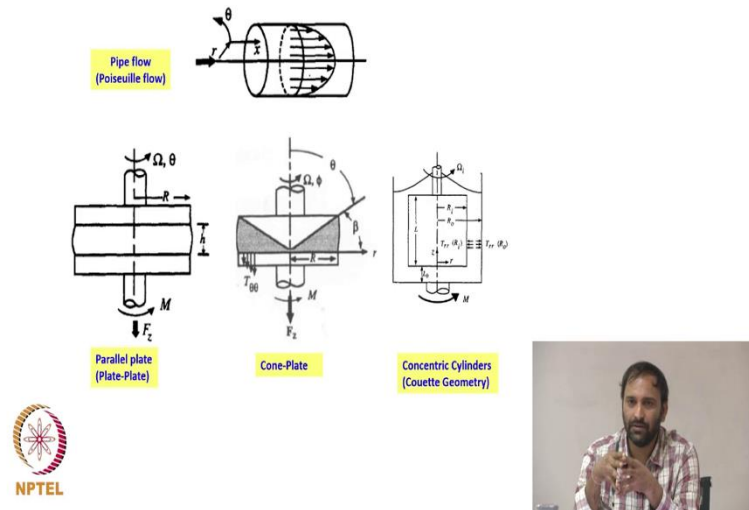


Fluid Mechanics
Prof. Madivala G. Basavaraj
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 19
Device for measuring fluid viscosity and flow behavior

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Flow Problems – Different Geometries

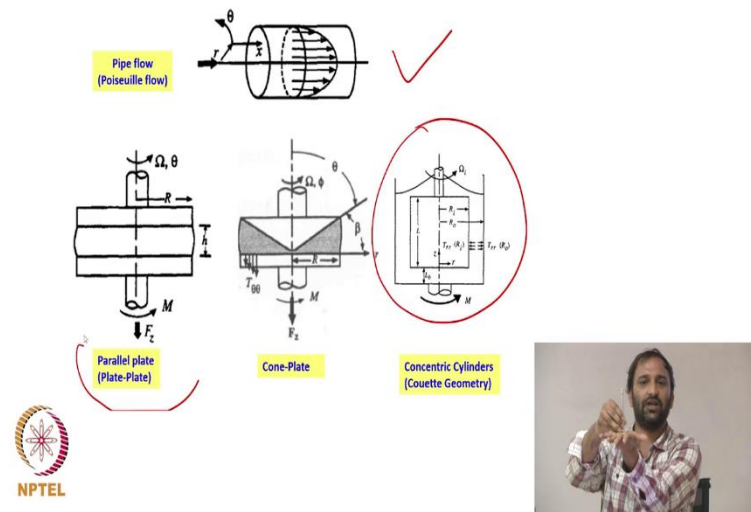


If you look at any textbooks in fluid mechanics right, you will see that you know irrespective of who has written it ok. You will always see these different class of problems that have been discussed ok. Now, I think Sumesh has already discussed about you know your pipe flow right. Plus we have also done; you know, this is a concentric cylinders right. You have two cylinders one inside the other. So, that problem also has been done ok. And then you will also see a problems where, there are two plates; that means, you know you have one plate at the bottom one plate at the top ok. And then the top plate is rotating. So, there are cases, where you kind of come across problems like this ok.

In the case of constructive cylinders what happened, whether you had a cylinder got an inner cylinder. And there was a way of rotating your inner or outer right depends on what kind of devices you want to deal with and. So, this geometry is called a parallel plate geometry. What you have is you have two plates ok.

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Flow Problems – Different Geometries



And I actually have an example of something like that. So, this is if you are interested in know a little bit more about it you can come to our lab. So, we have these devices. So, this is actually a kind of a shaft to which at the bottom you have a plate right. And then that is there is going to be a bottom plate, I am just going to keep you know this configuration. And your fluid is going to be filled between the plates ok; the bottom plate and the top plate ok. And there is a way of rotating this ok. That is you know this parallel plate geometry ok. Yeah ok, that is your parallel plate geometry ok.

Another example is something called as a cone plate geometry. So, again you know there is a; again a similar device like this instead of a flat plate here, it will be a tapered cone ok. And again there is a bottom plate ok. So, these are four class of standard problems that you do in different books ok. pipe flow, plate plate, cone plate and concentric cylinders ok. Now, when you do these problems ok, I mean all of you know the classic way to approach these problems right. You right you know momentum balance you know all of that right. So, you kind of know how to get the flow fields you know all of that right.

