Fluid Mechanics
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Lecture – 14
Circular poiseuille flow

So, this is how it is known as ok pipe flow or in. So, if you have flow between two parallel plates and if it is driven by a pressure gradient you call it a planar flow, with the name associated with him is Poiseuille law.

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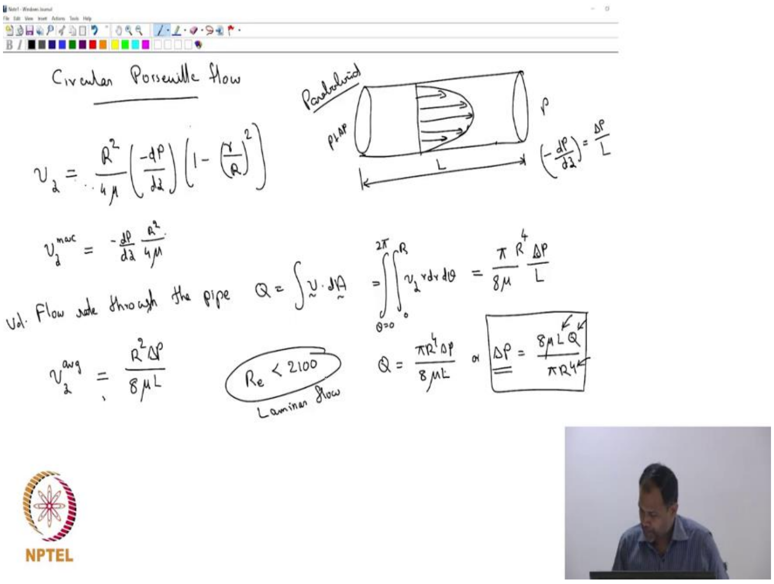

Or circular Poiseuille flow. So, what we have is a pipe right and fluid flow through it and we found that:

$$v_z = \frac{R^2}{4\mu}\left(-\frac{dP}{dz}\right)\left(1-\left(\frac{r}{R}\right)^2\right)$$

So, we will analyze this equation a little bit; the first thing that we want to do would be to find out what is the maximum velocity. Can you find out what is the maximum velocity?

$$v_{z_{max}} = -\frac{dP}{dz}\frac{R^2}{4\mu}$$

The next thing that, we want to calculate is the flow rate through the pipe. Yes volumetric flow rate yes, volumetric flow rate.

$$Q = \int \vec{v}.d\vec{A}$$

$$Q = \int_{\theta=0}^{2\pi}\int_{r=0}^{R} v_z r\, dr\, d\theta = \frac{\pi}{8\mu}\frac{R^4 \Delta P}{L}$$

I want v average. So, how do I calculate? So, I want to; so we find that the velocity profile is like that right, it has a shape of a parabola, but actually it is a paraboloid because it is a 3D parabola. It is a paraboloid with velocity being like that ok. The velocity is maximum at the center and we also calculated the volumetric flow rate, it is also good to know what is the average flow rate. So, v average is given by:

$$v_z^{avg} = \frac{R^2 \Delta P}{8\mu L}$$

Also, pressure drop is given by

$$\Delta P = \frac{8\pi L Q}{\pi R^4}$$

This is one of the most used expressions ok. Why is it one of the most used expressions? Because given a flow rate, and pipe length, and pipe radius you will know which pump you should use to pump a fluid of viscosity mu. That is what that expression is given giving you. Therefore; it is used highly ok. what else yeah. So, when; can we use this expression always?

There are original like assumptions ok, that u r is equal to 0 u theta equal to 0, there can be situations where these thing can develop ok. And one of the simplest place, where that can develop would be a turbulent flow ok. Turbulent flow meaning, flow is highly fluctuating, it will have all components ok. So, there is a limit in which all our analysis is correct and that is decided by the Reynolds number. When is the flow laminar in terms of Reynolds number in case of a pipe?

$$Re < 2100 \rightarrow Laminar\ flow$$

So, whenever you use this expression, it is important to check whether Reynolds number is actually less than 2000 or 2100. If it comes out to be less than that, your analysis will be ok. You will be finding out the correct pressure drop, otherwise you should be changing your analysis ok. Let us do one more thing, let us go back to our non dimensional equation.

