Fluid Mechanics
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Lecture - 22
Choice of Scaling parameter

First let us just do it and then see what it means ok. So, I am going to suggest so how many parameters are there in; how many variables are there in this problem?

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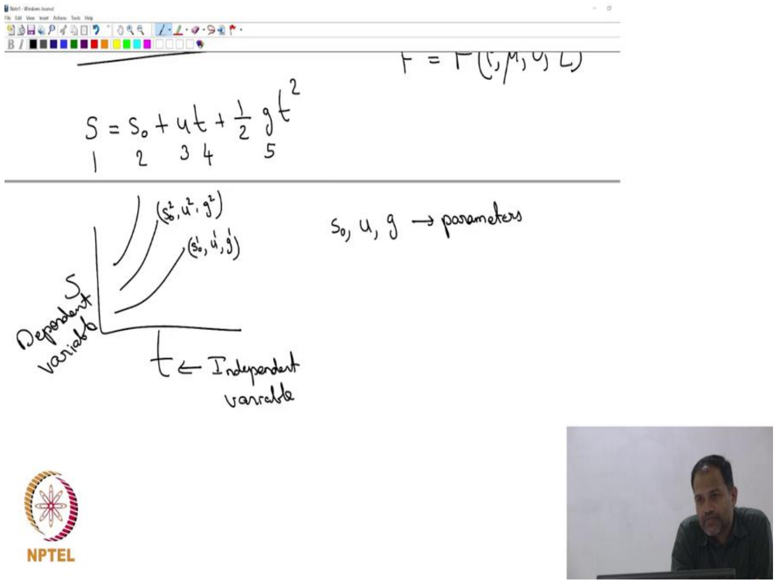

S, ok S0, u, t, g there are 5 things right, suppose you did not know this equation or you want to represent this equation, but as a data set. Not as an equation what would you do? Your interest is to prescribe the location of the particle as a function of time. So, if you were to plot a graph, you will actually plot is s versus t right.

You will plot s versus t as a graph, but then you have to tell for what value is s 0 what value of u and what value of g you would be plotting it? The moment you say change s 0 u and g the plots are going to be different agreed. So, you can say

$$S = S_0 + ut + \frac{1}{2}gt^2$$

$$S \rightarrow Dependent\ variable$$

$$t \rightarrow Independent\ variable$$

$$S_0, u, g \rightarrow Parameters$$

The moment you change the parameters you are going to get different different plots. So, this may be for a given set of parameters s 0, u and g ok, let us say 0, 1 u 1, g 1 you might get it for a 0 2 u 2, g 2 and so on for each of them you are going to get different different plots ok.

Or if you were to do experiments, without you knowing this relation, you would have change s 0 g and u multiple times get this plots ok. So, you have got in this particular problem 5 you know really variables out of which you have identified a dependent variable and an independent variable and 3 parameters ok.

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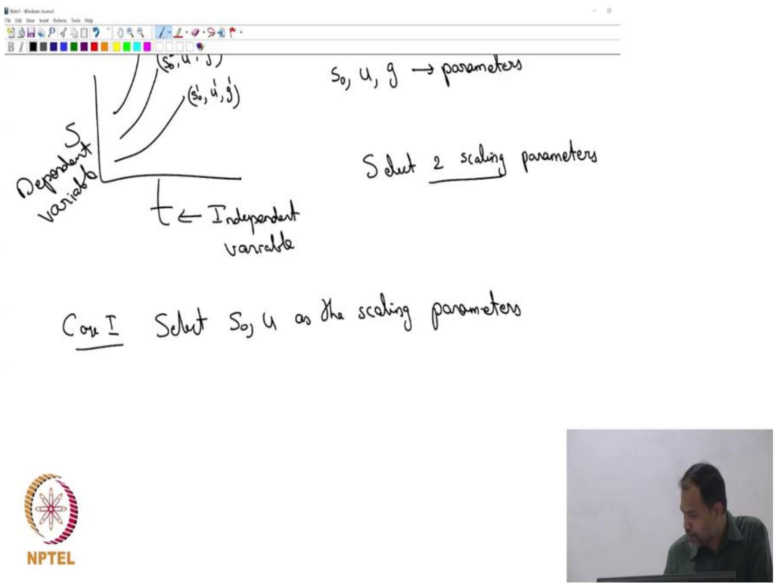

Out of which I will say we will select two scaling parameters select two scaling parameters. Now, what is meant by scaling parameters? Is essentially you going to use the scaling parameter to non dimensionalize your other variables.