Now, when you do problems like this, what is your thought? Do you think these are very academic oriented problems or do you think there are some applications these things, there is some applications ok. What would those applications be? How do you think we can use some of these geometries that, you know you guys learn you know in every course you know or every course in fluid mechanics how do we use them ok. That is going to be the

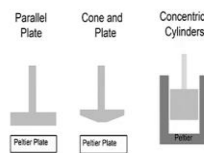
remainder of you know the today's you know lecture ok. So, I am going to show you some examples of, how we can actually exploit you know some of these things for measuring a few things ok.

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Measuring viscosity using Rheometer



Rheometer



By appropriate choice of dimensions of the geometries it is possible to generate well characterised flow fields



So, that comes to you know the measurement of viscosity ok. So, of course, there are several devices for you know measuring several ways of measuring viscosity of fluids ok. in my opinion one of the most versatile, you know instrument which can be used for measuring viscosity. Not only viscosity a lot of flow properties of materials ok. Because if I take for example, like say syrup ok, versus ointment right. They are very different right ok, one easily flows other one does not flow right.

Now, when you have complex materials like that ok you will you know you will see, that you know I cannot use something like say devices that are typically meant for measuring viscosity of fluids cannot be used of course, for the measurement of you know flow properties of the ointment right. So, in that context rheometers are one of the most versatile devices, which can be used for a range of samples all the way from liquid to solid like you know for example, toothpastes and ointments and stuff like that.

So, and of course, there are lot of different flow geometries that people use, but three most commonly used geometries are a kind of you know mentioned here, that is your parallel plate, cone plate and your concentric cylinders ok. Again if you want to know how to use

it if you want to see the instrument you can of course, drop by the lab we can show you a few things as well.

Let us just look at now. So, when I say that you know these geometries are kind of used for measuring flow properties, there is some you have to do something you know what I mean by that is that. So, Sumesh talked about, he showed some videos right. You know the videos were, if you had like say concentric cylinders there is fluid in between them right. You know when you are rotating you know you talk about Taylor number right if when you are rotating one of the cylinder at a very low you know rotation speeds you know it looked like the flow was all laminar right and at some point there were some instabilities and stuff like that right.

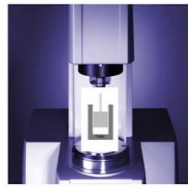
Now, and of course, when you do flow problems related to this ok. we said that you know you kind of assume some conditions that you know whether your v_r is 0 or v_θ is 0 you know you have to figure out right. Now, when you work with these geometries under specific conditions ok what I mean by specific conditions is that, if you choose geometries of appropriate dimensions ok.

Then it turns out that, the flow field that you generate in the fluid that is in the gap ok. Between these geometries is very well defined ok. And for one to use rheometers that is the most important criteria ok. What I mean by that is, if you are trying to use concentric cylinders ok you are; if you talk about R_i which is the radius of the inner cylinder and R_o which is the radius of the outer cylinder. It turns out that know all these well defined flow fields are going to be you know generated, if the ratio of R_i you know versus R_o is very close to 1.

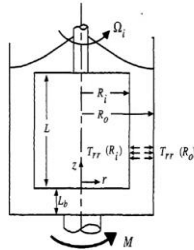
That means that is at the limit of a very very narrow gap ok. So, therefore, the point that I want to make is that you would have to work with geometries of appropriate dimensions ok. Only then you develop a flow field ok, which can be described by some of the math that you have done in the class ok, that is most important thing ok.

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Measuring viscosity using Rheometer



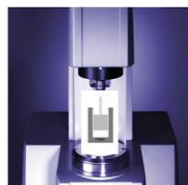
Rheometer



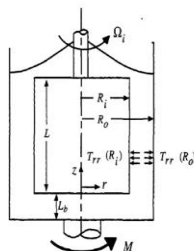
So, and typically these measurements are done as I said right, there is a basically a shaft ok.

