, the world had never seen something called a nuclear bomb ok. So, people had developed nuclear reactions and so on some theory was available. Then theoretically, it was found that it is possible to make a nuclear bomb ok, but how strong is the theory will it actually happen?.

(Refer Slide Time: 08:37)



So, this was the time of Second World War so, now why this story I told suddenly there is a famous name associated with it and that is G I Taylor. We have already heard this name right when we talked about Taylor Couette flow this guy was a fluid dynamicist he is famous for several things that is in fluid mechanics. So, you heard about Taylor number Taylor Couette flow there are lots of other things like Taylor's instability, Taylors Vortices Taylors scales and so, on ok. So, lot of things he has contributed.

(Refer Slide Time: 09:13)



So, what he said is this look what is bomb it is basically a release of energy. A lot of energy that was concentrated at a particular place essentially gets expanded to a large amount there is a wave of energy that is going to come spherically in all direction that is what for him the bomb was. So, he said if that is the case let us write down what might be the relevant variable involved and that is a first step of doing non dimensional analysis.

So, let us see what are the various that he considered.

(Refer Slide Time: 09:49)

Physical variables involved

- E - the energy released by the explosion
- R - the radius of the fireball
- t - the time since detonation
- P - the pressure inside the fireball
- P_0 - the pressure outside the fireball
- ρ - the density of air inside the fireball
- ρ_0 - the density of air outside the fireball

7 variables, three dimensions, 4 dimensionless ratios

He said energy because energy is very concentrated it is going to be released in a spherically symmetric form. So, let us say E the total amount of energy involved is 1 of the variables that we want to worry about. So, that is E then R the radius of the fireball which is getting expanded and he definitely wanted to know how fast it is going to grow and how much it is going to grow ok. So, R is a the radius of the fireball t is the time over which this is happening.

Then he said there is a huge pressure that is going to be develop in the fireball right which is what is going to expand the whole thing and its going to work against the external pressure. So, he said let us consider the pressure inside the ball and the pressure outside the ball as two relevant parameters and he also said we will also worry about the density of air.

So, the density of the air is going to be a. So, it is will be a super heated air ok. So, you are going to consider the density of that and you are going to consider the density of the outside fluid against which it is expanding ok. So, this is because it is really that the high pressure is working against the atmospheric pressure to expand it. So, that must be the mechanism that is controlling how fast it is going to expand. So, let us say if these are the variables involved then what could be the relation that exist between these variables.

Now, you know how to do that because you know Buckingham pi theorem or you can use Ipsen's method find out what is the non dimensional number what are the non dimensional numbers that are going to come up.

(Refer Slide Time: 11:27)

Dimensionless groups

$$\Pi_1 = R\left[\frac{Et^2}{\rho_0}\right]^{-1/5}, \quad \Pi_2 = \frac{p_0^5 t^6}{E^2 \rho_0^{-3}}, \quad \Pi_3 = \frac{\rho}{\rho_0}, \quad \Pi_4 = \frac{p_0}{p}$$

The required law is then (again by the theorem)

$$F(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0.$$

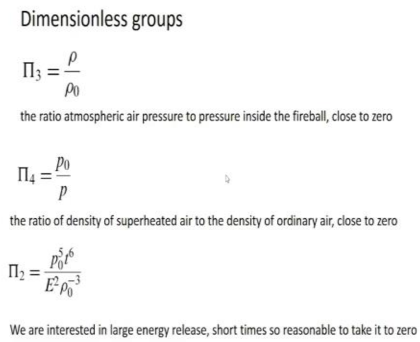
or equivalently,

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4).$$

So, that is the way Taylor chose to represent, but you might probably get some combinations of this you can actually verify that some of. So, we as we said it can be represented in many ways depending upon what it uses here repeating variables or what is select as your fundamental dimensions, we will get different different combinations, but they would represent the same thing. So, if you are getting let us say pi 1 and pi 2; if you combine pi 1 and pi 2 in a some particular fashion, you are going to get a same non dimensional number that he has written there.