So, remember when we were doing the pipe flow, we non-dimensionalized our equation in some fashion we chose a characteristic length we chose a characteristic velocity ok. Similarly we are going to select certain things and then you are going to use that to non dimensionalize your equation. So, here we do not know how many we should have actually chosen at the moment. Let us do not worry I am just telling you that let us select two of them. So, in this case I have a s 0, u and g has the parameters we have to select two of them, so let us say in case 1, select s 0 and we use u as the scaling parameters ok.

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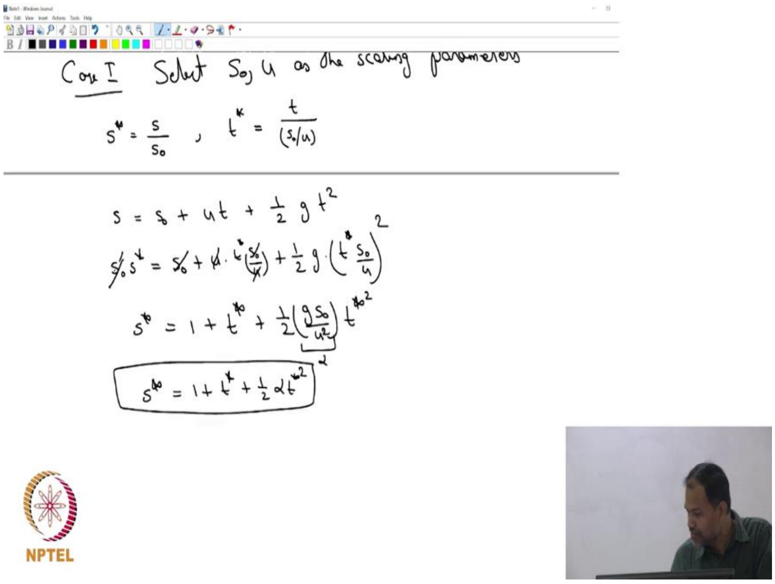

So that means, I will define:

$$S^* = \frac{S}{S_0}\ ; t^* = \frac{t}{S_0/u}$$

So, our original equation was:

$$S = S_0 + ut + \frac{1}{2}gt^2$$

$$S_0 S^* = S_0 + ut^*\left(\frac{S_0}{u}\right) + \frac{1}{2}g\left(t^*\frac{S_0}{u}\right)^2$$

$$S^* = 1 + t^* + \frac{1}{2}\left(\frac{gS_0}{u^2}\right)t^{*2}$$

$$S^* = 1 + t^* + \frac{1}{2}\alpha t^{*2}$$

That is equation that you are ending up with right. So, α has come out has a non-dimensional parameters, so it is as α dimensional or non dimensional it is g times s 0 divided by u square ok.

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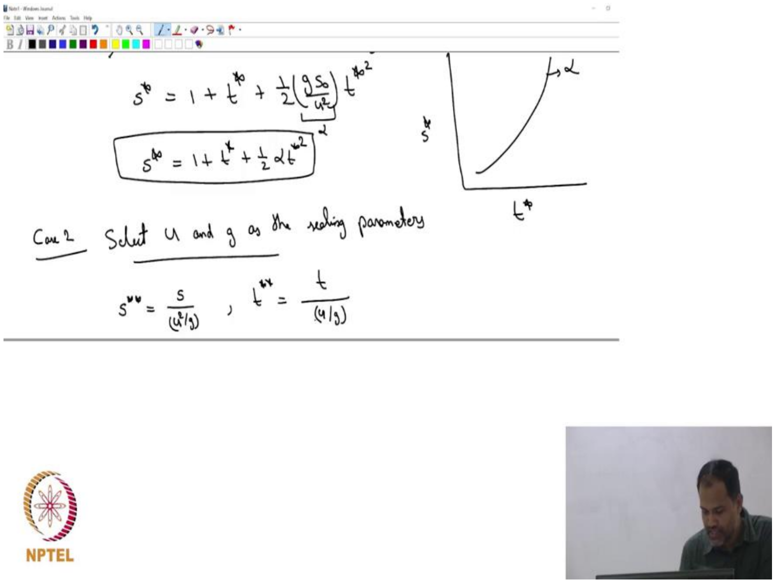

So, that has non-dimension, so that is the non dimensional parameter that is actually characterizing this equation; in which case you would have been able to just plot s * as a function of t * for just one parameter α ok. That tells you what the curves curve is you do not have to worry about 3 different parameters now. So, you have actually simplified your description agreed ok. Now, we have been able to do this because we selected two scaling parameters right.

