Fluid and Particle Mechanics
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Lecture – 26
Non-Dimensional Analysis - 4
Trinity Test

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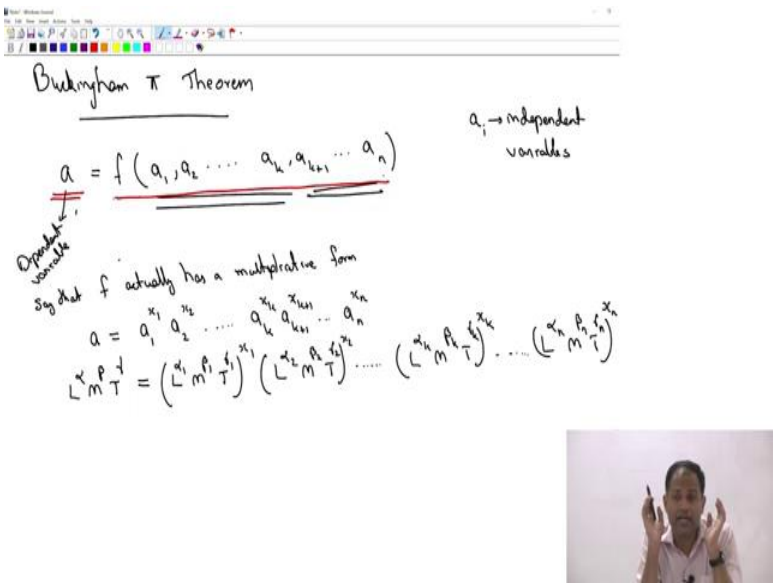

Let us start by saying that we have some physical process and we know some variables involved in that physical process and we are trying to find out a relation between those variables, without really solving anything about the system.

Let us say that a is a function of a 1, a 2 etcetera a k, a k plus 1 etcetera, a n where so a i are all the independent variables in the system and a is the dependent variable. So, this is analogous to if you think about the fluid flow pass the object that has been the problem that we have considered.

$$a = f(a_1, a_2, \dots, a_k, a_{k+1}, \dots, a_n)$$

So for example, the small a is basically the force is a function of all other variables like length, velocity, viscosity and so on. So, the everything that you see on the right hand side are your independent variables and they decide some dependent variable in the problem and let us say this function represents some physical process that is where we have started with is that clear.

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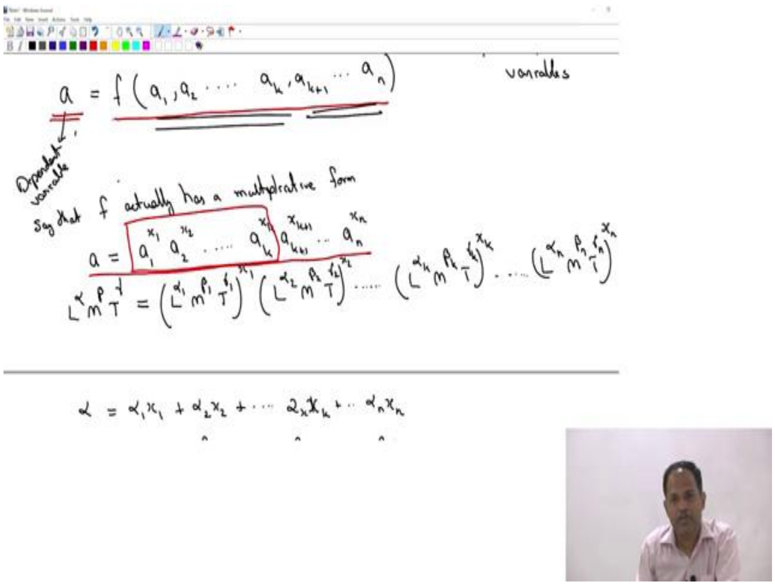