So, he got these four of them pi 1 at some Et square by rho 0 to the power minus 1 by 5 then there was a pi 2 a pi 3 and a pi 4. And of course, as we have been seen there exist a relation between these four and that is what he wrote and that is not surprising to anyone more.

(Refer Slide Time: 12:21)

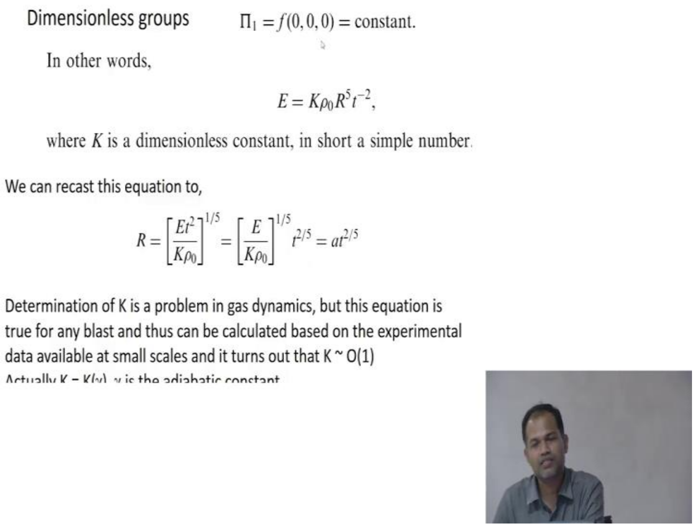


Now, he since he found that these four then he had to make some conclusions out of it. So, he thought what could be the best thing that he can do and he said let us look at some of those non dimensional numbers. There involved a non dimensional number that was ratio of the densities ok.

So, if you basically say look at it is density of air outside air by density of that fireball that is going to be extremely small because you have large amount of gas in a small amount of space. So, is presser ok. So, 2 non dimensional groups are going to be extremely small and he also said look at this pi 2 one of the non dimensional group that he go which actually had this particular combination and he said he would look at things, when time is very small; that means, at the very beginning of the explosion ok.

So, this is t to the power 6; that means, it is going to be extremely small quantity. So, in other words what he said he said I will look at that situation in which pi 3 pi 4 and pi 2 are very small ok. If you do that then what you so, originally you had pi 1 written as a function of pi 2 pi 3 and pi 4 and now you are saying that you are going to make pi 2 pi 3 and pi 4 very small or in other words that would come out to say that pi 1 is some function which is not dependent upon anything else. And therefore, pi 1 is a constant or some number whatever it is.

(Refer Slide Time: 13:49)



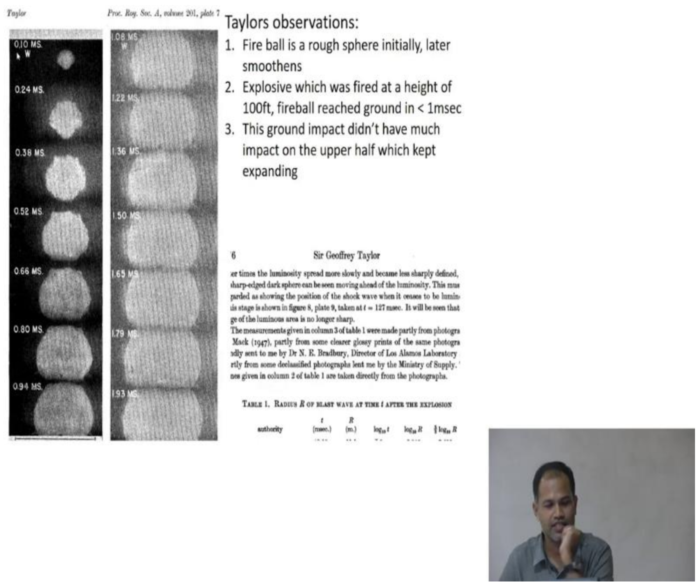
So, pi 1 was this expression which contained radius E t square and so on. See he wrote it in a nice form then that E is some constant times rho 0 times R to the power of 5 t to the power of minus 2 alright ok. So, where K would be some constant which we do not know what is going to come out or if you rewrite this equation as R as a function of t to which of course, you can do you will see that R is a into t to the power of 2 by 5 ok. So, he said that is how basically the radius of the fireball is going to grow it will grow with time as a function of 2 by 5 at least at the very initial movement wherever he could neglect other non-dimensional numbers.