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Measuring viscosity using Rheometer



Rheometer



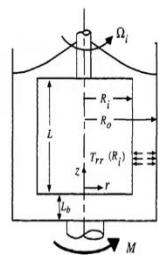
And so, only thing that you control during your measurement is that, you basically control the rotation speed that is all you do ok. There are some ways by which I can tell the instrument and I can say; hey you rotate it at a particular rotation speed ok. That is the control that I have ok. And in terms of the measurement ok, what the instrument basically measures is it measures something called as a torque ok. There is something called as a

torque sensors ok. Whenever some things are rotating right, you can think about you know the generation of the torque ok. This instrument has something called as a torque sensor and it basically accurately picks up what the torque that the instrument is applying ok. And this is common for any geometry, whether you use a parallel plate you know or a cone plate or a quad geometry ok.

So, the configuration you know is clear to you guys right. So, that is you know your rotation speed right. And M is basically the torque that you are you know; that is being measured. And of course, R_i is the inner you know radii in inner cylinder radii and R_o is outer cylinder and that is your length ok. So, this is a. So, basically this is a geometry of well defined dimensions ok.

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Working Equations



$$\frac{M_i}{R_i} = \tau_{rr}(R_i)(2\pi R_i L) \Rightarrow \tau_{rr}(R_i) = \frac{M_i}{2\pi R_i^2 L}$$

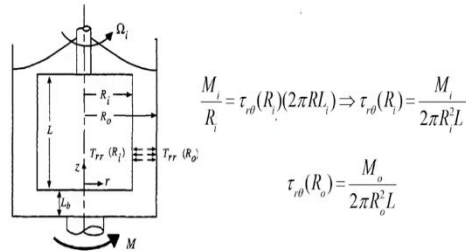
$$\tau_{rr}(R_o) = \frac{M_o}{2\pi R_o^2 L}$$



That is all I wanted to say now. So, now, when you have a some cylindrical object ok, when it is rotating ok.

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Working Equations

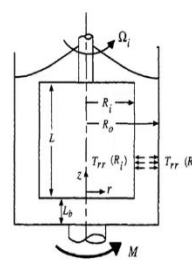


Now, what I can actually obtain is, what is the shear stress ok. What is the shear stress that the wall of the rotating cylinder is facing right or you know what is the shear stress? That this your rotating object is feeling right and so now, if I know. So, you know that you know this is your torque ok. Torque divided by distance will give you the force right ok.

Now, your $\tau R \theta$ is the shear stress that is being applied onto the your cylindrically rotating geometry right ok. Because it is a function of R and θ right it is independent of Z right. No matter you know, because that is your Z axis right and it is rotating like that ok. So, what you are basically measuring is at the wall of the rotating cylinder ok. Your shear stress multiplied by you know the dimensions ok. It is going to give you your force ok. Therefore, if I basically know if I because I am measuring M_i which is the torque that I am applying I have a way of calculating, what is the shear stress right. Because I know what I am applying, I know the dimensions, I can get your $\tau R \theta$.

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How to measure viscosity?



$$v_\theta = \Omega_i r_i \frac{R_o/r - R_i/r_o}{r_o/r_i - R_i/r_o}$$

$$\tau_{r\theta} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$$

$$\tau_{r\theta} = \mu \left(\frac{\Omega_i r_i}{r_o/r_i - R_i/r_o} \right) \left(\frac{-2r_o}{r^2} \right)$$

Evaluate above equation @ R_i and calculate viscosity



Now, you worked out the; you know your in the previous couple of class ago right you worked out you know expression for v_θ right. Yes or no you did right and you kind of got this expression that you know it depends on you know there is a mistake here right. So, it has to be capital R so right. So, basically you know it depends on your rotation speed you know our inner you know radii and outer radii and you know your radius right is a function of R right.

Now, you can go back and actually find out that you know. So, whenever you do a fluid flow problems you are going to think about shear stresses right. And shear stress is a tensor there are nine components and depending upon the problem that you are dealing with there are some are going to be 0 some are going to be nonzero ok.

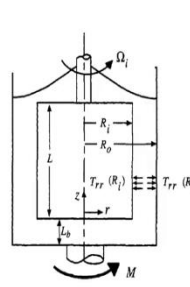
So, what I have done is I have taken an expression for $\tau_{R\theta}$ you know from one of the standard books ok and because I know v_θ ok. I know that this is 0 right. So, I can actually substitute for v_θ , I can actually get $\tau_{R\theta}$ ok. I am going to give these slides. So, you do not have to write it down So, I can substitute for v_θ and I can actually calculate $\tau_{R\theta}$ ok. And $\tau_{R\theta}$ right it goes as μ times you know all these parameters.