Now, one thing that we could check is there let us say that f actually has a multiplicative form. Now once you do the proof you will find that this has nothing to do with this assumption is not really important, this is just to determine some number what do you mean by say that saying the let us say a be given by a 1 to the power of x 1, a 2 to the power of x 2 and so on ak to the power of x k, a k plus 1 to the power of x k plus 1 etcetera a n to the power of x n. So, each of them are raised to some powers and then multiplied and let us say that if that is the functional form.

$$a = a_1^{x_1} . a_2^{x_2} \dots a_k^{x_k} a_{k+1}^{x_{k+1}} \dots a_n^{x_n}$$

This particular dependent variable has on the independent variable and if I write down the dimensions again I will only worry about mass, length and time. So, let us say so the they dimensions on the left hand side should match with the dimensions on the right hand side.

So, let us say the dimensions on the right hand side is L to the power of alpha, M to the power of beta, T to the power of gamma is equal to a 1, let us say L to the power of alpha 1, M to the power of beta 1, T to the power of gamma 1, raise to the power x 1 times L to the power alpha 2 M to the power beta 2, T to the power gamma 2 raise to x 2 and so on L to the power of alpha k M to the power beta k T to the power gamma k raise to x k and so on, L to the power of alpha n, M to the power of beta n, T to the power of gamma n raise to x n.

$$a_i = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}\ ; i = 1,2, \dots, k, \dots, n$$

What have I done I have just written down the dimensions for each of this variables, of course I do not know what those way dimensions are for a I said it basically is some length to the power of something, mass to the power of something, time to the power of something. For a 1 I said it is L to the power of some other x power other power L to the power of alpha 1, M to the power of beta1, T to the power of gamma 1 and the whole thing is raised by x 1 and so on for each of the independent variables that is ok. And the thing that I am going to do is I am just going to equate the powers for each of the dimensions.

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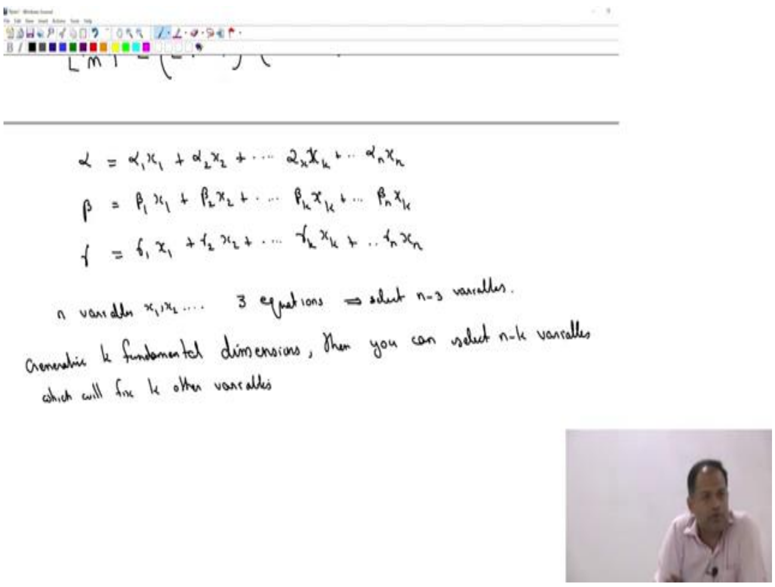

If I equate for alpha I will find sorry for length I will find that alpha is equal to alpha 1 x 1 plus alpha 2 x 2 and so on alpha k x k alpha n x n. If I equate the powers of mass I am going to get beta equal to beta 1 x 1 plus beta 2 x 2 plus etcetera beta k x k x n gamma is equal to gamma 1 x 1 plus gamma 2 x 2 plus etcetera right just 3 equations where we do not know what are x 1 x 2 and so on.

$$\alpha = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_k x_k + \dots + \alpha_n x_n$$

$$\beta = \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \dots + \beta_n x_n$$

$$\gamma = \gamma_1 x_1 + \gamma_2 x_2 + \cdots + \gamma_k x_k + \cdots + \gamma_n x_n$$

So, if you look at it what is this so we have n variables which are x 1 x 2 and so on and we have got 3 equations right. So that means, I can select n minus 3 variables independently and then that will fix the rest of the three variables right. I have got linear equations right which has got n variables in it, but I have got only 3 equations so that means I can select n minus 3 variable and the rest of them basically get fixed correct.

