Fluid Mechanics
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Lecture – 18
Viscosity and Momentum Transfer

Last few days you have seen flow in various contexts. We started with looking at flow between two parallel planes; we have also seen flow in a cylindrical channel that is actually a pipe or flow between two concentric cylinders. You know also sometimes you know the flow was you know it was along the long axis of the channel along the axis of the channel, you have seen flow you know in circular flows. And, in each width depending upon the geometry, now you are familiar with how to go about it you know there are certain assumptions that you can make then you can question those assumptions and so on ok.

So, in each of these cases there was one property of the fluid which came up again and again. What was that property that always that, you know it was coming up? What fluid property was there in most of the equations that we derived?

Student: Viscosity.

Viscosity ok. So, why is viscosity so important? Where does viscosity come from or can we understand viscosity a little bit more in detail? Or, how does you know or what is the molecular origin of viscosity? That is what we are going to actually look at today and then you will see that viscosity is not the only property that you need, there will be many other properties that you might need in certain situations. So, we will see a few of them in the next couple of classes ok. So, that is the idea for the next two three classes. So, we are going to look at what is viscosity.

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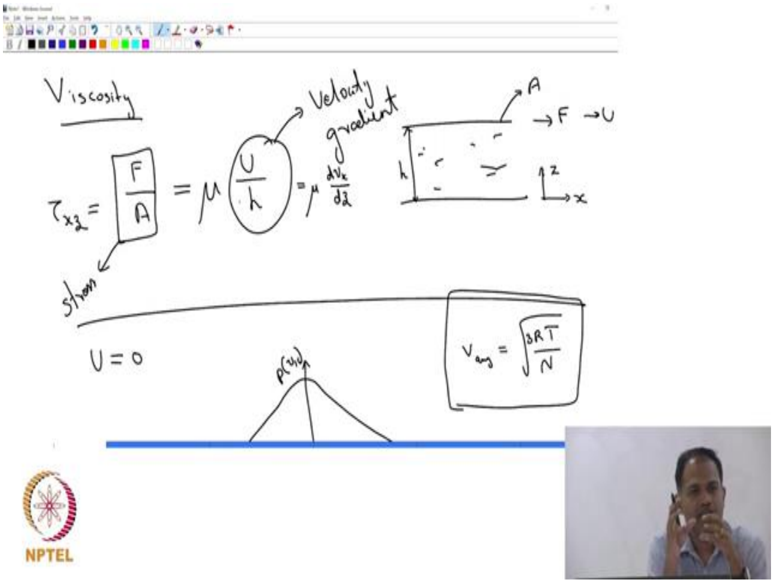

So, how does one define viscosity? What is the impression that one has? That it is a resistance right; it is a resistance to the fluid flow a little more precise is it is a resistance for the deformation ok. So, if a fluid element is undergoing deformation, then you know whether the fluid resists that deformation or not that is basically the idea of viscosity right.

And, if you go back to your older notes what you would have seen is that you take let us say fluid between two plates right and let us say that you apply some force F on the top plate there is fluid in between. And, let us say that this that area of the top plate is A because you have applied the force the top plate would start moving with some velocity which let us call it capital U and let us say if h is the distance between the plates.

You would say that F by A is essentially proportional to U by h and you would have made it an equal sign by defining a coefficient of viscosity μ that is what is done right and this left hand side is what is called as the stress and U by h is the velocity gradient.

$$Stress: \frac{F}{A} = \mu \frac{U}{h}$$

So, this is what Isaac Newton has postulated and that is what is known as the Newton's law of viscosity. Now, so, if you let us say select this as the x-direction and that as the z-direction we can write our shear stress let us say as τ x z the force that is acting in x direction acting on an area which has a direction in along the x z direction. So, to get why this viscosity comes into the picture let us actually replace the fluid. So, the fluid that is between the plates let us imagine that fluid is a gas ok. So, we say that now we have a gas that is between two plates. So, when I say fluid it could be either gases or liquids generally. So, we take a gas and then let us say that we start by a situation where U is equal to 0.

