Fluid and particle mechanics

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Lecture – 23
Non dimensional analysis

So, last week you found or you were exposed to what is meant by fluid properties? What are the various fluid properties that could be important in fluid mechanics? Right; so, if you look at what we have been doing we took simply situations and then we try to calculate what are the; how does the flow look like right. The tube flow the flow between two parallel plates. Then, we talked about how the fluid properties are important and how it is important in determining the velocity profile.

Now, you know that the equation that you used to calculate the flow profiles they were really complicated and they were complicated because on one hand they were partial differential equations then, they are also non-linear equations right. And therefore, you have to make a whole lot of assumptions every time to simplify the equations and then, you did get some profiles and you also found that could be reasonable in certain cases, but they may not be reasonable always. And the way therefore, you should be attacking a problem you just cannot dependent theory always ok. You need to look at other options ok.

There are two other options that come frequently and one is to do a numerical simulation. In other words, you have got a partial differential equation. You have done numerical methods last semester right. So, you basically use any of those methods and try to solve these equations ok. So that field is actually known as computational fluid dynamics. So, that is one of the option.

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So, one option that we have seen so far was essentially looking at the theory in which we try to get analytical solutions, but we may not be always able to do that and then, we could try doing CFD ok; so, the computational fluid dynamics. The other option of course, is to see what is going to happen directly and that is by doing experiments ok. So, these are the three options that you will have when, you want to look at a flow problem.

Now, this is straight forward because you have already seen that these two there is an issue; the issue is that you know let us see you doing an experiment you do it under certain conditions. Let us say you keep the temperature something you fix take some particular liquid and do it and then, you find some solution you find some flow fluid, but you do not know whether that is always going to be the case.

You do an experiment with water. Now, you change that to some oil; you do not know whether that is what should be expected. While in case of theory you did not have a problem because the entire problem is defined in terms of certain parameters ok. So, is the case with CFD ok? When you are doing a numerical solution, you will be doing it for a certain given numbers, even not actually solving the equations; you will be doing it for number.

So, from both from CFD and from experiments you are actually going to get numbers not equations and therefore, this numbers are for a given set of conditions and then you would know how to interpolate or extrapolate for that matter; that is clear? So, when you are doing CFD or experiments you are you actually going to get lot of data. Now so, when you get lot of data ok. So, let us look at a simple example and see what that means. Let say, you have some complicated shape object, some object ok.

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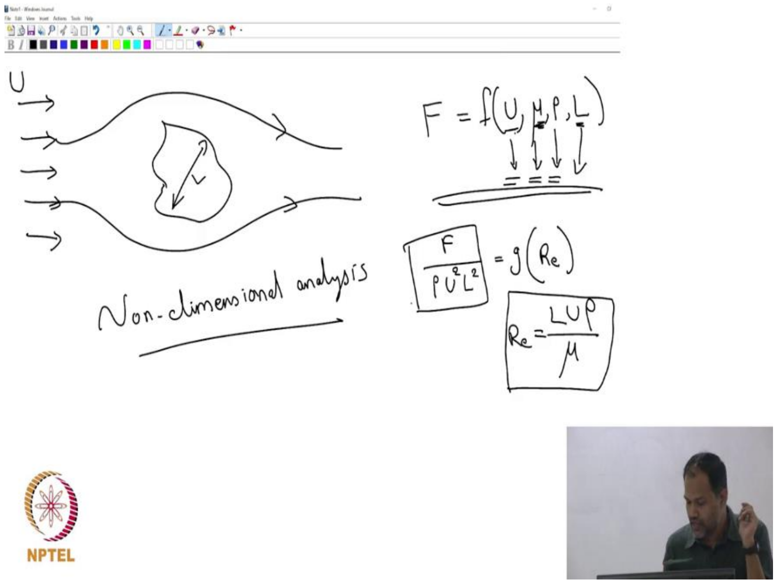

And there is a fluid that is going to come with some velocity U ok. So, the fluid will come; it will go pass this object right and your interest is in finding out what is the force that this fluid is going to exert on the particle or you could ask the other question how much drag the fluid will experience because of the presence of the particle right. The object is there, it is going to put some restriction on the fluid motion which the fluid will experience as a drag force or you could see that the particle is going to get you know force because of the moving fluid ok.

Either way you are interested in finding of what is that force ok. So, you are interested in finding out, what is the force? Let us say exerted by the fluid on this particle that you have put there ok. You would expect that it would definitely depend upon what is the velocity with which the fluid is coming. So, you would say maybe that is a function of U, the fluid properties right. What are the two fluid properties that would be relevant? Viscosity and density is the property that comes into the picture and then, of course, you know how big the object is it does not have any shape. Let us say that you know some particular length which might be some characteristic length of this object ok.

$$F = f(U, \mu, \rho, L)$$

Let us say these are. So, we expect that these are the things may be that would be important and you want to determine the force. What would you do, if you are doing either you know a numerical solution of equations or if you doing an experiment? You would choose a particular value of U, a particular value of mu, a particular value of rho and a particular object to run the simulation or run the experiment and calculate the value of force, right.