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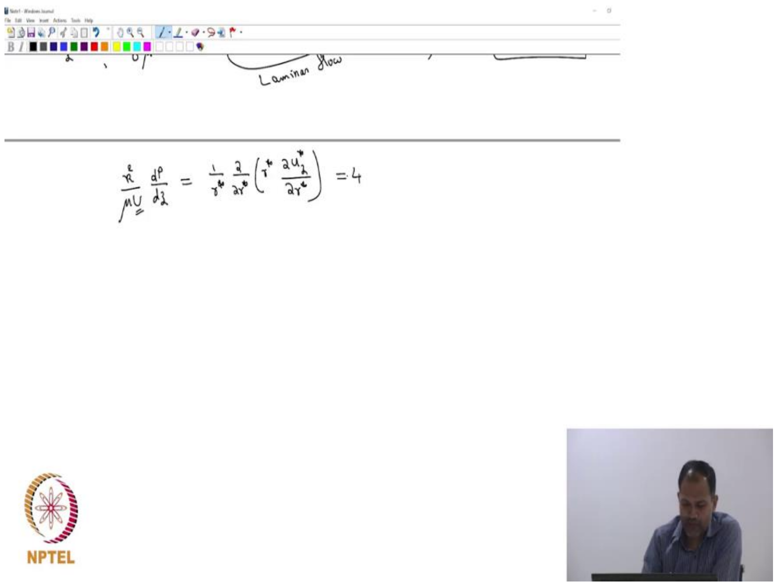

$$\frac{R^2}{\mu U}\frac{dP}{dz} = \frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d}{dr^*}(u_z^*)\right)$$

Substituting in the expression we get,

$$\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d}{dr^*}(u_z^*)\right) = 4$$

So, in fact so this is our what non dimensional equation looks like right k one. So, this is very interesting, because if you look at this; this is really the governing equation, that told you what the velocity profile was right. And this velocity profile has no information about the fluid properties, no information about the pipe dimensions. You see that so that means, this is like you know whatever you know a pipe you take as long as it is going to be a cylinder. Whatever fluid you take ok, if it is in the laminar regime this is always going to be the governing equation, you are always going to get a parabolic shape with a maximum of. So, what will be the value of U z star maximum value of U z star?

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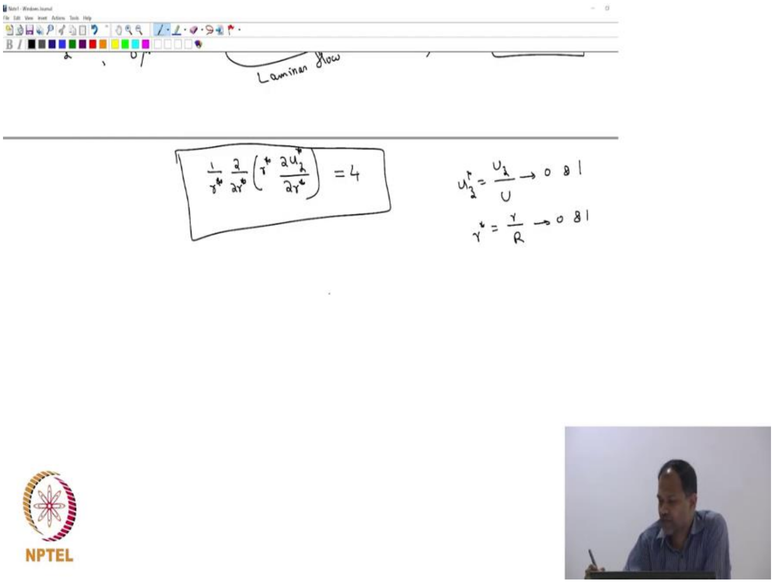

We defined U z star as U z divided by U r star as r divided by capital R, U z star will vary between 0 and 1 r star will vary between 0 and 1 ok. So, the velocity profile is always going to be a parabola with a minimum value of 0 a maximum value of 1, in a space which will go from 0 to 1 ok. As long as you non dimensionalize that profile with these things ok. So, that is what it says irrespective of the fluid property and so on.