Fluid Mechanics

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Lecture – 16
Flow between two concentric cylinders

So we started looking at the two concentric cylinders with fluid in between and we essentially calculated the velocity profile that is all we did yesterday, right. And, we also looked at the limit how to take the limit of such a problem, so that you know if you want to talk about a thin film or thin gap how does it look like. We will look at that a little bit more in detail.

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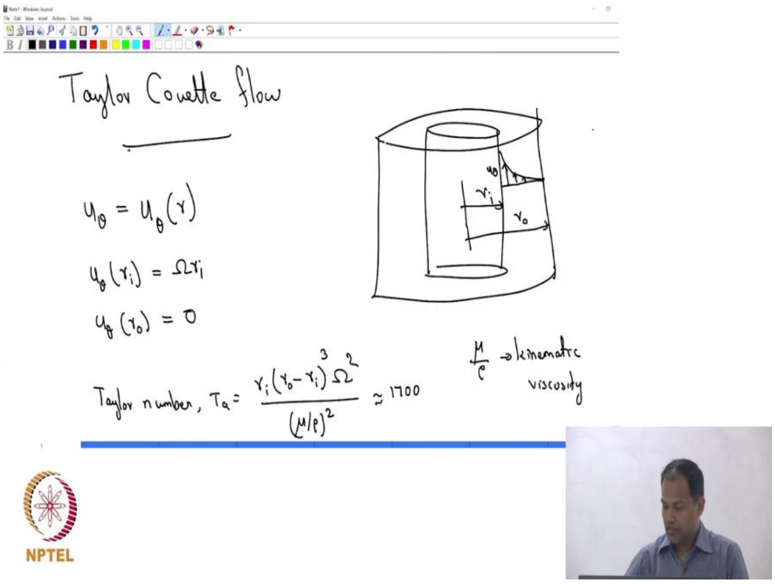

So, this flow is known as Taylor Couette flow. So, right and we found that there is only one component of velocity which was u θ right and then you found out an expression for that which happened to be a function of r.

$$u_\theta = u_\theta(r)$$

The inner cylinder was rotating with certain velocity omega which means:

$$u_\theta(r_i) = \Omega r_i$$

$$u_\theta(r_0) = 0$$

So, if you look at the velocity profile; the velocity profile would basically decay something like that that you like to actually plot and see, but that is what. Now, this profile is not a very stable profile. In fact, what happens is that if you define something called a Taylor number:

$$Taylor\ number, Ta = \frac{r_i(r_o - r_i)^3\Omega^2}{\left(\frac{\mu}{\rho}\right)^2}$$

$$\frac{\mu}{\rho} \rightarrow Kinematic\ viscosity$$

Kinematic viscosity is it is basically defined as the ratio of dynamic viscosity to the density of the fluid and it is a very useful quantity and it is like diffusivity in the. So, you have to heard about mass diffusivity, you have heard about thermal diffusivity, have you heard about? Ok. You will hear more about in the mass transfer and the heat transfer course and this quantity is a analogous quantity this is mu by rho is more like diffusivity, it is called a mu by rho is called a kinematic viscosity.

So, what happens is that if this quantity is approximately 1700 or below then whatever you derived is going to be the flow profile when you do it experimentally. But, if it exceeds that, then this flow happens to be what you call unstable in other words you will start seeing more or different things.

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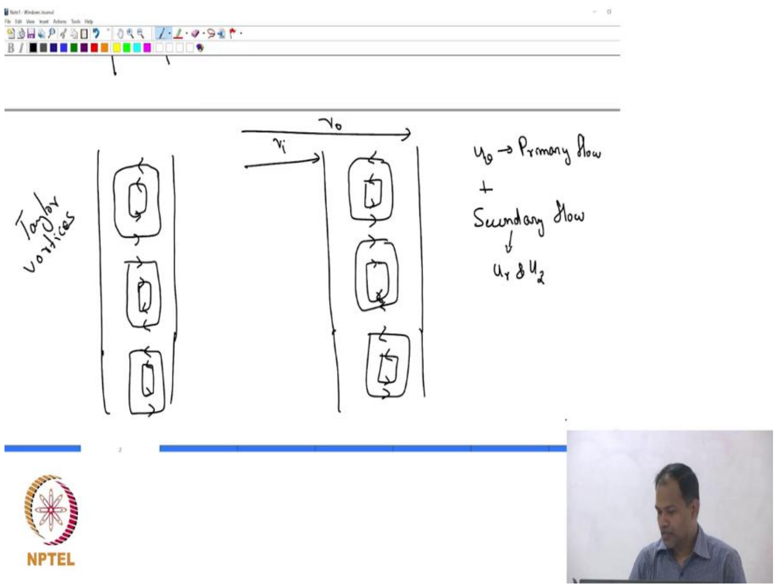