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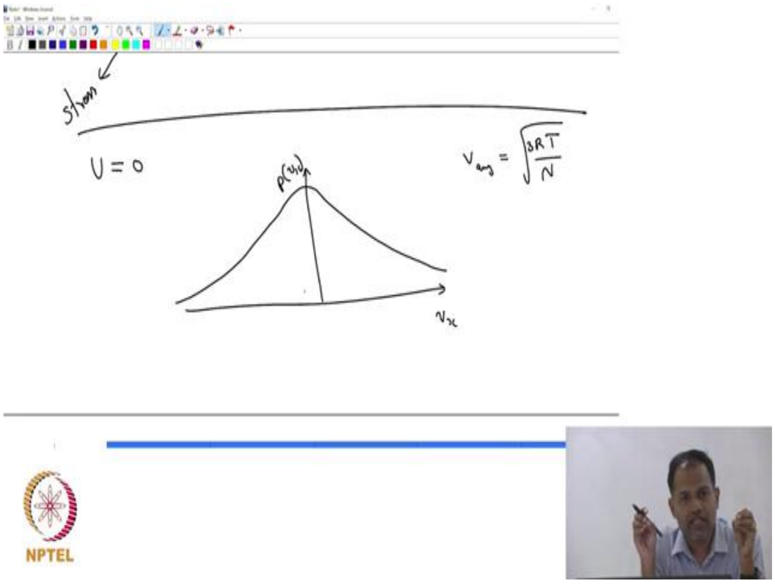

So, we will put U is equal to 0. So, the moment you put U is equal to 0; that means, the gas that you have is essentially a static thing right it is not moving hm. And, you start you know you know zooming into the gas that is you know filled up in between the plates. What would you see? You zoom in and zoom in till you start seeing the molecules that is making the gas ok.

So, you go to that level where you start seeing the gas as molecules. Would those molecules be having a 0 velocity or they would have some velocity? They have some velocity right. How do you characterize that velocity? Some root mean square velocity of a gas that you would have learned, does anybody remember the expression the average velocity of molecules in an ideal gas? What is that?

$$v_{avg} = \sqrt{3RT/N}$$

So, that is the average. So, what is R?

The ideal gas constant; T is the temperature and N is the number of molecules that you have. Now, so, that means, that each of the molecule is moving, but the net motion is 0, right. So, molecules are moving in all different directions it just so happens that oh a net is 0. So, if one looks at the individual molecules and look at their velocities then in fact, it would look something like this. So, what I will draw now is on x-axis I will plot let us say v x ok. So, that is the velocity of the molecule in x-direction and on y-axis I will plot probability of finding molecules with velocity v x and the picture would look something like that ok.

So, the molecules which has no velocity the 0 velocity there is a high probability and then there will be few which are moving in positive x-direction with some velocity there will be few which are moving in negative x-direction. And, if you say look at the number of molecules which have got large velocities that fraction is going to be really small ok. So, there is so, what I am trying to say is that the molecules are not moving with uniform velocity, they are random in all direction. It also has different speeds ok; some of them will have 0 speed, some of them will have a large speed and what you had calculated or what you knew is some average speed of all the molecules, that is clear? Ok.

So, that is the picture that you have when you have contained this gas between two solid plates which are not moving ok. Now, so, the only thing that happens is that suppose you start moving the you know moving the plate, so; that means, that the fluid start moving the gas is moving ok. Then, this picture essentially tilts a little bit tilt. So, it is not really tilting what I meant to say is that here the mean was at 0 velocity that is because there is no net velocity.

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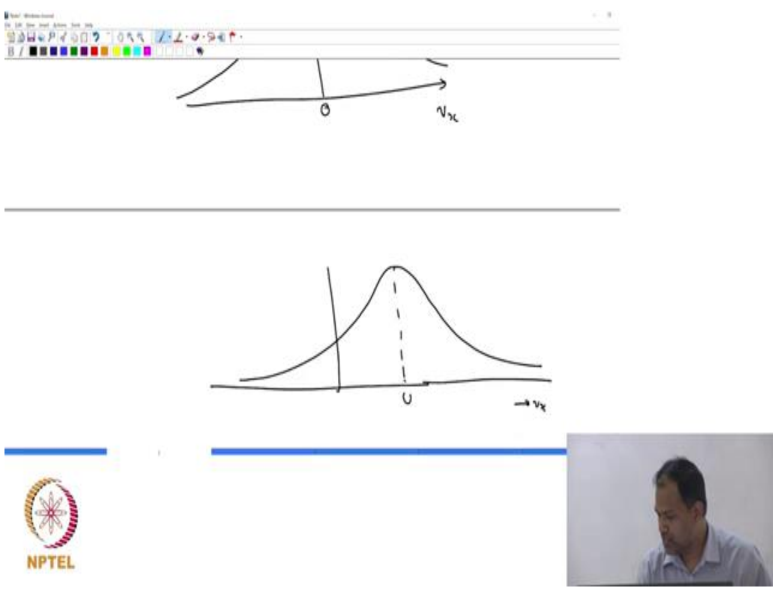

