Fluid Mechanics
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Lecture – 02
Fluid statics

Now, let us talk about pressure. So, pressure is a special kind of stress, its also defined as force per unit area.

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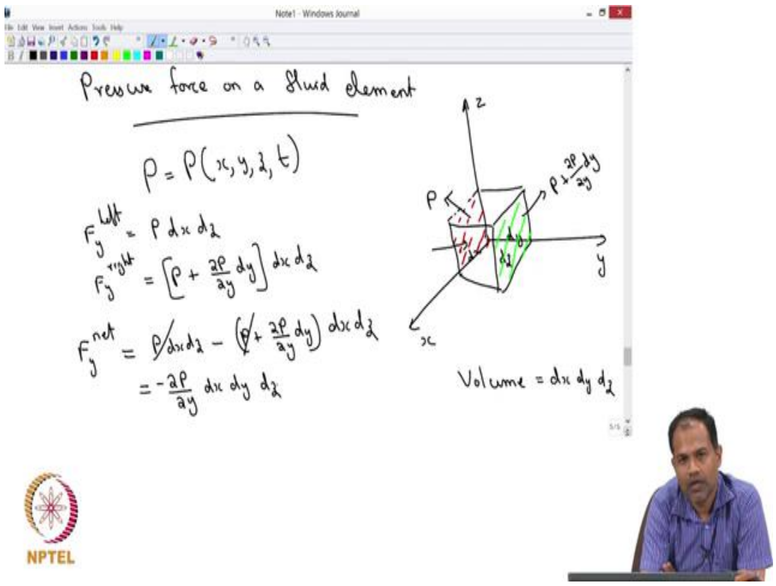

So, let us look at pressure force on a fluid element. Let us take a coordinate system. So, let us say this is x that is y and that is z and let us take a fluid element. So, this has got a length d x in x direction, d y in y direction and d z in z direction. So, that the volume of this fluid element:

$$Volume = dx\,dy\,dz$$

Now, let us say this fluid element is in a given fluid where there is a pressure distribution, so; that means, the pressure is changing from one point to another and let us say pressure is given as a function of x, y, z and t.

$$P = P(x, y, z, t)$$

So, pressure is changing at every point as well as in time ok, at any given point x, y, z and t you know what is the pressure and what you are going to do is you are going to now calculate what is the pressure exerted on each face of the fluid. So, let us say we are going to look at this face. So, the face that I have marked with red color and I want to calculate what is the force on that side. So, I know that on this side of this face, I can calculate the force because pressure is defined as force per unit area therefore, if I want to talk about a force which is acting in the y direction on the left side of the play on the left side of the cube is simply going to be. So, let us let me write left is simply going to be:

$$F_y^{left} = Pdx\,dz$$

So, that is the force that is going to be acting on this side of the element. Similarly, you can calculate what is the force on the other side. On this side this exactly opposite side of the red side that I have marked. Now, we do not know what is the pressure there, but we can let us say do a Taylor expansion and write down the pressure there.

So, let us say if the pressure here is given by P, then the pressure here can be written as:

$$P + \frac{\partial P}{\partial y}dy$$

So, that is a simplest approximation that you can do to represent pressure on that side. So, therefore, F y on the right side is given as:

$$F_y^{right} = \left[P + \frac{\partial P}{\partial y}dy\right]dx\,dz$$

And your interest is to calculate what is the net force acting in the y direction, which you will get if you subtract one from the other. So, if your interest is to calculate F y net which is the force acting in the y direction is given as :

$$F_y^{net} = Pdx\,dz - \left[P + \frac{\partial P}{\partial y}dy\right]dx\,dz = -\frac{\partial P}{\partial y}\,dx\,dy\,dz$$

So, that is the force that is arising in the y direction due to a pressure. We can continue, we can do the same thing in x direction as well as in z direction.

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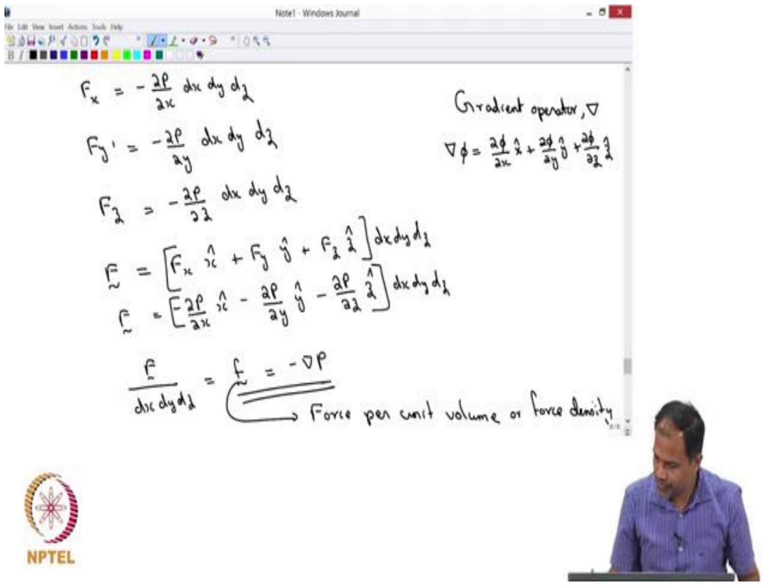