So, what did we do we selected two scaling parameters one was s 0, other was u and then we then we non dimensionalize the equation. Let us say you are going to select a second case, Case 2:

$$S^{**} = \frac{S}{u^2/g}\ ; t^{**} = \frac{t}{u/g}$$

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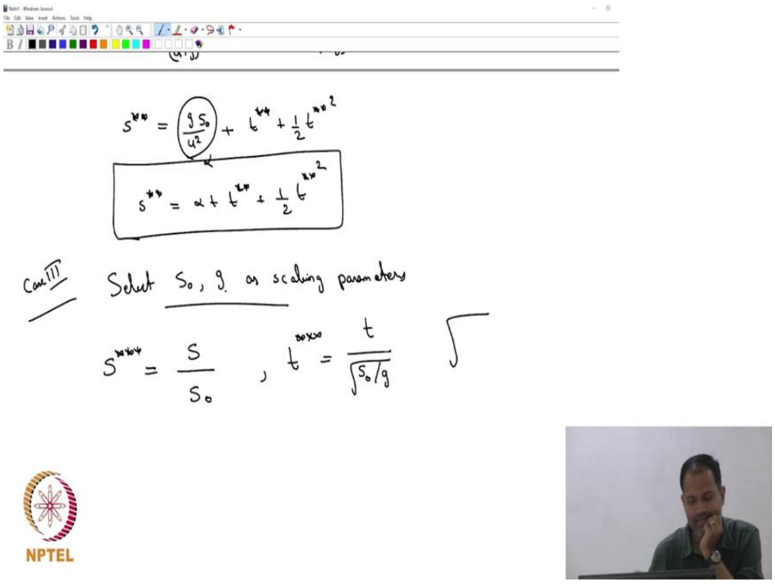

On non-dimensionalization we get:

$$S^{**} = \alpha + t^{**} + \frac{1}{2} t^{**2}$$

Correct ok. So, this parameter is what we had originally defined, identified as α right. So, we have really gotten s double * is equal to α plus, t double * plus half t * square ok. Again we have got a non-dimensional equation ok, which is again parameterized only by their parameter α ok; though it appeared differently because we have now scaled it differently, we have chosen different set of scaling parameters ok. We will do one more and then we will conclude case 3:

$$S^{***} = \frac{S}{S_0} \; ; t^{***} = \frac{t}{\sqrt{\frac{S_0}{g}}}$$

$$S^{***} = 1 + \frac{u}{\sqrt{gS_0}} t^{***} + \frac{1}{2} t^{***2} = 1 + \frac{1}{\sqrt{\alpha}} t^{***} + \frac{1}{2} t^{***2}$$

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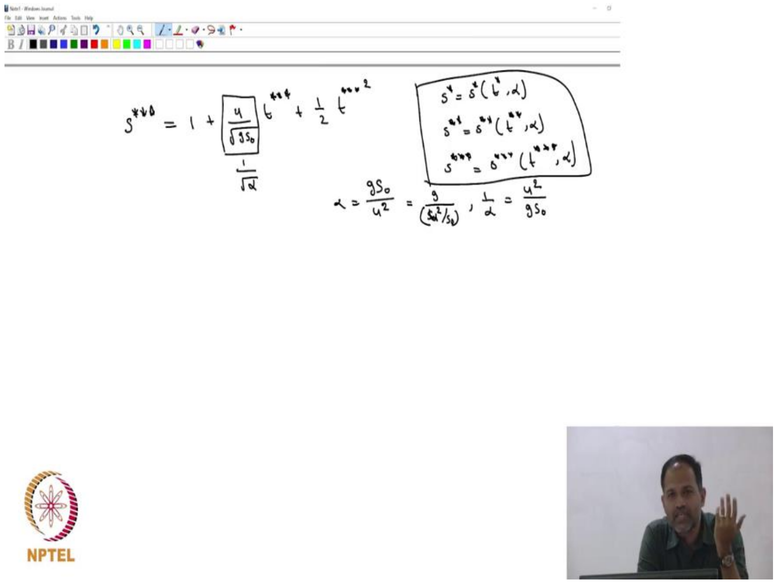

So, again we have been able to get it, in all cases

$$S^{*} = f_1(t^{*}, \alpha)$$

$$S^{**} = f_2(t^{**}, \alpha)$$

$$S^{***} = f_2(t^{***}, \alpha)$$

So, α is the parameter that has always come out in some form and therefore, α is the only non dimensional parameter that is in the problem. It does not matter which scaling parameters you select you will always end up with that parameter the right non-dimensional parameter it may just come out in a different form. In fact, if you look at in the first case we had where did α appear in the first case.