Instead of that you would see that this entire picture is something like that. So, here I have got v x. So, if I am looking at gas which is moving at a velocity U it is just that the entire distribution gets shifted ok. So, most of the molecules have a velocity U some of them have got velocity larger than U, some of them have got velocity smaller than U and on an average you will see that you know there it has got a velocity of capital U. So, that is what is it if you say that a fluid element or a gas element is moving with a velocity U. So, that is really the picture that you have ok.

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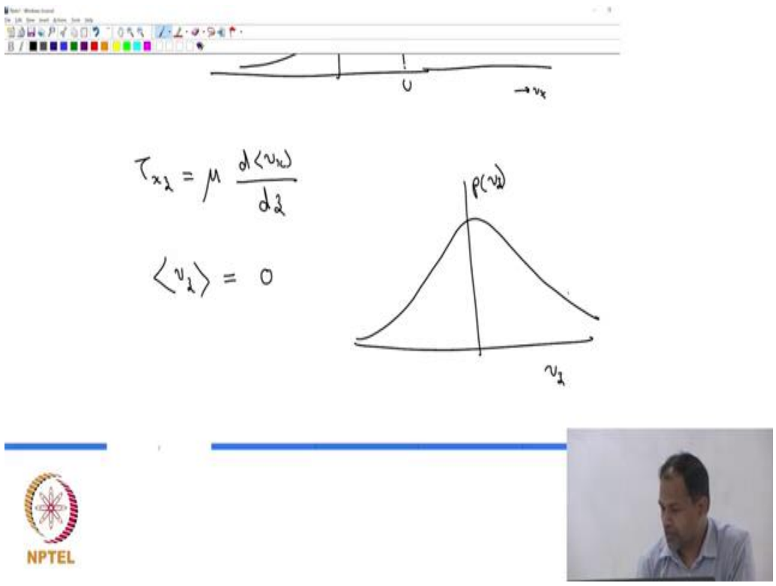

So, in that sense when you defined your Newton's law of viscosity which you defined us τ x is z is equal to μ into ok. So, we had defined it as U by h right U by h which is nothing, but μ into d v x by dz correct. So, that v x that we have defined is the average velocity of many molecules that you are considering right. So, U so, what we defined as U by h is essentially the velocity gradient ok.

So, the velocity at every point now we have realized that it is actually not just a single value, it is just that the average comes out to be a single value there are things which are changing from there. So, I should have really defined this v x as an averaged v x. So, the right way to represent it is that for a gas you should have said it is really:

$$\tau_{xz} = \mu \frac{d < v_x >}{dz}$$

Now, in this case what is going to be average v z? There is no fluid net motion perpendicular to the plate. So, it is definitely 0, but that does not mean that the molecules are not moving. The molecules are still moving right in particular if you plot v z as a function of you know sorry the probability of finding a molecule with velocity v z as a function of v z I am still going to see that right. So, there are molecules which are going up you know perpendicular to the plate there are molecules that are coming down right perpendicular to the plate.

Now, that is actually a crucial thing and that is what leads to the definition of viscosity. So, why do I say that?

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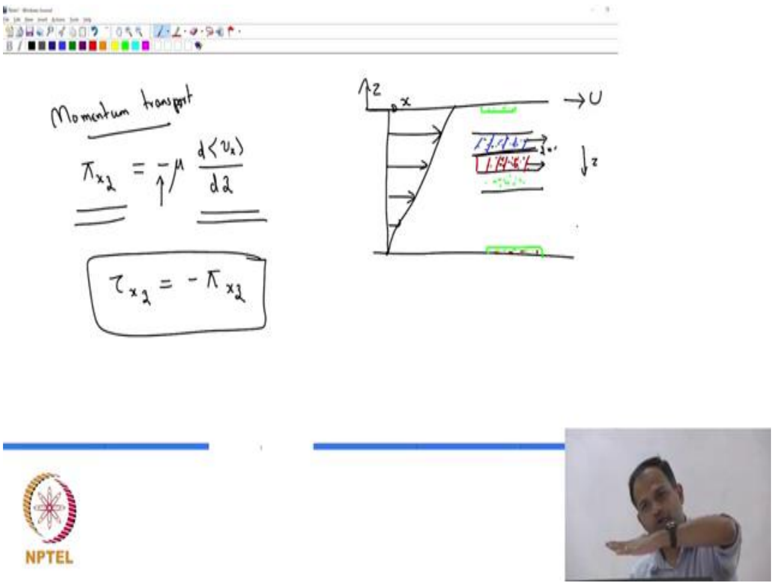