So, we can do the same exercise and then you find that:

$$F_x = -\frac{\partial P}{\partial x}\ dx\ dy\ dz$$

$$F_y = -\frac{\partial P}{\partial y}\ dx\ dy\ dz$$

$$F_z = -\frac{\partial P}{\partial z}\ dx\ dy\ dz$$

So, in the given coordinate system these are the forces that are generated, I want to write force as a vector. So, this is the component of force in x direction the second F_y is the component of force in y direction and F_z is the component of force in z direction.

So, I can write the net force:

$$\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z} = \left[-\frac{\partial P}{\partial x}\hat{x} - \frac{\partial P}{\partial y}\hat{y} - \frac{\partial P}{\partial z}\hat{z}\right] dx\ dy\ dz = -\nabla P\ dx\ dy\ dz$$

So, what is grad? Grad is nothing, but the gradient operator gradient operator that is the grad. So, that when grad is acting on some quantity that is nothing, but or we will use a different one let us say ∇ phi; that means:

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{x} + \frac{\partial\phi}{\partial y}\hat{y} + \frac{\partial\phi}{\partial z}\hat{z}$$

So, that is the definition of the gradient operator. So, we get:

$$\frac{\vec{F}}{dx\,dy\,dz} = \vec{f} = -\nabla P$$

So, what is this f now? This f is force per unit volume ok. So, in other words what we have said so far is that pressure gradient differences in pressure leads to a force or more precisely gradient on pressure gives rise to a force density. So, force per unit volume or force density. You have shown now that gradient in pressure gives rise to a force density on the fluid element ok.

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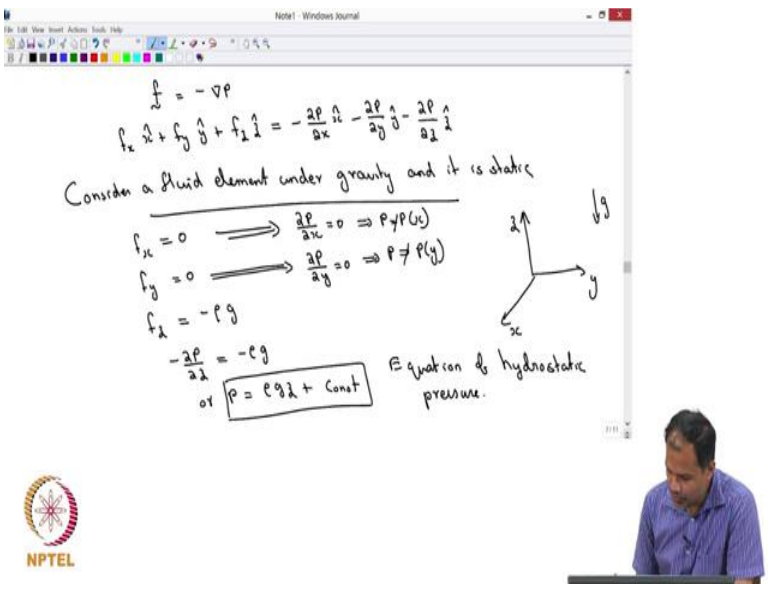

So,

$$\vec{f} = -\nabla P$$

$$f_x\hat{x} + f_y\hat{y} + f_z\hat{z} = -\frac{\partial P}{\partial x}\hat{x} - \frac{\partial P}{\partial y}\hat{y} - \frac{\partial P}{\partial z}\hat{z}$$

Let us consider a fluid element under gravity and it is static. So, the fluid is not moving the fluid element is static, the only force then that is acting on the fluid element will come from gravity ok. So, in which case you can say that. So, we will use the same coordinate system, so that is your x, that is your y, that is your z and gravity is acting in the downward direction. So, there is no force that is going to act horizontally. So,

$$f_x = 0, f_y = 0, f_z = -\rho g$$

So, you know that, so gravitational force is nothing, but weight ok. So, the weight is given by mass times g, but here we are going to write mass. So, weight as mass times g or it can it can be also written as volume times density times gravity.