So that means, I would be able to select n minus 3 variables this is because I selected three fundamental dimensions or if you generalize by saying that if you have k fundamental dimensions then you can select n minus k variables which will fix let us say k other variables, this is what you would have done if you were solving the simple system of linear equations.

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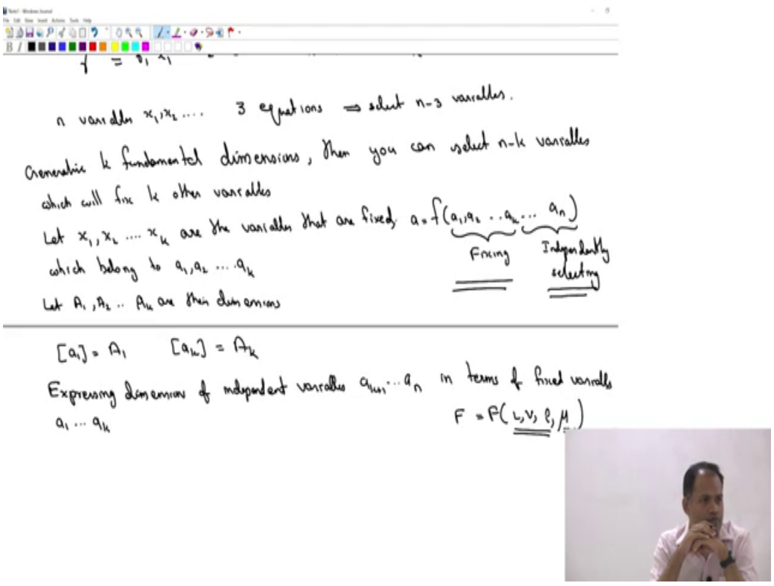

Let us x 1 x 2 x etcetera x k are the variables that are fixed. So, let me just try to tell again what are we trying to do we have got a dependent variable a which is a function of lots of independent variables, by writing down the dimensions I have found that if I cannot have any arbitrary number of powers that I can assign to any of the variables. If I talk about three fundamental dimensions then I am allowed to take only n minus 3 I can independently choose n minus three variables and that means I will be fixing three of them right three of them will be uniquely determined.

So, in general case if I have k dimensions I am going to be basically fixing k quantities as fixed variables and n minus k will be independently chosen. So, in other words what I am going to do is that I am going to say that the first k variables are the ones that are going to be fixed and n minus k variables are the ones which can be independently chosen.

So and if you choose that if the first k variables x 1 x 2 etcetera up to x k are the powers of the first three variables, remember the functional form is a equal to f of a 1 a 2 etcetera up to a n. I am going to be fixing these and these from k plus 1 onwards I am going to be independently choosing. So other variables that are fixed which belong to a 1 a 2 etcetera up to a k. So, a 1 a 2 etcetera up to a k are the ones which I am going to fix and let x 1 x 2 x k be their powers which we have already taken and let a 1 a 2 etcetera up to a k are their dimensions. But simply what I am saying is that a 1 is nothing but a 1 a k is same as a k.

So, for the first k variables let us say capital A 1 capital A 2 and so on are their dimensions and A k to a n or a k plus 1 to an are the independent variables that I have and I am going to now represent the dimensions of these independent variables in terms of dimensions of the fixed variables. What am I trying to do I am expressing dimensions of independent variables a k plus 1 etcetera up to a n in terms of fixed variables a 1 etcetera up to a k.