So, to illustrate that so, let us say let us just look at the gap now, so I mean it write in the next page ok. So, that is my r i, that is my r o. So, if I am increasing my Taylor number which is defined in that particular way, so Taylor number just includes the what you call the inner radius, the gap the angular velocity and some fluid properties. So, let us say you are doing an experiment and you calculate this and it came out to be let us say less than 1700, you will see the flow profile exactly what you derived in the last class.

But, if it happens to be bigger than that, then the flow profile changes. For example, if I take a cross section ok, a perpendicular cross section and look at what is the flow that is going to look like there will be a primary flow which will actually have a u θ which would be like that, but it also develops a secondary flow. So, u θ will be there as a primary flow and then it in addition it develops a secondary flow and that secondary flow will have both u r and u z components. And, the flow in fact, looks something like this and so on this side.

So, that means, the fluid is not only going like this it is also trying to do rotate like that. So, I am just looking at a cross section and you will see that there are like you know things are going circle in the in round and round. So, each of these are called Taylor vortices; vortices, so when you say vortex you know you have some feeling of vortex right; vortex is something which is going round and round. So, here you see that the fluid element will actually start going in round and round. So, these things are called Taylor vortices each of them.

Can you imagine how a fluid element will actually be going? So, so see when it was less than 1700, it only had a u θ component ok. Now, I am saying that if I am looking at a cross section with a Taylor number greater than 1700, you would actually start seeing things going round and round, but in that is only a secondary flow. The primary flow is still in the u θ direction. So, what happens if this is a fluid element it was originally going like that, but now it had to do a rotation. So, it would actually go like that; it would go like a helix ok.

So, any section you look like any section you take if you only look at that radial cross section it would look like a you know vortex, but in addition things have to go round and round, so overall it would be helix, but then this is how the cross section would look like you know it will form bands and you can see that this has go to u z component, u r component and a u θ component. So, all of them will come into the picture. Is that clear?

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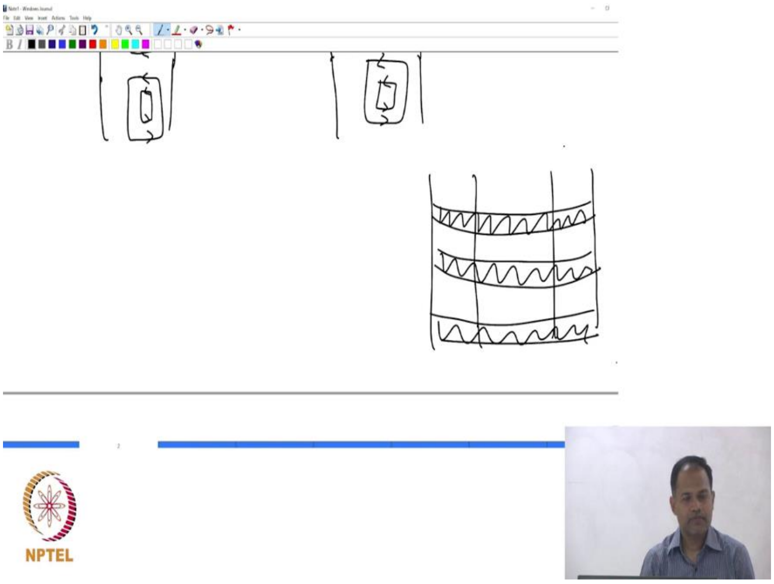

Now, this is not the end so in fact, when you look at the experiment you would see that you know if this is the gap you will see that you know there are these bands kind of thing that has formed. So, each of these band would represent these Taylor vortices. There will be just you know arranged next to each other, just a representation each of them, so in a one vortex band, next vortex band, next vortex band it forms. Now, if you increase the Taylor number further even this becomes unstable ok.