So, let us have the plates that is moving with a velocity U. I will draw the velocity gradient ok. So, if I look at let us say two layers ok. So, this is I do not know some z is equal to some number some z at some z. We are looking at a plane there are molecules here, there are molecules there. These molecules have a x-directional velocity, the molecules here have got an x-directional velocity. It just so happens that because what this profile indicates is that the molecules that you see at the bottom have a slower average velocity compared to the molecules above right.

But, these molecules not only are fluctuating when they are moving in this x-direction they are also fluctuating in the z-direction. So, that means, that some of the molecules that you are seeing here; so, let me give two different colors, the red ones and the blue ones. They are identical. It is just for convenience that I have given different colors. The blue ones have lower v x, some of them when they are you know executing their random walk they will go to the reach above the that plate right. So, not above the plate this region; so, some of the red molecules will actually intrude into the region of the blue molecules, some of the blue molecules will come from the region into the red molecules.

What would be the consequence of that? The v the red molecules actually have got a lower momentum compared to the blue molecules right because on an average the red molecules are at a lower velocity right. So, because these are moving up and down, some of the red molecules that have gone towards the region of the blue molecules is essentially transporting x-directional momentum in z-direction. Sorry, they are. So, I should do the other way. The blue molecules which have got a large x directional momentum rather than the red molecules; when they come down they will essentially transport the momentum in the negative z-direction.

So, the blue molecules have got a larger x-directional momentum because they happen to move in z-direction or rather minus z-direction. So, we have defined z up. Some of the momentum gets transported from the blue region to the red region and that is what you call as momentum transport right. So, that is the definition of momentum transport and you simply say that momentum gets transported across the layers and it happens because of this molecular motion ok.

Even though there was no net velocity in the z-direction the random you know components or the where fluctuating velocity that the molecules have it is sufficient to carry the momentum from one layer to another layer. So, that is the molecular mechanism of momentum transport and if you think about red to blue it will be essentially in the other direction because when the red molecules are go into region of blue it is going to be taking a lesser momentum in that sense ok. Because this from blue to red it will be a larger momentum from red to blue it will be a slower momentum or you could say the momentum in the opposite direction if you were in a center of mass frame ok.

So, the momentum is getting transported from regions of larger v x velocity to regions of lower v x velocity. So, the direction of momentum transport is in the minus z-direction. Is that clear? Doubts? So, if you look at momentum transport which is typically a written as π x z ok. So, that is the amount of momentum that is getting transported:

$$\pi_{xz} = -\mu \frac{d < v_z >}{dz}$$

So, why would you write that because this is our x-direction, that is our z-direction dv x by dz is so, vx is increasing in the positive z-direction, but momentum is getting transported in the minus z-direction.

So, you have to recognize that fact and when you recognize that fact you basically say that the momentum that is getting transported is really proportional to the gradient in the velocity and it will really be in the negative direction of the gradient and velocity. So, that is why momentum transport is defined like this. It is just the way you would have done your heat transfer. So, in your heat transfer you have learned Fourier's law of heat conduction? You have learned yes, no? Heat flux is proportional to temperature gradient right you would define it as some heat flux is equal to some diffusivity times the temperature gradient and the heat basically flows in the direction of decreasing the temperature. So, similarly here the momentum gets transported in the direction of the smaller velocities ok. So, that is why you have put this negative sign to recognize that fact.

Now, if you look at this the momentum transport and our original you know equation of what we have defined as the shear stress they are essentially same with just a negative sign ok. So, in another word of another way of saying is that you would define your τ x z has just negative of π x z ok. So, shear stress is typically defined in the other way. Now, you can also see that it is just a matter of convention.

$$\tau_{xz} = -\pi_{xz}$$

Doubts? Go ahead.