$$R = at^{2/5}$$

(Refer Slide Time: 14:55)

Now, the test happened exactly witness the test also, but he did not get the details he had no idea about the numbers. So, he could not actually verify any of the calculation. And then, in 47 there was a magazine called this magazine life magazine which carry an article about this trinity test or the pictures that may release were of this form ok. So, basically you see the explosion and that was enough for Taylor because there was a length scale and a time scale present in that photograph you see the ok.

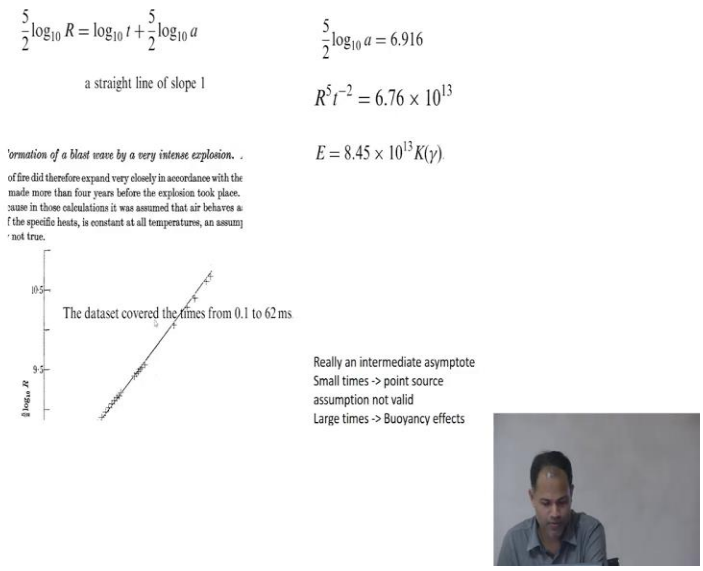
(Refer Slide Time: 15:35)



In fact, the magazine actually contain a series of them in which its see at 0.10 milliseconds, this was the how we look like and then it had information up to 1.93 milliseconds.

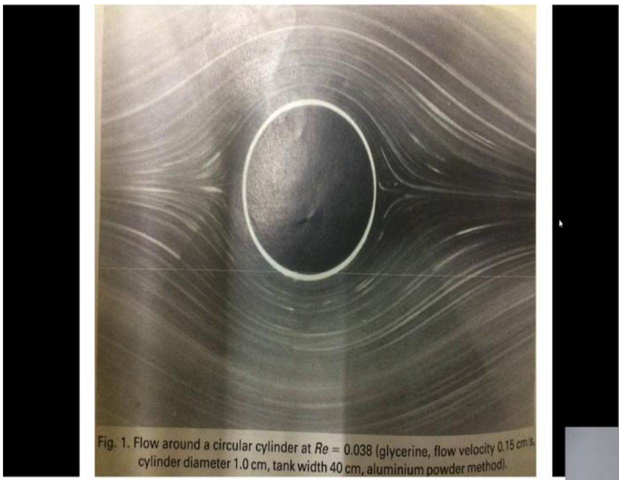
So, he got basically time and radius and that is all he needed he went back to his equation which so, you had derived that R is a function of a sorry some constant which is basically dependent upon other quantities t to the power of 2 by 5 where this constant a will now contain what is the amount of energy that is involves right. So, he fitted r versus t calculated a and calculated t the energy of the bomb which was actually classified information there ok.