So, the first k th variables you have fixed the dimensions, so let us take the problem of force is given by let us say mu rho L V. So, there are or I will write it in a different way L V rho mu, what I am saying I will say L V and rho I am choose going to select as the fundamental dimensions. So, that I can represent the dimensions of mu in terms of L V and rho and I will be able to do that right.

$$[a_1] = A_1 \,;\, [a_k] = A_k$$

$$F = F(L, V, \rho, \mu)$$

So, similarly I am saying that if I have you know M variables I will skip the first k variables I will select this for dimensions of the first k variable and then the n minus k th variables can be represented in terms of first k variables. This is the way you can represent the viscosity in terms in the terms in dimen in the in terms of dimensions of L V and rho.

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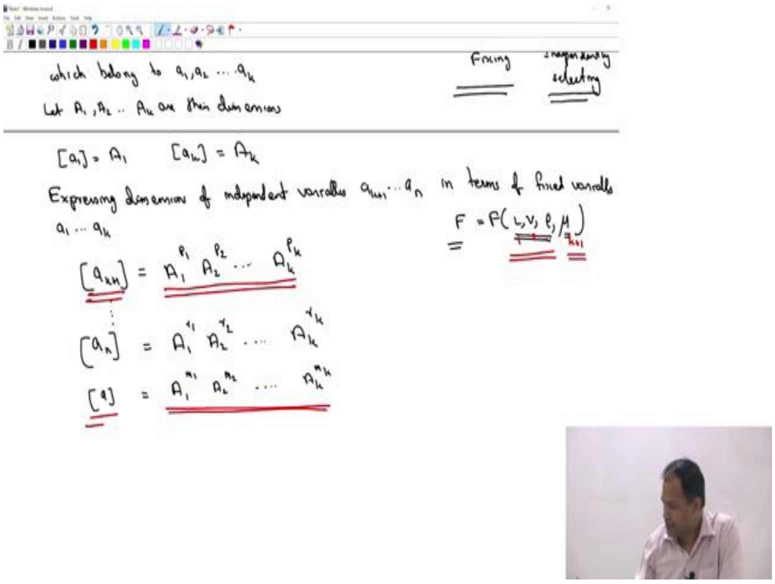

So that means, if I have a k plus one dimensions of a k plus 1 can be written in terms of dimensions of the first k variable let us say A 1 is the dimension of small A 1 which will be raise to some arbitrary power let us say p 1 A 2 to the power p 2 and so on A k to the power p k.

$$[a_{k+1}] = A_1^{P_1} A_2^{P_2} \dots A_k^{P_k}$$

So, I have A 1 this capital A 1 capital A 2 etcetera up to capital A k are the dimensions of the first k variables and you are representing the k plus 1 th variable it is dimensions as a power of other variable other k variables that you have already chosen and so on you could do it. All the way up to let us say a n minus sorry a n which will be another some A 1 to the power of r 1 A 2 to the power of r 2 so on A k to the power r k.

Similarly, you will also be able to write your dependent variable again, so let us say F you want to write F in terms of L V and rho. So, small a which is the dimension of the dependent variable again in terms of other dimensions let us say A 1 to the power n 1, A 2 to the power M 2 and so on A k to the power M k. I will take a pause and ask the questions if you have.

$$[a_n] = A_1^{r_1} A_2^{r_2} \dots A_k^{r_k}$$

$$[a] = A_1^{m_1} A_2^{m_2} \dots A_k^{m_k}$$

Should I go through the procedure once again or you are clear or you are lost either say yes or no should I go through it again. So, we are trying to prove Buckingham pi theorem and we want to do it in a general form, we do not want to take five eq five independent variables three dependent variables and so on. So, I have taken a general function a with that so I have taken a general function a which is a function of a 1 a 2 etcetera up to a n, so that is a form that I have considered. Now the first thing that I want to do is to find out how many dependent and how many independent variables that I can select.

To do that I am going to say then let us say that this function is actually of a multiplicative form, that is I am saying that if small a the dependent variable is expressed as a power as a multiples of powers of each of the variables. So, a 1 to the power x 1, a 2 to the power of x 2 and so on and if I do that and if I write down the dimensions of all the variables let us say the dimension of small a is in terms of L M and T raise to some arbitrary powers a 1 raise to the power.