We are talking about force per unit volume or in other word or force per volume and that is just going to be ρ g and I have given the minus sign because the minus sign tells you that its actually acting in the minus z direction, because this is the diagram that you see on the right that is the coordinate system that we have chosen. Therefore,

$$f_x = 0 \rightarrow \frac{\partial P}{\partial x} = 0 \rightarrow P \neq P(x)$$

$$f_y = 0 \rightarrow \frac{\partial P}{\partial y} = 0 \rightarrow P \neq P(y)$$

Therefore, P is only a function of z. This the equation of hydrostatic pressure.

$$-\frac{\partial P}{\partial z} = -\rho g$$

$$P = \rho g z + Constant$$

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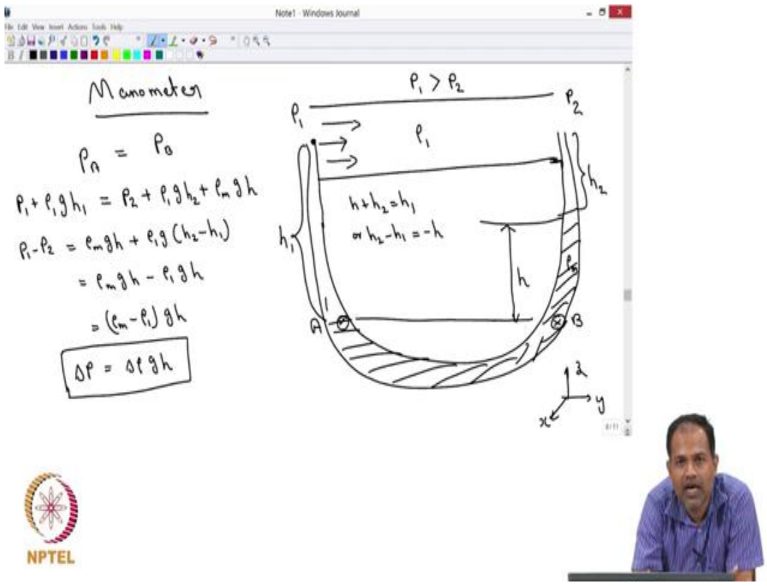

Let us see where we can use this, let us look at application of a manometer. So, where do we use manometer? Manometer is used to measure pressure differences between various points. You know let us consider simple case where you have a pipe and let us say a fluid is flowing in the pipe and for the fluid to go from the left side to the right side you need to apply a pressure difference. So, let us say you have applied a pressure P1 here and let us say the pressure on the other side is P2 and obviously, P1 has to be greater than equal to P2 for the fluid to move from left to right and some point you wanted to calculate what is the pressure difference between these two points.

So, the simplest way to do it is that you can connect a manometer. So, let me draw a manometer, I will draw a U-tube manometer it is a big manometer and manometer has a manometric fluid which let us say is as shown by the shades of that ok. So, the fluid inside the pipe has a density of $\rho 1$, let us say the density of the fluid of the manometric fluid is ρm and there will be a height difference between two limbs in the manometer let us call that h.

Now, from our previous analysis we have found that, so this manometric fluid right now is a static fluid, even though there is a fluid that is flowing in the pipe ok. The manometric fluid is a static fluid and therefore, the calculations that we just did is applicable for the manometric fluid. So, let us use that fact and we found that the pressure does not vary in, so we will choose the same coordinate system, so x y z ok. So, pressure can vary only in in z direction because that is the only direction in which gravity is acting or weight is there to balance your pressure forces, pressure does not change in x or y direction.

So, if I mark let us say this point as A and a similar point here at the same elevation as B, then pressure cannot change between A and B. So that means,

$$P_A = P_B$$

But I also know that now I can write the pressure A as pressure at this point which is P 1 plus the pressure that is going to come from this height let us call that height h1. So,

$$P_1 + \rho_1 g h_1 = P_2 + \rho_1 g h_2 + \rho_m g h$$

So, this is the pressure force balance or rather the pressure balance at point A and at point B and our interest is to calculate what is P1 - P2. So, I will write

$$P_1 - P_2 = \rho_m g h + \rho_1 g (h_2 - h_1)$$

Now, if you look:

$$h + h_2 = h_1 \rightarrow h_2 - h_1 = -h$$

So, I will use that as:

$$P_1 - P_2 = \rho_m g h - \rho_1 g h = (\rho_m - \rho_1) g h$$

$$\Delta P = \Delta \rho g h$$

So, pressure difference between two points is given by the difference in density times gravity times the height and this is the height that we are interested in. And that tells you how you can use a manometer to measure pressure difference between two points.

In other words the pressure difference just got translated into a height difference and height difference is something that you can measure and from the height difference, you can go back and calculate what is the pressure difference. So, this is a nice application of the equation of the hydrostatics that we derived just now.