Now, I can evaluate this $\tau_{R\theta}$ either R_i or R_o depending upon inner one you know whether I am rotating the inner one or a outer cylinder ok. I can actually evaluate $\tau_{R\theta}$ right. Now, because I have already know, what $\tau_{R\theta}$ is in terms of the torque, I can substitute that into this equation. I can actually measure what is your viscosity ok.

So, all that is done in these devices is that there is some way of correlating the torque that the instrument measures ok, into the shear stresses ok. And if I know the shear stresses, I can actually back calculate what my viscosity is.

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How to manipulate strain rate or shear rate?



$$\begin{aligned} \dot{\gamma}_{r\theta} &= \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} = r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \\ &= r \frac{\partial}{\partial r} \left(\frac{r\Omega}{r} \right) \\ &= r \frac{\partial \Omega}{\partial r} \\ &\sim \frac{R_o \Omega}{R_o - R_i} \end{aligned}$$

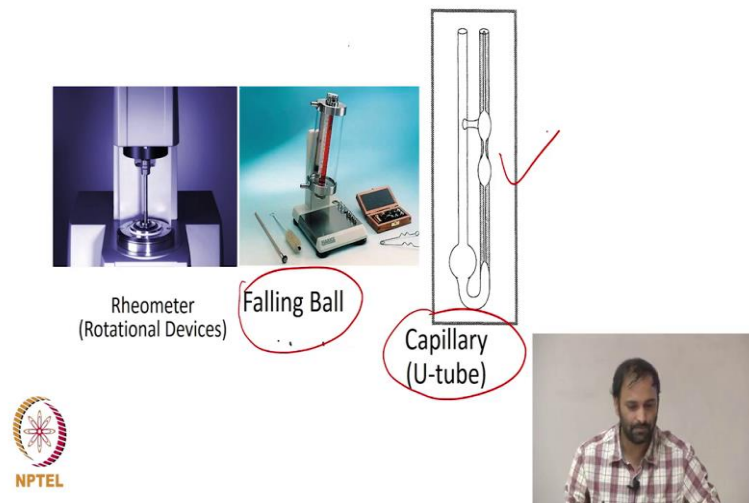


And similarly you can also again, you know your shear rate would also depend on you know the your average radius you know the rotation speed and you know inner and outer cylinders.

So, basically what I can do with these devices is, I can take a fluid ok. I can put them in the gap between you know the cylinders are the bottom plate on the top plate, I can just change your you know the rotation speed. I can change both the shear stress and I can also change the shear rate. I can measure viscosity yet you know every value of shear rate and shear stress. I will get information about how does the viscosity is changing as a function of shear rate or shear stress ok. That is what is done in a in rheometer ok. Any questions?

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Typical Devices or Instruments

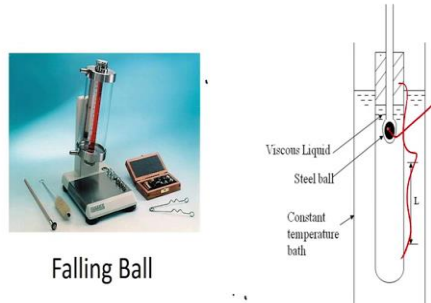


So, there are other devices right, I mean rheometer as I said is one of the most versatile devices for doing you know, what is called as a rheological studies or flow behavior of materials ok. And there is also something called as a falling ball viscometer and something called as a capillary or a U-tube viscometer.

In this capillary viscometer, what is actually exploited is the fact that you have a thin capillary. And through which a liquid is flowing and that is a classic case of flow through pipes right. So, basically whatever equation that you develop in flow through pipes, is going to be kind of useful here.

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Falling Ball Viscometer



Falling Ball



In the falling ball viscometer what you basically have is a liquid columns. So, that is you know; that is you know a spherical object ok typically a steel ball or some other material.

And that is there is a column of liquid, that is the liquid is filled in this column and of course, it has to be a constant temperature bath, because you know that in a viscosity is a function of temperature as well right. So, right and then what you do is, you make this object fall in the fluid.