Student: Sir, how does it saying momentum transport is proportional to velocity gradient?

It is just like this. So, so here you have red here you have red molecules and you have blue molecules right. So, when the blue molecules and red molecules let us say are at the same velocity then there is no reason there will not be any net change in the momentum right. So, whatever momentum that is going to be transported is simply going to be proportional to the difference in the velocity at two different points. So, when you take a small element it essentially becomes the gradient. So, that is all.

So, you are essentially assuming that the momentum transport is linearly proportional to the velocity gradient that is a simplest relation that you could think about and that is what actually Isaac Newton has postulated and most of the fluids do obey that. You had some doubt?

So, we are right now talking about only fully developed flow, but this momentum transport mechanism has really nothing to do with it. See, what you are really looking at is you are looking at a region in the molecular scale you have zoomed in to that scale and we are seeing that if there are two regions with different average velocities because you know the molecules you know go you know go in some random direction and mix there is a way in which the momentum transport happens.

So, what happens is that see the blue molecules right some of them will go this way, but then we have green molecules here ok. So, some of the blue molecules are going to come here some of the green molecules are also going to come up there. So, the green molecules which are going to come up will have a lower momentum ok. So, on an average nothing happens. On an average the velocity of the layer would still remain like that and that would happen all the way down here. So, for example, if you look at the molecules which are very near to the wall they would not be moving at all while the region the molecules which are very close to the top wall will be moving with a large velocity ok.

So, since we have assumed an incompressible fluid there is not going to be any change in the density. So, that means, the number of blue molecules that will come towards the red region will be same as the number of red molecules that are going to go to the blue region and that will be through true throughout any z that you take.

Student: Sir how will we realize that τ x z is minus of π x z?

No. So, we define τ x z as just μ times velocity gradient. Now, for the momentum transport we have just said that π x z because it makes sense to define it with a negative sign because we have defined it as momentum transport or momentum flux is proportional to negative of the velocity gradient. So, we rather define our shear stresses negative of momentum transport or momentum flux. See, this negative sign is just to account for the fact that fluxes always in the direction of the decreasing gradient.

It is just a definition, that is all, yeah. Yeah. So, in one case see in one case you defined it as a force in the other case you have defined it as a momentum flux and you have connected the two now. In fact, you know why would you define this moment and transport as a force? Actually it is easy to see. Look the blue molecules there right, the blue molecules when they actually intrude into the regions of the red molecules, what is it really trying to do? It is trying to increase the momentum of the red region right. So, that momentum transport is as if this; so, let me color the regions ok.

So, that is what I am trying to say. There will be an equal number of molecules from the green region which will also come up there ok. So, there is no reason to expect that the number of the velocity would change. Now, what is the consequence of that? See, the blue molecules which have got larger velocity when they are coming into the region of red you know the red region what it is trying to do is it is really trying to speed up your red region right. So, it is as if the blue region is exerting some force on the blue region that is why it is interpreted as a force ok. So, that is why momentum and transport is exactly equivalent to shear stress.

Or thinking of the other way, if the red molecules are going towards the region of the blue molecules, then the red molecules are really trying to bring down the velocity of the blue region. So, that is as if the red region is exerting a force in the opposite direction onto the blue region. So, that is why it is a shear stress and that is why shear stress and momentum flux are equivalent quantities.

So, the topmost layer for example, these green ones they are getting continuously collided with a wall a solid wall which is moving. So, every time a molecule goes and hits that solid wall it will gain momentum from the top wall. Similarly, the molecules that are there at the bottom whenever it is hitting that bottom wall it is actually losing momentum because it was originally moving some velocity, but since it has hit a static wall it will lose some momentum. So, there will be a continuous transport of momentum from the top wall to the bottom wall. Anything else? Yeah.

Π is in. So, we have just defined π as a momentum flux ok. So, that is momentum is getting transported in some direction ok. So, here momentum is getting transported from the top wall to the bottom wall so, that is getting transported in the minus z-direction. d v x by dz we have defined it as a positive quantity and therefore, we just put a negative sign.

