Fluid Mechanics

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Lecture – 15
Shear stress Distribution

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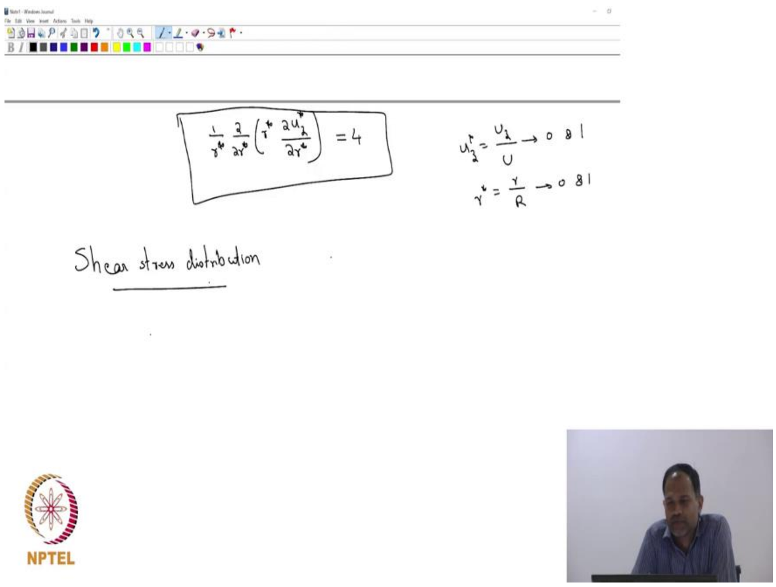

So the next thing is Shear stress Distribution. So, Tom Joseph yeah; so, what is shear stress Tom Joseph? Stress is force per unit area; so what is shear stress?

Shear force per unit area that is all it is ok; it is essentially nothing, but the viscous forces. So, we want to calculate how is shear stress distributed and looking at that is going to be little more useful because it will tell you know some nice detail about the flow.

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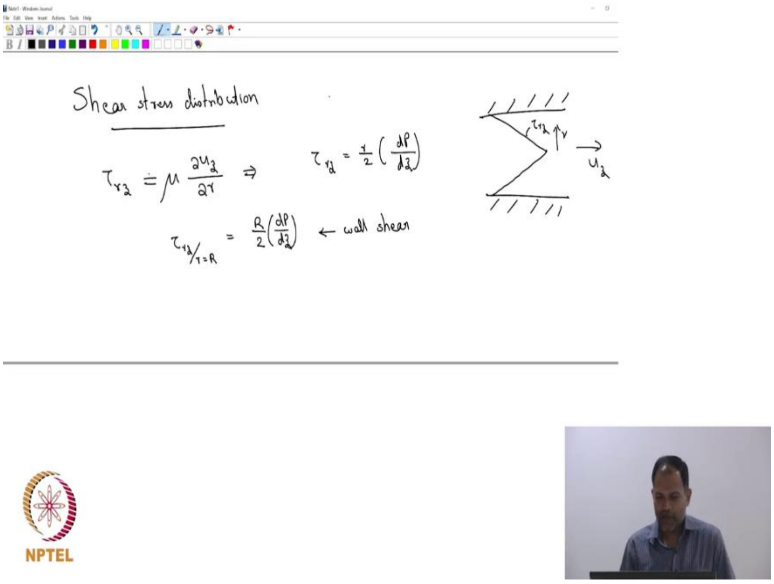

So, can you calculate; so what is the relevant shear stress that I will have. So, shear stresses has how many components? It has nine components in general be in a 3d system ok; so because we are talking about a cylindrical coordinate system; we will have $\tau_{rr}, \tau_{r\theta}, \tau_{\theta\theta}, \tau_{rz}$ and so on ok.