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Falling Ball Viscometer

$$\frac{du}{dt} = F_g - F_B - F_D = mg - \left(\frac{m}{\rho_p} \right) \rho g - 3\pi\eta u_t D_p = 0$$



$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18\eta}$$

Terminal Settling Velocity



And of course, you are going to learn a little bit more about it, whenever you have a fluid that is going to that is falling in a you know a column of, whenever you have a an object which is falling in a column of fluid and if the fluid is stationary ok.

So, you can think about writing a simple force balance ok. And there is a net gravitational force acting on the particle, which is your gravity minus buoyancy and there is a drag force ok. So, you can put up expression for each of these things and. So, therefore, you can actually ultimately get an expression for something called as a terminal settling velocity ok.

That means. So, the moment you drop this you know object in the fluid initially there is some acceleration ok. At some point that ball is going to move at a constant velocity. And that constant velocity is basically governed by the balance between the net gravitational force and your drag force. And we know that you know your drag force depends on your viscosity right. As it falls down the presence of a liquid, basically slows down the ball right

And therefore so what you can do is I can get this terminal velocity and if I have some way of measuring terminal velocity. If I know what is the radius of the or the diameter of the ball I am using ok. If I know what is the density of the particle density of the fluid; if I have some way of measuring u t I can actually get the viscosity ok. So, it is a simple concept ok. But people still use, I mean there are commercial instruments available which kind of exploit this technique ok.

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Falling Ball Viscometer

$$\frac{du}{dt} = F_g - F_b - F_d = mg - \left[\left(\frac{m}{\rho_p} \right) \rho \right] g - 3\pi\eta u_t D_p = 0$$

Stoke's Drag

$$N_{Re,p} = \frac{D_p u_t \rho}{\eta} < 1$$

$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18\eta}$$

Terminal Settling Velocity



So, and it turns out that are this particular you know approach you can only use for cases where, you know the expression that I took for drag force ok. That is what is called a Stoke's drag ok. And that is only true you know if your Reynolds number is less than 1 ok.

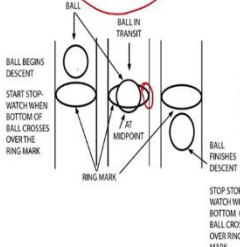
So, therefore, this only works, if you are working with you know very small particles or very small objects ok. If I take like say meter sized you know spherical objects if I make it fall it is not going to work, because you know you have your diameter of the object in the numerator. Its going be really large number ok, you are this condition is not going to be satisfied therefore, you would have to work with appropriately small you know dimension objects ok. And then you would have to ensure that you know, it is in the what is called as a laminar flow conditions ok. And that is when you know you can use this.

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Falling Ball Viscometer

$$\eta = \frac{gD_p^2(\rho_p - \rho)}{18u_t} = \frac{gD_p^2(\rho_p - \rho)t}{18l} = K(\rho_p - \rho)t$$


$\eta = t(\rho_p - \rho)KF$



The diagram illustrates the setup of a falling ball viscometer. A vertical tube contains a fluid. A ball is dropped from the top. Key points labeled include: 'BALL BEGINS DESCENT', 'START STOP-WATCH WHEN BOTTOM OF BALL CROSSES OVER THE RING MARK', 'RING MARK', 'AT MIDPOINT', 'BALL FINISHES DESCENT', and 'STOP STOP-WATCH WHEN BOTTOM OF BALL CROSSES OVER RING MARK'. The ball is shown at different positions: 'BALL IN TRANSIT' and 'AT MIDPOINT'.

Legend:

- η dynamic viscosity [mPa·s]
- t travelling time of the ball [s]
- ρ_p density of the ball according to the test certificate [g/cm³]
- ρ density of the sample [g/cm³]
- K ball constant according to test certificate [mPa·cm³/g]
- F working range constant

 NPTEL



And how do people measure. So, what they will typically the instrument would look something like this right, I think I have shown this here. So, there is basically the central column that you see here right, that is your fluid column.

Now, there are some graduations in the column ok. Now, it turns out that when you want to measure you are going to have this experiment in the lab next semester ok. The objective would be you drop a ball ok and you look at the velocity of the ball at different sections in the column. It turns out to you know if you plot the velocity as a function of time ok, it is

going to increase initially and its going to remain constant right that is when your terminal velocity is reached right.

So, therefore, I would have to; have some way of measuring velocity of the ball that is falling at some successive positions ok. So, they are basically there are markers ok. there are markers like that, that is a you know graduation that you see in the ball ok. Now if you are what you do is you have basically identify at, what is the instant of time at which the ball intersects this ring ok.