No, no. I am saying that the effect of the effect of the momentum transport is as if to reduce the speed of it. If you look at it is not really reducing it because there is an equivalent thing that is coming from the top layer ok, but these so, if you want to think about it as a problem in which you start everything static and then you are you know moving the top layer then you can see that a layer of molecules which are going to come down to the next one is really speeding up the next layer. And, that would continue to happen till it reaches the fully developed flow and we are really looking at right now as a fully developed flow. But, even otherwise this is the picture. Let us try to write this thing mathematically ok, that is going to give us an expression for viscosity in fact.

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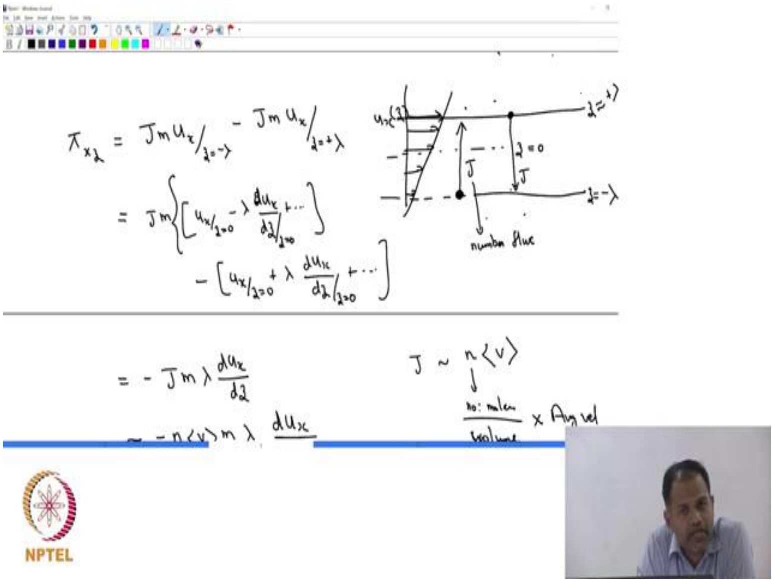

So, let us look at some region let us call that as z is equal to 0. There are molecules here, there are molecules above. Let us say some of these molecules are going up and let us call j as the flux of molecule crossing a plane at z is equal to 0 and we do know that the flux of molecules which are crossing downward would also be equal to j correct. So, π x z the momentum that is getting transported is going to be equal to what the net momentum in that direction to the upward direction minus the net the momentum that is getting transported in the other direction right. So, let us write that.

So, we need to know, so, let us define J number flux. So, that means, the number of molecules that are crossing per unit area ok. So, if that is the case then it is equal to J times m these two layers ok. So, the molecules; so, the molecules that are crossing z is equal to

0, why do they do that? The picture is that the molecules are hitting each other they undergo collisions and then they you know move in a random fashion right. So, that means, that if I look at z is equal to 0 plane, the molecules that are going to cross z is equal to 0 plane would have undergone some collision before crossing that way and do you know how often the collisions happen? You have heard about mean free path right? What is mean free path?

So, that means, I assume that all of you know. So, the mean free path is essentially the distance between average distance between two consecutive collisions. So, we can actually imagine that the things which are going to cross z is equal to 0 would have undergone a collision somewhere at a distance you know approximately of the order of λ from z is equal to 0 or in other words this could be we can define it as let us say approximately minus λ and this one is going to be approximately plus or plus λ ok.

So, that is the typical distance with which these things these things would have moved between collisions. So, all the molecules that are crossing z is equal to 0, would have undergone collision at some distance close to λ before it is crossing it and so, the other way. So, in other words one could say that we write momentum flux as:

$$\pi_{xz} = JmU_x|_{z=-\lambda} - JmU_x|_{z=\lambda}$$

Now, what would be an approximate value for J? We will correct it ok. So, right now I am just assuming that it is going to be of the order of λ. We can write that as:

$$\pi_{xz} = Jm\left\{\left[U_x|_{z=0} - \lambda\frac{dU_x}{dz}|_{z=0} + \cdots\right] - \left[U_x|_{z=0} + \lambda\frac{dU_x}{dz}|_{z=0} + \cdots\right]\right\} = -2Jm\lambda\frac{dU_x}{dz}$$