The only quant only shear stress that is really relevant for us is here because you have u_z as a function of r. So, the only shear stress that we need to really care about is:

$$\tau_{rz} = \mu \frac{\partial u_z}{\partial r}$$

And that so and that will be a non-zero quantity because u z is a function of r anything else you are going to take most likely we will end up being 0 or a constant. So, that is the quantity; so can you go ahead and calculate

$$\tau_{rz} = \frac{r}{2}\left(\frac{dP}{dz}\right)$$

So, that means, the shear stress varies linearly in the pipe ok; the velocity varied quadratically, the shear stress is going to vary linearly in the pipe. So, if these are walls of the pipe and this is at the center; you will find that it is basically going to. So, what will be it at the center? What will be the shear stress at the center? It is 0; why is it 0?

Because you have an r max; you have a velocity maximum there. So, dv by d r has to be 0 that is why the shear stress is 0. So, shear stress is 0 and then it will decrease sorry it will increase with a negative sign linearly towards the wall. So, that is how it really the shear stress distribution is going to look like. So, this is really; so that is my r this is my pipe that is my velocity u z and the profile would for τ r z would look like that any doubt, Rakshitha? So, shear stress at the wall:

$$(\tau_{rz})_{r=R} = \frac{R}{2}\left(\frac{dP}{dz}\right) \rightarrow Wall\ shear$$

So, this is really the shear stress at the wall and this is known as wall shear. And it is an important quantity because that is what actually is you know resisting; your flow. You are actually pushing the fluid with a pressure gradient, the wall is opposing it. The wall is opposing it by applying a shear stress on the fluid.

So, typically when you want to talk about a force that is resisting your fluid motion; it is really the wall shear ok. Now that is true now in case of a laminar flow and this is a quantity that is going to change when you are going to talk about other flows. Like you know turbulent flow or you will see few more examples later, but wall shear is actually an important quantity that is also clear.

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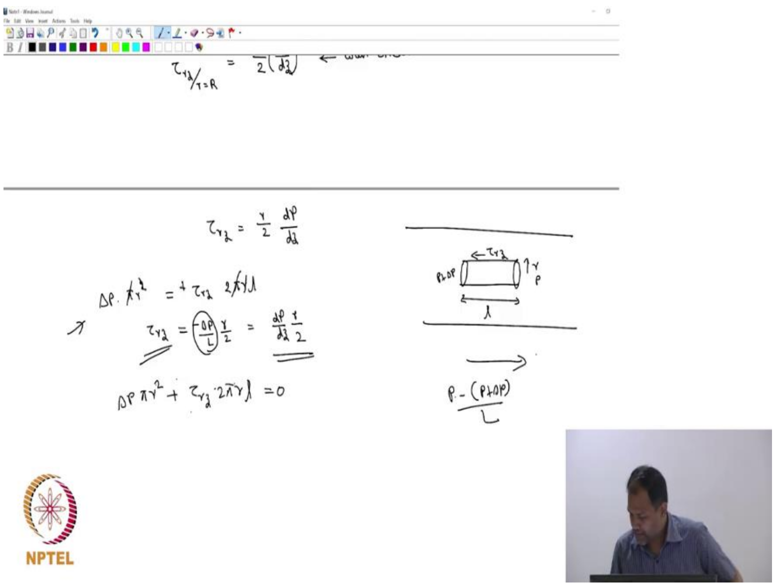

Let us write down that from a cell balance. So, you have done the cell balance approach in your continuum mechanics; is that simpler than what we do here? Or is it was a difficult?

It is actually not very hard, let us just do a quick one ok. So, we have fluid flow through the cylinder the reason I am trying to do that is because it gives you a little more insight on what is happening. Let us say the pipe and let us consider a cylindrical fluid element; a cylindrical fluid element of radius r of length let us say small l ok; you have applied a pressure gradient.

So, let us say on this side the pressure is P plus delta P and on this side the pressure is P ok. And what happens because it is basically in a fluid; the surrounding fluid will exert a shear stress on the cylindrical element, which is externally going to be τ_{rz} which is acting in the opposite direction everywhere on the surface of the cylinder right. Then you can say

$$\Delta P.\pi r^2 = \tau_{rz} 2\pi r l$$

$$\tau_{rz} = \frac{\Delta P}{l}\frac{r}{2} = \frac{r}{2}\left(\frac{dP}{dz}\right)$$

So, it is one and the same thing that is what I wanted to tell you; that if you have it actually started from a cell balance; you would have ended up with the same equation. You would have written down this equation and then you would substitute τ r z using Newton's law of viscosity, you can find out then what is the velocity profile; that is clear?