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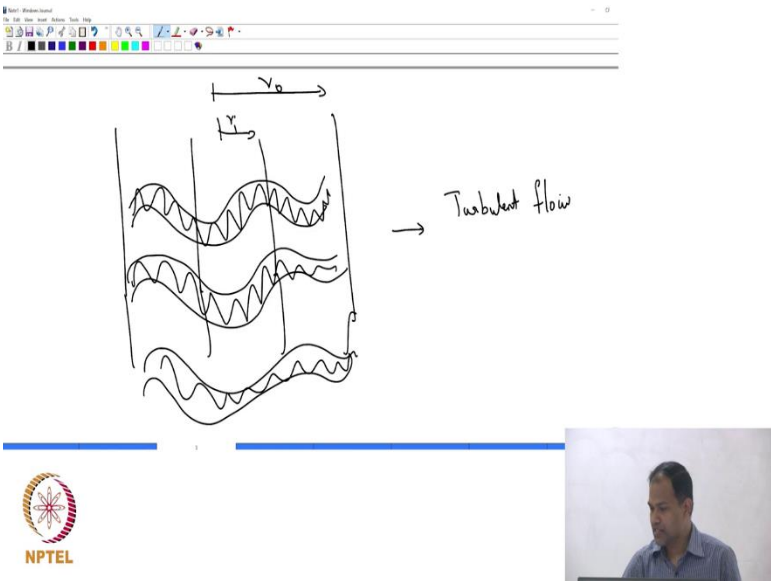

So, then in addition ok; so, that is my inner and outer cylinder that is my r i, this was my r o. So, this is, so this was the you know this is the one which you see on the right hand side this was what the known as the Taylor vortices and the bands. These bands become unstable and the bands would look something like this.

So, each of these bands will not just go in one direction like that, each of the band will start doing like this go around. So, that is even more complicated flow profile, that is what is going to happen. I can further increase my Taylor number because it is essentially a property it is a number that you know consists only of geometrical parameters and a fluid properties. If I increase it further that is actually going to give you a turbulent flow between the plates..

This is not turbulent yet. Turbulent flow would mean that you know everywhere you are going to see huge fluctuations. This is still laminar ok, the flow is still laminar, but the flow has got all three components and there is nothing that is fluctuating it is just that you know there is a you know some kind of a wave that you would see. In fact, each fluid element would act like a wave which is traveling, so that is what.

So, all these three states the one which we derived yesterday which is the primary state that is happens is a you know simple flow which would mean that it has just u θ going round and round, that is also laminar flow. With further increase in Taylor number you will see start seeing Taylor vortices which means u r, u θ, u z all of them are there it will form bands. The flow is still laminar and then further increase ,the bands become unstable and will form some waves it is the flow is still laminar and then it forms turbulent flow.

So, there are 2-3 points that I want to drive from here ok. One is that we keep making assumptions right we say that oh I will take del by del θ to be 0, I will take my u r to be 0 ok. Are they always true? So, in this case it clearly says that it is not right with our assumptions we could get only one of the flow states, if we had we if we were able to solve the equations fully without making those assumptions, then we would have been able to you know get any of these flow states ok, but we are not going to get that. So, you can always question the assumptions that you make. Is that clear? Ok.

Second is that the you know it is the same governing equation right they are ordinary the differential equation that you started with, it is always same. It is just that you have got a now four different solutions and that is because the equation is a non-linear equation and a non-linear equation is expected to have multiple solutions. So, you are ending up with multiple solutions for the same governing equation because the equation is non-linear. Is that understood two points? Ok one is the existence of multiple solutions, the second is actually questioning the assumptions that we make Clear. Any questions?

So, so you know the point is that you can do theory always, but you do need to see experiments ok. Unless you if you were not able to do experiments you would not have seen any of these states ok. So, it is really experiments that has actually given insight into what these flow profiles are. How would you do experiment, if you want to see flow? You can take two cylinders that is not a big deal, you can rotate one of them put fluid and in order to see it just put some particles ok. Just the way you know when the particles are there you can see that the particles are moving just like you know you can see or visualize how smoke is going.