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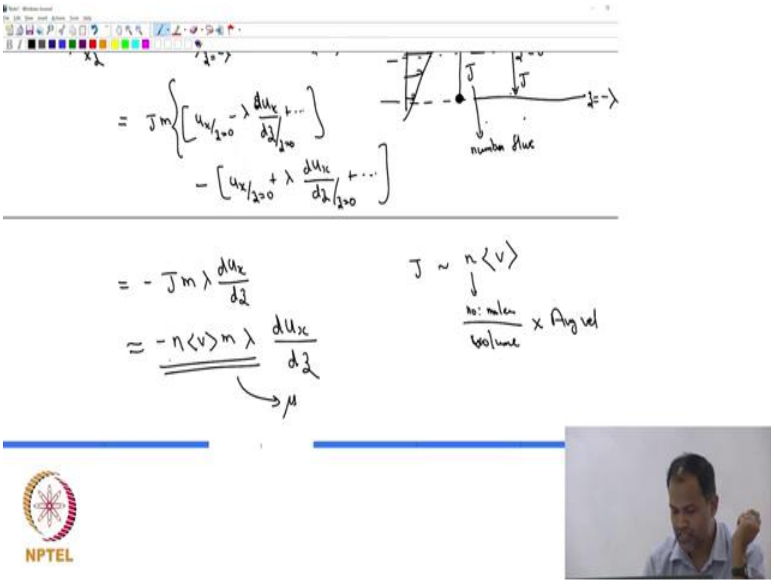

So, J is number flux and some so, a reasonable way of telling J is the J is equal to the molar concentration. So, essentially the number of molecules per unit volume into let us say some average velocity. So, this is the number of molecules per unit volume into some average velocity will give you some average flux at various points and therefore, this quantity is :

$$\pi_{xz} = -2n < v > m\lambda \frac{dU_x}{dz} = \mu \frac{dU_x}{dz}$$

Now, there is a factor that should really come in because I have just said that you know it goes from minus λ to plus λ and so on which is not really correct. But, the picture that comes out is going to be correct because it says that my viscosity my viscosity is going to be proportional to the number of molecules that you have its average velocity, the mass, the mean free path and so on. So, let me repeat ok.

So, what I am trying to say is that now you want to. So, you have understood what is momentum transport and you want to write down a mathematical picture based on that ok. Now, it is very hard to write down an exact picture you do not know at any moment where the molecules are. But, you can sort of hand wave and write down something which might give you some decent picture and that is what we have really tried to do here by saying that let us look at any you know a plane let us say this z is equal to 0.

And, let us say that the layer above it is moving with a larger velocity ok. So, that is your really profile velocity profile the molecules above z is equal to 0, are moving with a larger velocity; the molecules that are below are moving with a lower velocity because of the z-component they are going into each other's regions. You are trying to write down what is the momentum that is getting transported because of this you know molecules going up and down ok.

So, what is that momentum going to be? It is nothing, but the number of molecules that are getting transported times the velocity of those molecules and we are worried about now x-directional momentum getting transported. So, we are writing down everything that is. So, we need mass number of molecules that are going per unit area and then the velocity in one direction versus that in the other direction except that we really do not know whether they are coming from.

So, I am just saying that I will assume that the ones which are going you know through this z is equal to 0 plane they have come from a distance of minus λ and the ones which are coming from plus is they are coming from a distance plus λ because those are some numbers that I have some idea about. So, I just if I use that then I basically end up seeing that my momentum transport or momentum flux is n into v into m into λ times d U x by dz. So, that is what comes out.

So, that means, viscosity is basically coming from this picture and it is dependent upon these quantities such as mean free path, mass and number density and so on.

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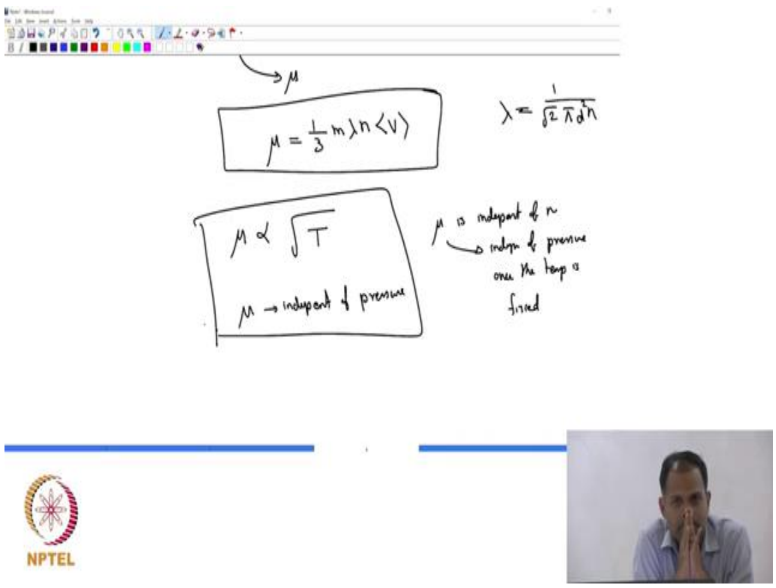