In the first case it was really s * is equal to 1 plus t * plus half α g t * square. So, α was coming in the place of gravity ok. So, you really can think about when you are changing α there, you are really changing your gravity. So, you can really say that ok. So, there α really represented how if I were to change gravity what would have happened? That is the physical interpretation that would have come out. On the other hand in the second case α came out at the place of the initial displacement. So, α was really playing the role of the initial displacement.

In the third case it would have come out as the initial velocity u ok. So, u could interpret that non dimensional parameter in any fashion you like because α always contained these 3 things; g s 0 and u ok. And you can actually interpret in multiple ways and that is because its essentially a non-dimensional quantity. So, how did we define α? A was defined as g s 0 divided by u square no? What is the other way that we defined this way right.

So, so this contains g s 0 and u square, so I could write this as g divided by s u square divided by s 0; in that sense we are saying Oh its gravity to some other acceleration in the system. Or I could have written it as g by u square divided by s 0 inverse, then I would say that or maybe I should do 1 by α, so let us see here 1 by α I will write it as u square divided by g s 0.

So, there it would carry the meaning of some velocity square by some other velocity square. So, all non dimensional numbers you will be able to interpret it in multiple ways ok, whether you want to interpret it as strength of gravity, whether you want to interpret it as strength of the displacement anything ok. And what this exercise has shown you is that it does not matter what the scaling parameter that you select you will always end up with the right expression. In all these cases we ended up with right expression, even though the precise form of the equation was different in each cases ok.

So, non dimensionalization will not able to tell you what the precise form is it will only tell you what are the relevant parameters involved; is that clear? Any questions, should I repeat anything?

Yeah.

Depending upon what you want to do it right, if you worried about flow here then you do not have to care about. But, if you have worried about flow between what is happening at equator and what is happening at pole you might or you want to talk about what is happening in you know moon and what is happening here you want to talk about right? So, that is why so you want to call it a dimensional consider a dimensional variable depending upon your problems.

So, you look at this; this simple equation has now given actually told you everything that you have to know really about non dimensional parameter and non dimensional analysis ok; except that we just do not know we got a situation how to do that ok, so that is our next goal. I will just say few things and then, we will stop and will do the rest in the next class; power law expression.

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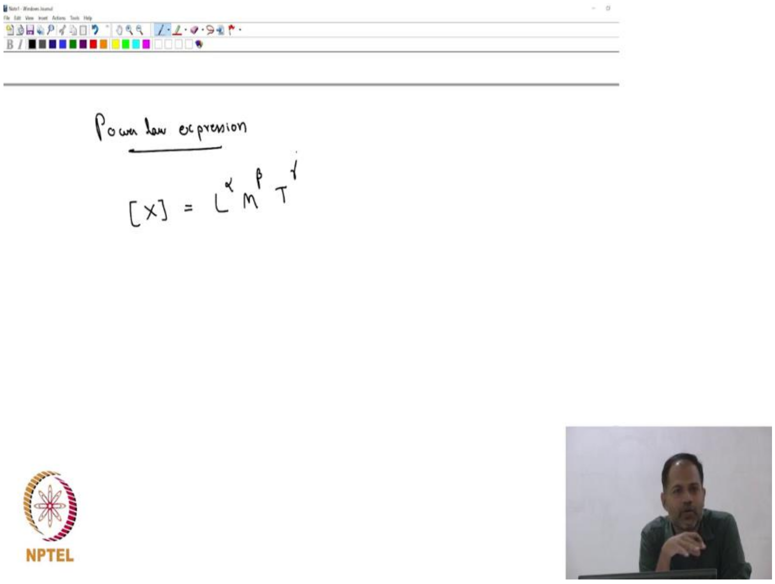

Given any physical quantity ok, any physical quantity if you look at its dimension, so we only variable let us say you know fluids and fluid flow any parameter that you are going to look at let us say x, its dimensions will have the form:

$$[X] = L^{\alpha} M^{\beta} T^{\gamma}$$

$$L \rightarrow Length, M \rightarrow Mass, T \rightarrow Time$$

Is that what is true for any of the properties or any of the variables any of the physical quantities that you are familiar with?

You are agreeing or disagreeing I am confused; you want to say something go ahead.

So, let us so if you want temperature you want to include that I am going to worry about things that relevant for flow heat transfer he will keep it aside always variable constant temperature right. But how do we know whether it is always correct, we know that this is what we have always observed right velocity, length, mass, force, power, work right. So, we are going to prove that this is always going to be the case form which we will derive something called Buckingham Pi theorem. Have you done Buckingham Pi theorem? No ok. So, we will derive that and that is the theorem that we are going to use to get non dimensional equations, which we will do next class.