Look at this force balance; this force balance tells that any fluid element you are going to see; it experiences a force because you have applied a pressure from one side. So, the fluid element is actually trying to move in presence of the rest of the fluid, but when it is trying to move in presence of other fluid elements, it is experiencing a shear stress in the opposite direction. Because viscosity does not like things to move with a differential velocity; viscosity does not like that viscosity will try to resist it, it will try to resist it by applying a shear stress ok.

So, this pressure is trying to push it in one direction; the shear stress is trying to push pull you know push it in the other direction and that fluid element therefore, is in a equilibrium state whether it is; it is not being acted by a net force. But it is still continuing to move because we have already assumed its steady state; so, it is moving with a steady state velocity. So, that is what is actually happening its clear.

So, it is just a convention ok. So, in this pipe we are saying that this is going to go with that way; that means the pressure you have applied is larger compared to here ok. So, in this direction we have pressure that is decreasing ok. So, when we define dP by dz this was our positive z direction. So, dP by dz is just you know or other way to think about it; it is really P minus P plus delta P divided by L because the pressure on that side minus pressure on this side.

Depending upon whether you want to talk about whether the fluid element is exerting shear stress on the fluid outside or it's the other way; there exist one convention. There also exists another convention in which whether you want to interpret shear stress as a force or momentum flux ok. So, the minus is essentially to do with that, but the point is that; the when you consider the fluid element, pressure is the one which is going to push the fluid on the right side, shear stress is the one which is going to push the fluid in the other side and it is a balance of that.

We will do quickly comment about pressure now. So, I kept saying that we will you know always do this; I mean I have been doing this with the horizontal pipe right. And many of you actually had this concern that why aren't we talking about gravity. And I kept saying that ok let us talk about horizontal pipe let us in ignore gravity.

But gravity is always there; the pipes need not be horizontal and you know that you we have gravity present in that equation, but we need not solved again because there is a nice way of interpreting things. So, let us just look at that interpretation and before moving on.

So, we will start with again Navier stokes equation, but we will stick to Cartesian coordinates because that is simple and we will just look at a 1d equation.

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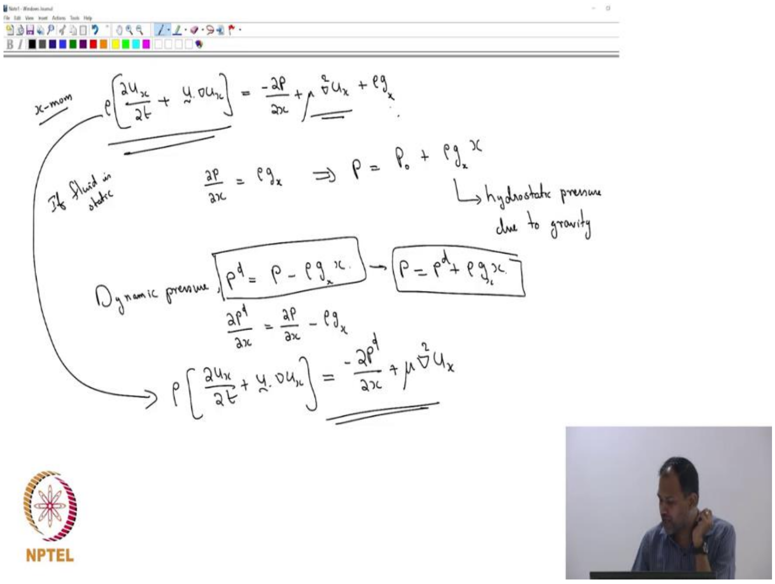