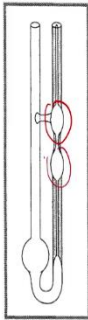
And what instant it basically travels and you know goes outside this ring ok. Therefore, in that sensor, it will travel the distance that is equal to the diameter of the particle in some time right. So, therefore, I have some way of measuring, what is the distance it travels ok. And what is the time that it has taken for you know a traveling that distance right.

So, basically you know this expression ok. And you know I can measure what is the distance is travels and that is your length, time right. And because I learn these are all constants ok. So, everything is constant I can actually write it as a K times ρ_p minus ρ times t ok. Now, your ρ_p is it is kind of kept here, because you know depending upon the kind of fluid that you going to work with ok. You may be working with you know objects of different materials you can have a steel ball, glass ball and things like that ok.

So, in that sense you know you can. So, basically your viscosity you know is actually some constant K that is something called as a ball constant ok. It depends on the properties of the ball and things like that and ok. So, you measure the time and you basically get your viscosity ok. That is how the falling ball viscometers work.



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Capillary or U tube viscometer



Measure the time required for a given volume of fluid through a capillary of given radius and length

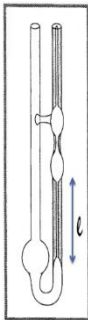
Capillary (U-tube)

So, in the capillary viscometers, what you do is you have some blobs right, something like that here alright. So, what you do is, you basically fill these blobs with the fluid of known volume and then you basically find out, what is the time that it takes for draining out ok.

(Refer Slide Time: 21:11)

Capillary or U tube viscometer





$$\frac{V}{t} = \frac{(\rho g l + \Delta P) \pi R_c^4}{8 \eta l}$$

$$\frac{V}{t} = \frac{(\rho g l) \pi R_c^4}{8 \eta l} \Rightarrow \eta = \frac{g \pi R_c^4}{8 V} \rho t = A \rho t$$

A → Instrument constant

$$\frac{\eta_2}{\eta_1} = \frac{\rho_2 t_2}{\rho_1 t_1}$$

Capillary (U-tube)

So, again the working equation is something like this again you can derive this. So, your V by t, which is the volumetric flow rate. Can anyone recognize this equation does it look familiar to you?

Student: (Refer Time: 21:25).

Was that?

Student: Poisson equation.

Correct right; Poisson equation; now I do not know if. So, we did this in the class. you kind of derived the velocity profile for a flow through pipe right. Now, if I know what is the velocity profile, I can there is a way of getting your flow rates right. You would have to you know integrate your integrate the velocity profile multiplied by you know your area right. So, you can basically get your flow rate ok. That is your V by t ok.

So, volumetric flow rate. So, now, if you look at this equation there are actually two things right one is your Δp plus your $\rho g l$ that is there are two contributions to the pressure drop right. One is your, because you know you typically these experiments are done in a vertical column therefore, I would have to also worry about your hydrostatic pressure as well, that is why you have this term ok.

Now, when you are working with you know capillaries ok. Your pressure drop across the pipe becomes important, when you are working with pipe that are really large in size right. When the length of the pipe is really large your Δp becomes appreciable, but when you are working with a capillary, when you are working with small distances actually I can neglect that ok. And I can say that you know the pressure drop purely comes from you know your hydrostatic you know fx therefore, your V by t basically goes as you know $\rho g l$ times you know the πR^4 that is the R is your raised to the capillary and you know your viscosity in the length ok.

If I know the length of the capillary, if I know the diameter of the capillary, if I you know I have a way of measuring, you know your flow rates ok. So, basically I can substitute all of this and I can basically get your viscosity.

So, what people typically do is, you can express this as a some instrument constant times ρ times t ok. Where instrument constant, what you do is if I have two fluids ok, I can calibrate my instrument, I can actually get the you know the instrument constant ok. So, yeah that is what I wanted to say today ok.

So, basically the point is that there are different fluids different and you know depending upon, what kind of fluids are you dealing with, they exhibit different fluid properties ok.

And there are devices for measuring you know viscosities ok. And you should be aware of some of these things and some of these things you are going to use it in your next semesters lab ok. So, with that I will end, if you have any questions, we can take them.