(Refer Slide Time: 16:25)



Nobody knew, but he calculated and then. So, the graph actually because of this simple pattern during come well, but he also we can see the data points it came out to be a very nice straight line falling on t to the power of 2 by 5 ok. And then he back calculated what is the energy and this is the number that he came up with and in fact, it happened to be very close to what was actually used ok. So, that is the strength of non dimensional analysis without knowing anything you could still calculate some very relevant information.

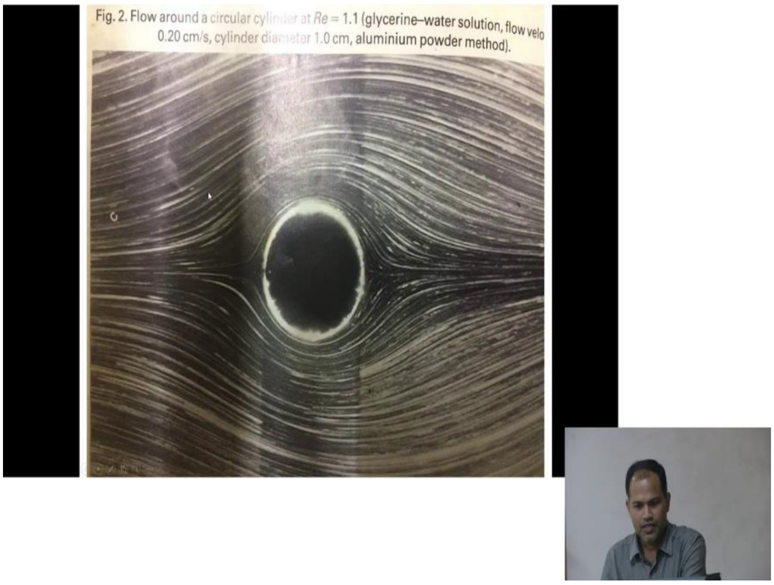
(Refer Slide Time: 16:57)



Yeah. So, this is I thought I will show you some pictures of the classic problem that we have been considering I have been telling that let us see we have a fluid you know that is flowing pass some object and we have been thinking about you know how to calculate the force on that object and so on these are from some experiments. So, these some experimental pictures and the way people do it I think, I mentioned sometime you just put some particles into the fluid and look at the motion of the particles that would tell you how the fluid flow is going to look like here you have this circle is actually cylinder.

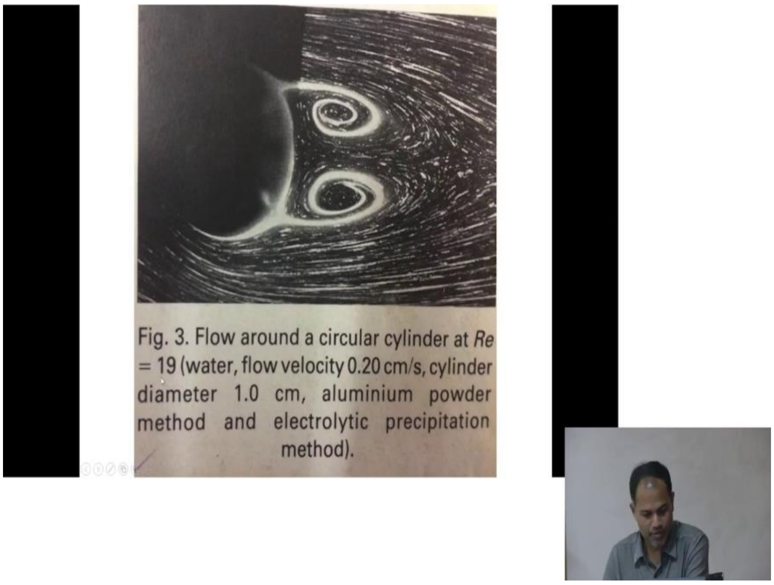
So, you have a cylindrical object fluid is flowing from one side and it is going to the other side ok. In this picture it is not obvious, but here the fluid is coming from the left side it basically you know deflects and when it goes back ok. So, these lines would be what lines this would be? Yeah these lines are stream lines in this particular case this is a streamline and this is this is a set a Reynolds number of 0.038 very small Reynolds numbers this is how you would see flow past a cylinder.