So x-momentum equation:

$$x - momentum\colon \rho\left[\frac{\partial u_x}{\partial t} + \vec{u}.\nabla u_x\right] = -\frac{\partial P}{\partial x} + \mu\nabla^2 u_x + \rho g_x$$

$$For\ static\ fluid\ in\ 1D\colon \frac{dP}{dx} = \rho g_x$$

$$P = P_0 + \rho g_x x\ [Hydrostatic\ pressure\ due\ to\ gravity]$$

So, it is as if there exists a pressure gradient that supports the weight of the fluid and that is what gives rise to buoyancy forces and so on right. So, what we can do is we can say; so what our intention right now is to take care of this gravity. And what I am going to do is I am going to bring this gravity in defining a pressure and that pressure is called dynamic pressure:

$$Dynamic\ Pressure\colon P^d = P - \rho g_x x$$

$$\rho\left[\frac{\partial u_x}{\partial t} + \vec{u}.\nabla u_x\right] = -\frac{\partial P^d}{\partial x} + \mu\nabla^2 u_x$$

So, dynamic pressure essentially is defined in order to accommodate gravitational force and this is the equation in fact, that we solved throughout right. So, the pressure that we have really calculated; even in presence of a gravity was really the dynamic pressure. If you get dynamic pressure and if you want to talk about the total pressure; now you know the relation, you can use this relation and find the total pressure. So, that is the way to easily accommodate gravity without really caring about gravity is that clear.

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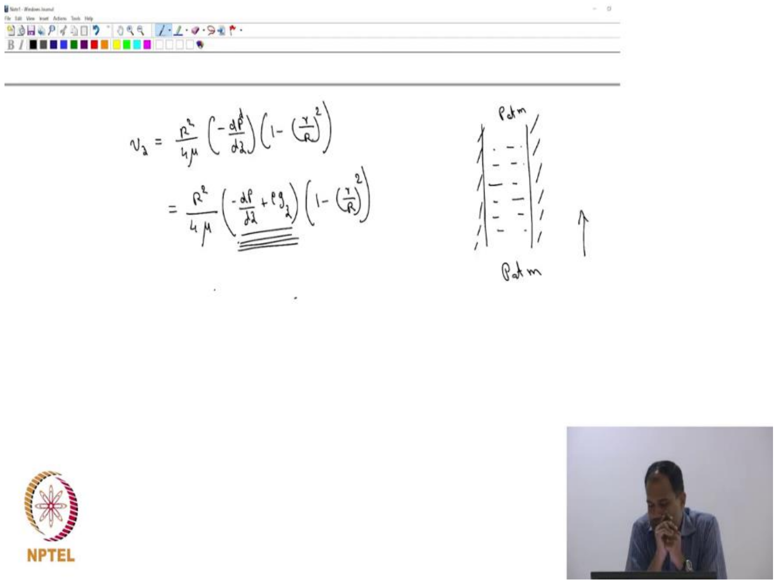

So, what would it look like if you write down let us say velocity profile for our you know pipe flow?

$$v_z = \frac{R^2}{4\mu}\left(-\frac{dP}{dz}\right)\left(1-\left(\frac{r}{R}\right)^2\right)$$

What pressure is this? Will this be absolute pressure or dynamic pressure? This is the dynamic pressure that we really used. So, can you substitute for in terms of absolute pressure and gravity what would that mean? That is what it is going to be right that is all it is ok.

$$v_z = \frac{R^2}{4\mu}\left(-\frac{dP}{dz}+\rho g_z\right)\left(1-\left(\frac{r}{R}\right)^2\right)$$

So, this is nice because let us say you consider two walls and then there is fluid in between ok. This side is P atmosphere, this side is also P atmosphere; that means, dP by dz is 0; you only have ρ gz because of which it is going to develop a flow and the fluid is going to go down right; that is all it says.

On the other hand, if you had applied a pressure gradient in this direction; let us say by putting something below like fluid in a container. Then what are you going to do? You are going to apply a upward force ok; you are going to apply a dP / dz upward and these two terms will cancel each other, when dP / dz is exactly equal to ρ gz in which case there is no flow. If dP by dz is more than the ρ gz then there will be an upward flow that is what it is.

So, it is really that competition that these two terms are showing; that is also clear, next problem.