So, the same idea is what is followed in an experiment also, you can take fluid and put particle and keep increasing the Taylor number. So, let us see what that looks like it is a nice movie.

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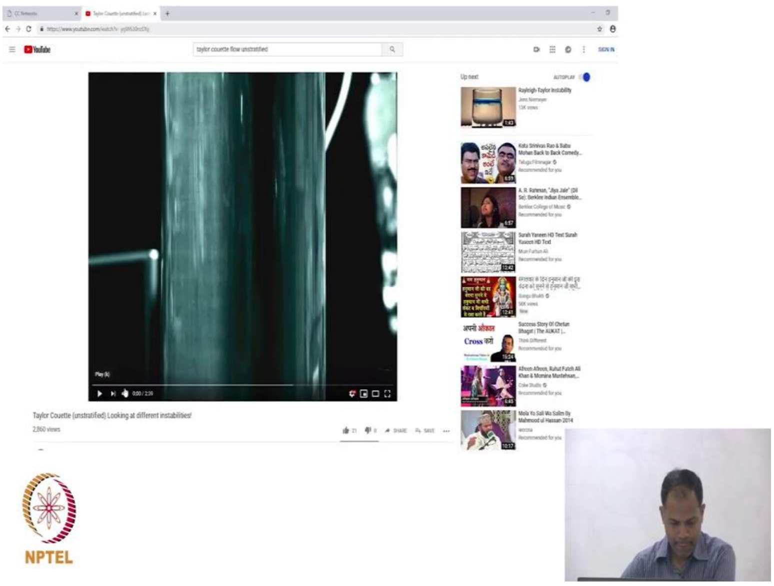

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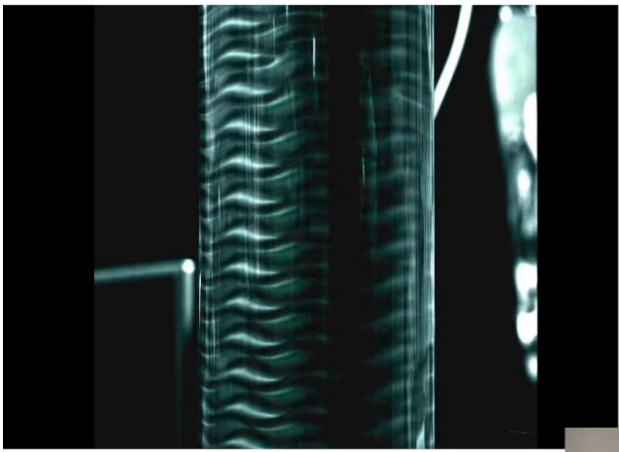

So, you have two cylinders and one of them is rotating, you may not see much because it is just the only the u θ things are just going round and round. So, you are not really seeing you can, if you look carefully you can see things moving right. Can you see things moving? Can you see fluid moving? So, you can. So, now, so, you are not. So, this is just. Yes. So, this is Taylor number is very small less than 1700.

Now, they will increase the speed of the rotating cylinder which would mean that you are increasing the Reynolds number and the Taylor vortices start appearing. The square vortices that I talked about they will look like bands that is what you are seeing now.

Now, with further increase in the Taylor number this bands beam will become unstable and it will start generating waves. So, the bands become thinner and thinner first. There you go, so the formation of the waves ok. These waves this is the third state that we discussed. So, the bands are forming the waves and the fluid elements actually are they become traveling wave in fact. You can see that clearly now and with further increase in the Taylor number it is going to go to turbulence.

So, just remember this flow state is still laminar and now, it is getting into turbulence, so there you go. So, that flow is turbulence you can see that there is lot of fluctuations in the velocity, you can see that all that nice structure is going away and it becomes very noisy ok. So, that is a turbulent flow. Now, what they will do is they are going to reduce the speed and you can see that as you reduce the speed that is equivalent to reducing the Taylor number and it will go to these transitions again. So, right now it is all fully turbulent flow.

Now, the speed is getting decreased; so, that means, the Taylor number is going down ok. So, it has come to that state where you have waves of bands. So, yeah, so that is about Taylor Couette flow.