So, that is the idea that I want to in fact, if you do the proper derivation what you find is that viscosity is:

$$\mu = \frac{1}{3} m\lambda < v >$$

So, it is just that you know a complete derivation will give you this picture where the only that pre factor changes. There is a 2 that will comes is it because you are adding two things yeah anyway it is a wrong pre factor. Yes it is for a 3-dimensional case. In fact, you can do calculations very rigorously which is probably beyond the scope of what we can do at the moment.

Student: It is a kinetic theory?

It is kinetic theory. So, we are really using the results of the kinetic theory and that means, we also know what is λ. Do you know what is λ from kinetic theory? The expression is:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

You can calculate the mean free path based on the molecular size and the number concentration that will help you what is the approximate value of viscosity that a fluid would exhibit. And, that is why viscosity is there and that is why whatever you know equations that we have derived is in that particular form that is the take home message.

So, yeah because that is what I am saying I do not know way up to what distance I should really consider, but the typical distance that you will have would be of the order of what you call λ because, if it has if it is hitting another molecule then this when it is going to come back. So, the average distance before it is undergoing to go a coalition is going to be of the order of λ. You could do it as λ by 2 if you like, but then you will see that that pre factor is going to change, but we know that that pre factors at the moment just hand waving thing. So, this equation is important in two aspects ok: one is that v average velocity how does it scale with temperature, how does it change with temperature.

Proportional to root T; so, that means, that viscosity is going to be proportional to root T. So, that means, as you increase the temperature of a gas the viscosity is going to go up not decrease ok. So, for most of the liquids what you find is that when you increase the temperature you will see that things are becoming loose and then viscosity would go down, but viscosity of a gas actually increases with temperature and that is an important thing because many times in factories and so on you would want to you know heat gas and do things and increasing the temperature is only going to increase the viscosity of the gases and now decrease it, but for liquids is the other way.

So, that also means that the picture that we talked about is not really or the derivation that we have done is not really true for liquids. The mechanism is similar there also. There is a momentum transport the only difference is that the liquid molecules do not undergo these so much fluctuations. They do not collide each other and go away somewhere else. So, that picture is not really correct and that is why we are not getting the right expression for viscosity, but the momentum transport is still there and the connection between momentum transport and sheer stress all those things is still true for liquids.

We assume that the molecules are transporting they are undergoing collisions you know basically transporting momentum. Even in case of liquids what happens is that you have large number of molecules and those larger because of the large number of molecules they do not really go somewhere else. They basically have a very vibrational kind of motion. So, that is the dominating mechanism which you will have to take into account if you want to really derive an expression for viscosity of liquids which people have not been able to do so very clearly, but there are some approximate theories which can now give you that; so, the fact.

Student: There are another model for that?

There is another model for liquids yes, but the fact is that the momentum getting transported because of the molecular mechanism is still true and that is the picture that is the take home message for us. The other thing is that this viscosity so, viscosity comes out to be proportional to root T. In fact, if you substitute this λ here μ is independent of n, you can see that right. So, μ is independent of n.

$$\mu \propto \sqrt{T}\ ;\mu\ is\ independent\ of\ n$$

Another way of saying is this μ is independent of pressure once the temperature is fixed. So, assume you have fixed the temperature; you are only changing the pressure ok. You would change both number density and mean free path simultaneously. So, that viscosity will have no effect. So, the pressure so, the viscosity of a gas is typically independent of pressure that is other and these things have been seen experimentally.

In fact, if you look at when you do calculations and if you are actually the fluid that is under consideration is a gas then you do not have to really worry about what is the pressure that you are working with ok, unless the pressure is extremely small or extremely large. So, that is another important thing that you can keep in mind while doing calculations. So, yeah I think that is what essentially I had to say. Any questions? So, then we will meet on Friday.