(Refer Slide Time: 18:13)



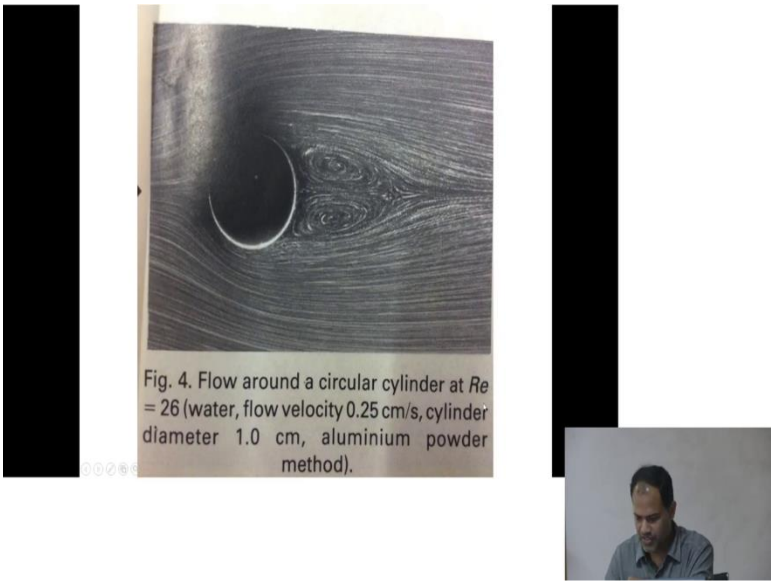
This is even at a slightly higher Reynolds number and you would see something very similar Reynolds number is actually 1.

(Refer Slide Time: 18:25)



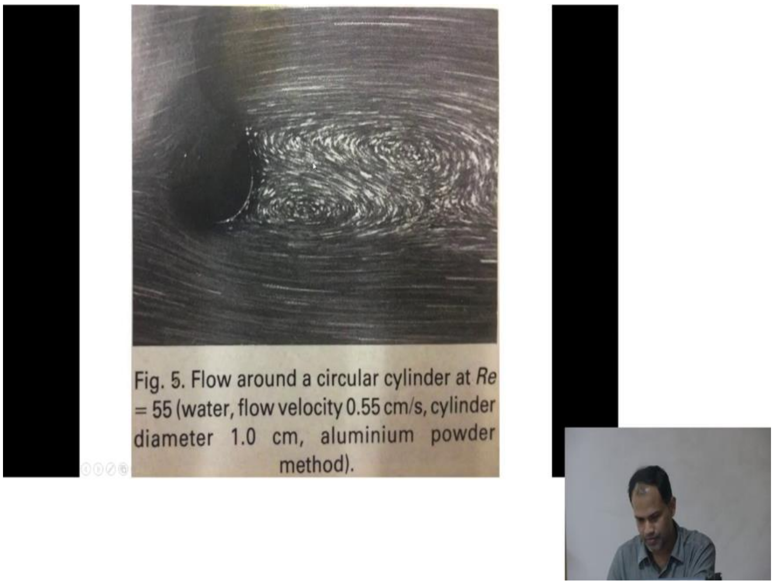
This is Reynolds number is equal to 19 when Reynolds number increases what happens is that the fluid is actually coming from the left side left side of your screen s and then it goes and then this circulations will develop at the rear end or the back side ok. These are called vortices and there is a lot of you know there is a strong fluid flow ok; that means, you actually lose a lot of energy because of the formation of this let us see that later.