Then, you would do that for various combinations right. So, for example, you could keep U fixed, mu fixed, l fixed; I will change sorry, rho fixed and you will change L. Let us say you will change L let say ten time; so, that you can get it for large range of length ok. Then, you would keep mu sorry U mu and L fixed and change rho and let us say you would change that for another ten times and you would keep doing that ok. So, you will get huge you have to do huge number of experiments or huge number of simulation, to find what is really the dependence of force on something. So, if like say for each of them; if you do ten times, 10, 100, 1000, 10000 experiments or 10000 simulations you will have to do, if you really want to find out what is the force that is going to look like agreed that is clear.

So, you are; so and the problem is that one is that you have you need huge amount of resources large amount of time and then, end up with the huge amount of data which you do not know how to really even represent because you have things which are dependent on four things and then you basically end up with huge number of tables. So, that is what is going to happen. So, the question is, is there a better way of doing this and that is what we are going to do and that is what is you know for the domain or non dimensional analysis ok. So, we are going to look at non dimensional analysis.

So, we will see how to do non dimensional analysis, but let me tell you what the result is going to be for this particular problem that we just talked about. For this particular problem, if you were to do non dimensional analysis, you would find that this equation can be recast into:

$$\frac{F}{\rho U^2 L^2} = g(Re); Re = \frac{LU\rho}{\mu}$$

$$Re \rightarrow Reynolds\ number$$

So, this is going to be a non dimensional number. What will be the units of F by rho U square L square? It is non dimensional. This is another non dimensional number ok. So, you get one non-dimensional number is equal to some function of another non dimensional number ok. So that you can easily represent, if you were to; you know have let us say graph.

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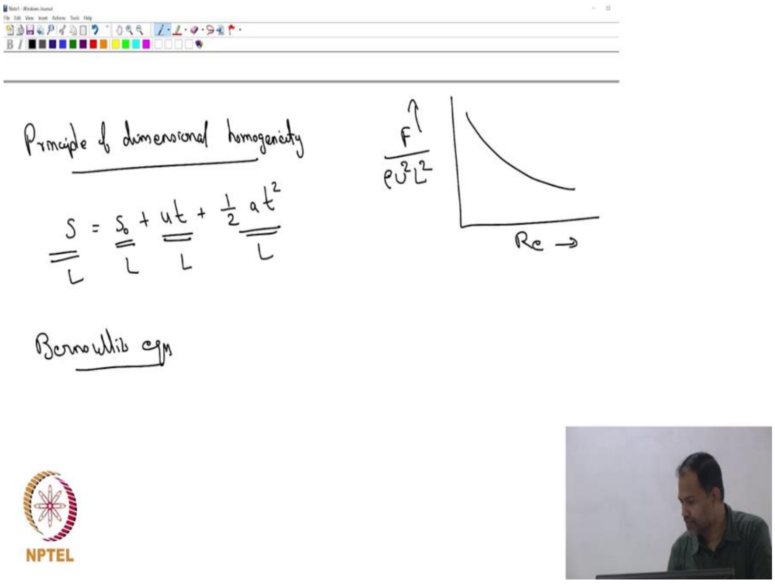

You will just say that ok I will just plot F by rho U square L square as a function of Reynolds number and then, you will get some data, some curve and thats all needed for this particular problem. You dont have to worry about 10000 points and so on representing space because the only quantities that are really going to matter in this case would have been Reynolds number and force this you know force divided with this particular quantities; so, a particular form of force becomes a function of Reynolds number alone.

You change fluid doesn't matter; it is only what matters is only this particular curve which you would have represented as a particular function ok. This is a nice way of presenting things; it is very compact and then, you do not need to worry about how to do the extrapolation or interpolation; clear. So, the question is how does one end up with such a relation? We may not know, what exactly g is?

The non dimension analysis not going to tell you whether this is you know this particular form is Reynolds square or is it exponential of Reynolds or this is sinusoidal of Reynolds; we will not get it. We will know the exactly what it looks like, but we will know what are the number that are going to be connected by a particular form of a function. And that is what we care about because that is enough in most of these cases because things basically get into a compact form. Doubts? Few things which you are very familiar with; I will just till tell principle of dimensional homogeneity ok; that means, given an equation each term in that equation will have the same dimensions, you all know that very well:

$$S = S_0 + ut + \frac{1}{2}at^2$$

The equation describing the motion or the trajectory of a particle right; S 0 is the initial displacement, u is the initial velocity and a is the constant acceleration and this equation is going to tell you where the particle is going to be at a given point in time. What would be the dimension of each of them? S will have the dimensions of length; S S 0will have the dimensions of length; u t will have the dimensions of length, metre per second into second; a t square will have the dimensions of length ok. So, all terms basically have the same dimensions and that is known as the principle of dimensional homogeneity.