So, again you know each of them in terms of L M and T raise to some arbitrary powers to the power x 1 because, I have assumed that it is the power of it is the function itself is a 1 to the power of x 1. So, I have just written each of them as you know some L M T to the power of something and if I equate the dimensions of each of them, then I basically end up with three linear equations.

Of course we did not know what x 1 x 2 etcetera to x n where and that is our objective we want to find out x 1 x 2 and so on and we are now we cannot do that because we basically have three equations and we will have n unknowns and we are ending up with n unknowns because we had n variables to start with right. So, we have n variables three equations, that means that we have to select n minus 3 variables independently and that will fix three variables as you make solutions right.

Because these are linear equations you will be able to solve them and you will be able to find three of them. So that means, in this problem if I take this the if I take this as the form I have to say that what is x k plus 1 x k plus 2 etcetera to x n, if I independently fix them then I will be able to tell what is x 1 x 2 etcetera up to x k right.

If I fix x k pl if I choose x k plus 1 x k plus 2 x a etcetera to x n then I will be able to tell what is x 1 x 2 etcetera up to x k right. If I have five variables and if I have three dimensions two of them I will independently select which will fix the you know the powers of the other three.

So, so that is what I am so we so the that means, we can select some of them independently and some of them as a fixed value. Now the idea is that we are going to represent all the fixed variables all the variables that we are going to select as the fixed ones, which means this particular set in terms of the independent sorry the independent ones that we are going to represent in terms of the fixed ones. So, the dimensions of k plus 1 we are going to write in in dimensions of a 1 a 2 and so on. So, that is what I was telling you this example that if you have force as a function of L, V, rho and mu. So, this is 1 this is 2 etcetera and let us say this is the k plus 1 th variable the k plus 1 th variable I will represent in terms of first k variables or to be precise the dimensions of k plus 1 th variable will be represented in terms of k other dimensions of other k variables.

So, because I can represent the dimension of mu in terms of dimensions of L V and rho, I can keep on doing that I can add any other variable to this system and I will be able to represent the dimension of the variable that I am adding in terms of dimensions of the first three variable I have chosen right. So, I have so in this system I have; I have taken a k fundamental dimensions.

So, all the variables that I am going to have from k plus 1 onwards will be represented in terms of dimensions of first k variables. So, this is the dimension of the k plus 1 th variable I have represented it as dimensions of all the first k th variables and the dependent variable a, I can again represent it as a in terms of dimensions of the first k th variables.

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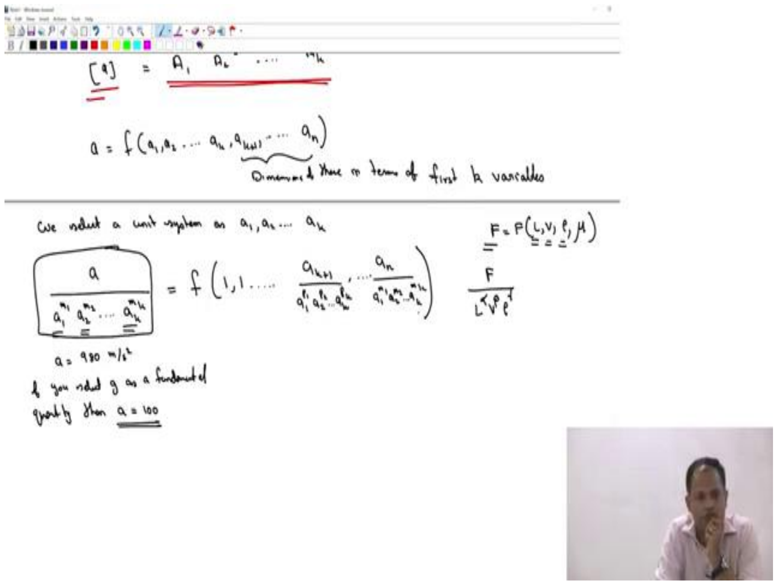