(Refer Slide Time: 18:59)



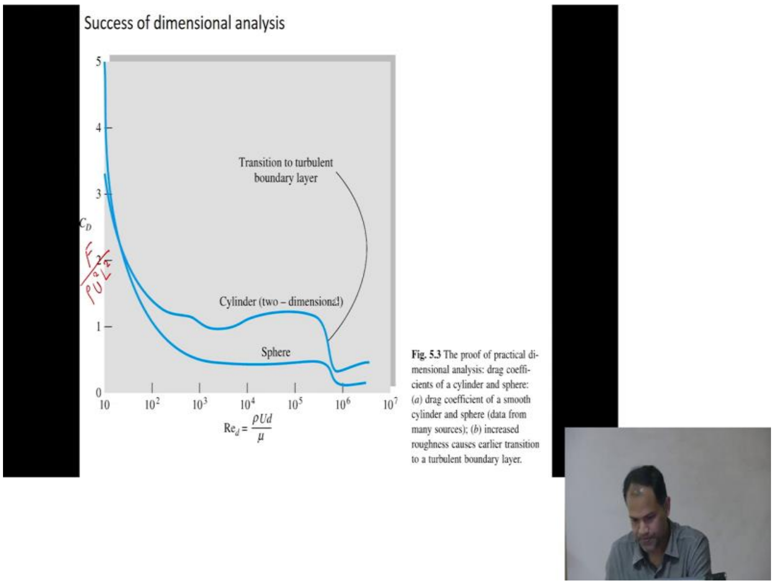
So, that is what would look like. It is another picture which is even at a higher Reynolds number. So, it basically comes and then it goes and this it goes, then this vortices actually become bigger this circulations.

(Refer Slide Time: 19:09)



And if you increase the Reynolds number further which is 55, this vortices will actually dislodge from the cylinder and then it actually start going away ok. So, this is a some picture of that then you can see that initially when flow was very neat there this nothing that you were losing ok, but know you basically have a large region with lots of you know vortices or recirculating region and you are actually going to lose a lot of energy because of this particular process yeah.

(Refer Slide Time: 19:47)



So, that’s what flow pass an object would look like and we know that you know you can actually calculate the force exerted by that object simply as a function of Reynolds number and what you see is essentially that graph on the left hand side its call the drag coefficient its the force then I should going to write yeah.

So, it is a drag force that he calculated over rho U square L square goes Reynolds number. That how you take this plot and this is actually plot that you get when you whatever experiential observation whatever relation you do, you can actually put it into the single line ok. So, there are 2 thing that you see one is that of the sphere and one is that of a cylinder; cylinder and sphere are 2 different objects. So, that curves would be different, but you can see that it is giving you a nice plot.

We still do not know how to calculate this theoretically there is no way you can calculate it except in some limits, but. So, this is the very important curve and you can see that as you change the object you are going to get different different curves like this. So, this is a way of representing.

So, here the flow is laminar, here the flow is laminar here also actually the flow will be laminar it would just be that there will be lot of circulation there will be lot of vortices like this, but that does not mean that the flow is turbulent ok. Here also I think the flow will remain turbulent by the time this reaches there is chances that some turbulent you know flows will develop in between the outside fluid will still continue to be laminar and remember that 2,100 has nothing to do with this particular case ok.

Here you can see that the Reynolds numbers are much smaller oh no this is different transition this is a transition called turbulent boundary layer which is not the Reynolds number at which it happen.

So, what happens is which we will see this region this region which is very close to the surface the we would expect the flow to be laminar right because it is very close to the surface there will be lot of friction coming from the surface the velocity will be all, but beyond some Reynolds numbers actually when that region will become turbulent. So, the region which is very close to the surface will become turbulent and that happens at that particular Reynolds number and this is just formation of process.