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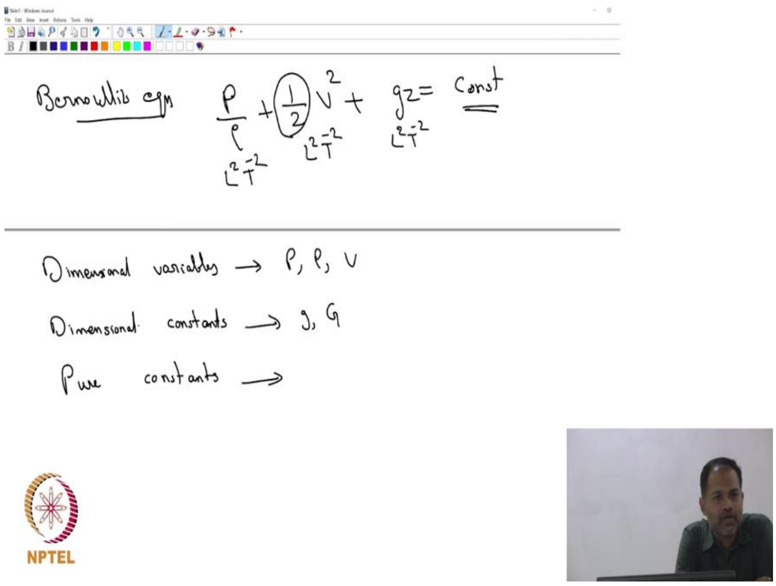

How about Bernoulli's equation

$$\frac{P}{\rho} + \frac{1}{2}V^2 + gz = const.$$

So, therefore, each of the terms have the same dimensions and that is one of the principle of homogeneity. Few other things. So, these are the other things that you will come up in any equation. Dimensional variables mean some kind of variable that would have dimensions. For example in Bernoullis equation it would have been pressure, density, velocity any of them ok.

Dimensional constant are constants, which may also have dimensions; for example, gravity, gravitational constant and so on. Pure constant like for example, the half we had there; things like that ok. So, these are the three things the typically come in any equation and pure constant are something which we won't really have to worry about throughout the dimensional analysis because we would never worry about that. We would be worrying about dimensional variables and dimensional constants things which have got dimensions ok. So, now before doing it ok; we need to worry about one another thing that is choice of variables.

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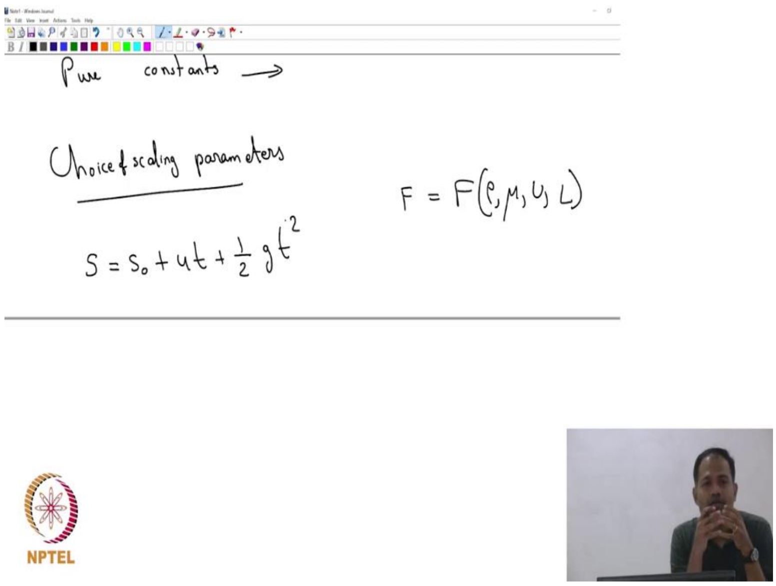

Or choice of rather scaling parameters; choice of scaling parameters; so, see we are going to have few variables ok. In this problem of which where, we talked about flow ok. Past some object; we said it is basically a function of rho, mu, U, L. We need to identify some parameters to non dimensionalize it ok.

Now, it would not be obvious, what are the parameters that we should choose. And it is basically, I mean there is no way to really do it. But in order to see what it means let us take a simple exercise where, we already know the functional flow which is again will go back to this equation :

$$S = S_0 + ut + \frac{1}{2}gt^2$$