If that is a case then so what is our form right now a is f of a 1, a 2 etcetera up to a k, a k plus 1 etcetera up to a n, this is the functional form that we have chosen and we have said that we are going to represent the dimensions of all these the dimensions of these in terms of first k variables that is all I have said so far. Now let us say we select a unit system as or as a 1, a 2 etcetera up to a k. So, we might deal with the unit systems like SI you we will deal with the unit systems like mk mks instead of that let us say we select a unit system which is given by the first k variables itself. If I do that then that means each of the variable that I have can be expressed in terms of a 1, a 2 and so on.

So, if I do this then I can write let us say a represented in terms of a 1 a 2 and so on is a 1 to the power m 1, a 2 to the power of m 2 etcetera up to a k to the power of m k. So, that is a dependent variable is what have I done now a is written in terms of the k variables that I have chosen. So, if you like to think about let us say; let us say you have a system in which let us say you have you are dealing with acceleration.

Let us say acceleration is given as 980 meter per Second Square. So, if you were chosen an SI system of unit and you say that your acceleration is 980 meter per Second Square. But if you choose a unit system in which g is a fundamental quantity, then what would be acceleration if g is the fundamental dimension that you have chosen then the acceleration would be just 100 right because it is 100 g.

So, if you are choosing SI as the coordinate system then you would have said 980 meter per Second Square is your acceleration. But if you select g you know as a fundamental quantity in your unit system then you will say oh my acceleration is 100 that will tell you what the acceleration is right. So, similarly if you select a 1, a 2 etcetera up to a k as the; as the fundamental variables or fundamental quantities in your unit system, then a has to be divided by each of those variables to get the number that you are looking for.

Just the way if a is equal to 980 meter per second square, if you select g as a fundamental quantity then your a is nothing but 100 because 100 g is going to be your ah you know your acceleration some number. Similarly if I have a any variable I can just divide it with whatever independent variables that I have chosen you know raise to powers appropriately and then I will get you I will. So, this is like this small a measured in a unit system which is panned by a 1 a 2 etcetera up to a k.

I have selected a unit system in which my fundamental quantities are a 1, a 2 etcetera up to a k. In this problem f which is a function of L V rho mu ok. I want to represent force in terms of a length, velocity and density and then I would write F. If I want to measure F let us say some 10 newton I want to measure 10 newton, but not as newton but in terms of things that are contributed by length, velocity and density. Then I would have written it is L to the power of alpha, V to the power of beta, rho to the power of gamma, of course L beta sorry alpha, beta, gamma are to be chosen in such a way that it will basically will have the dimensions of force, so that is all what I am doing here.

I am saying that I am selecting a 1 a 2 etcetera up to a k as my cor as my units. So, that I can measure any variable in terms of other k variables that I have chosen, so that means that is going to be equal to I will just write down the expression we will talk about it again. So, a 1 measured in terms of a 1 will be just one a 2 measured in terms of a 2 will be just 1 and so on, a k plus 1 measured in terms of a 1 a 2 etcetera up to a k will be again some a 1 to the power of p 1 which we do not know a 2 to the power of p 2 etcetera a k to the power of p k and so on.

All the way up to variables a n which will be measured as a 1 to the power of m 1, a 2 the power of m 2 some arbitrary powers, a k to the power of m k that is what we basically would get. If we were you know taking up unit system span by the first k variables let us stop there we will talk about it once again tomorrow. That is all actually Buckingham pi theorem it already says that what is that we need to do.

$$\frac{a}{a_1^{m_1} a_2^{m_2} \dots a_k^{m_k}} = f\left(1,1, \dots, \frac{a_{k+1}}{a_1^{P_1} a_2^{P_2} \dots a_2^{P_2}}, \dots, \frac{a_m}{a_1^{m_1} a_2^{m_2} \dots a_2^{m_2}}\right)$$