Fluid and Particle Mechanics
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Lecture – 13
Solutions of Navier Stokes in the cylindrical coordinate system – 2

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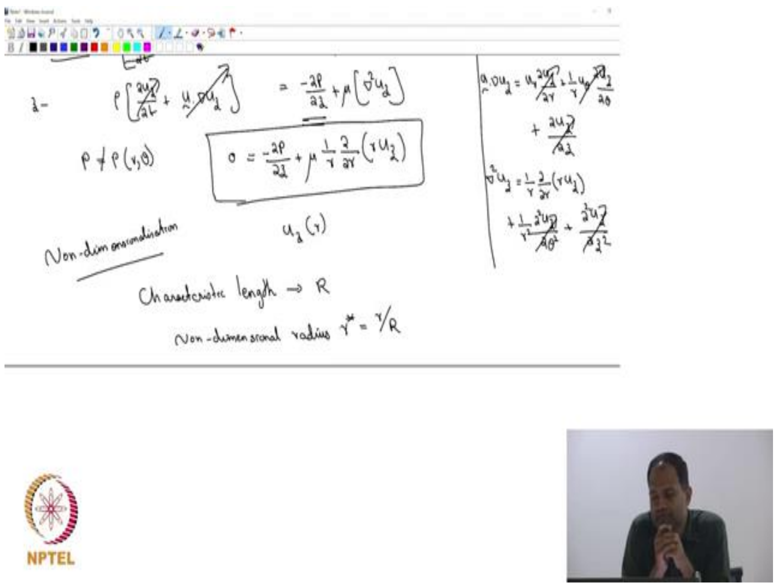

Before solving it let us do one thing, let us do something called non-dimensionalization. So, you have done non dimensionalization in the last semester's course, no you heard about non dimensional numbers, Reynolds number that is a non dimensional number right. So, we are going to see this the next part in which we will see how to do non dimensionalization, what is meant by non dimensionalization, but at this point itself we can actually start doing that and I think it is worth doing it at this point. So, before solving let us do something called non dimensionalization of equations.

So, what is meant by non dimensionalization? You are going to make all the variables non dimensional. So, for example, what is the dimensions of velocity? It is just length per time, you want to now make it a non dimensional quantity; that means, you get rid of the units from that that is what is meant by non dimensionalization ok. So, we will actually non dimensionalize each term. So, how do we do that, how can we get rid of you know dimensions of velocity?

You have to divide it with another quantity which has the units of velocity ok. So, that is one thing what is the other quantity that we need to non dimensionalize length r ok, radius see what are we expecting to see we are expecting to see u z as a function of r that is our objective ok. So, we want to find we want to non dimensionalize our velocity, we want to non dimensionalize our length.

So, now we have to do that we need to find out what can be used to non dimensionalize velocity first or okay let's non dimensionalize length first, what can be used to non dimensionalize my length. So, what we need to do is to choose something called a characteristic length ok, some constant length from the problem. So, one can you think about some constant length that we have in the problem right now.

Radius ok, it could be radius or it could be diameter depending upon whichever you want to do we can do that let us just choose radius. So, let us choose characteristic length as radius of the pipe. Then what we do, we will define a non dimensional radius

$$Non-dimensional\ radius: r^* = \frac{r}{R}$$

So, this r, small r was our original radius which had the dimensions of length we have divided it with the pipe radius and we have gotten an r star which would now vary from? 0 to 1 that is a good thing about another good thing about non dimensionalization, you have gotten rid of the length and you would basically say that things are going to change between 0 and 1. If you had taken diameter you would have found that it would go between 0 and half that is all ok, but it is always going to go in that range it is not going to go from 0 to 1000 or 10000 to some 100000 ok, that that does not come out it is basically going to be a number which is going to be of the order of 1, when I say also order of 1 it is around 1.

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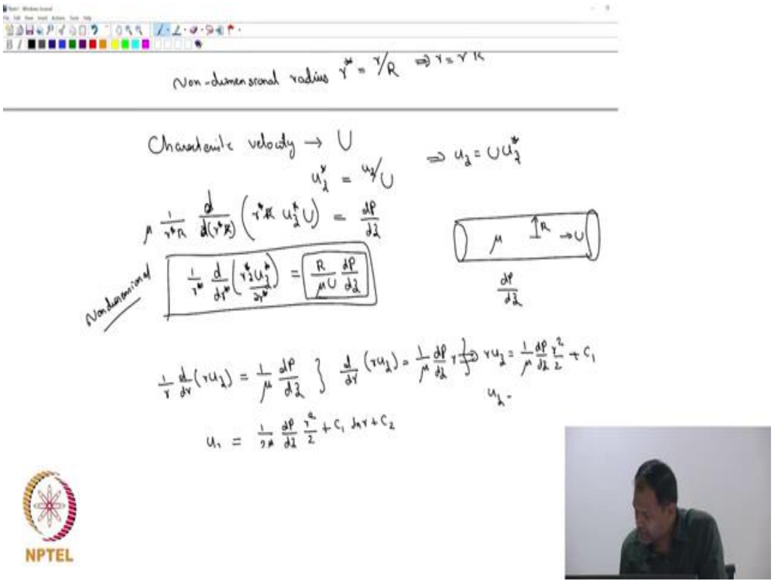

So, that so, radius is done, the other one which we need is velocity. So, we need to get something called a characteristic velocity, what can be a characteristic velocity pipe?

Maximum velocity is a possibility, but we do not know what it is at the moment, but let us say that let us just define a velocity say U which let it be the maximum velocity where does the maximum velocity appear? At the center ok. So, we will use U which we do not know, but let us just take it. So, and therefore, I can define my u z star the non dimensional velocity

$$Non - dimensional\ velocity: u_z = \frac{u}{U}$$

Now can you substitute this into your differential equation:

$$\frac{dP}{dz} = \mu \frac{1}{r^* R} \frac{d}{d(r^* R)} \left(r^* R \frac{d}{d(r^* R)} (u_z^* U) \right)$$

$$\frac{R^2}{\mu U} \frac{dP}{dz} = \frac{1}{r^*} \frac{d}{dr^*} \left(r^* \frac{d}{dr^*} (u_z^*) \right)$$

So, you look at this equation now. So, this tells a whole lot of thing ok, the right hand side r by μ u d p by d z. So, you have this pipe, you have applied a d p by d z you have applied it, it has got this radius R the fluid has a viscosity μ and it has a velocity maximum velocity

U, which must be related to the flow rate ok. So, this quantity, so, what will be the dimensions of this quantity? What would be the dimensions on the quantity on the left hand side, what is the dimension of r star?

Nothing what is the dimension of u z star? So, what is the dimension on the left hand side? Nothing. So, the dimensions on the right hand side also be nothing. So, the right hand side is actually a non dimensional number ok, it is a non dimensional number that tells you how the flow is going to be it is ok. So, the entire characteristic of the flow determined by that non dimensional number. So, that non dimensional number is nothing, but R by μ u d p by d z ok, if you say that number then the entire flow can be characterized in other words, you know u double r and then you change your you double your u also in some way then the right hand side does not change; that means, the left hand side does not change, the solution that you are going to get will have just this thing as a parameter.

So, if you it is like this. So, you have heard about parametric equations right, x is equal to r cos theta y is equal to r c sine theta is one of the parametric equation. So, what do we mean by that? So, it just says that, this is a differential equation this is an ordinary differential equation ok, there is only 1 constant in that entire ordinary differential equation which is this right hand side, which is a unit less constant it tells you everything about the flow. Originally, when we had we did not know this.

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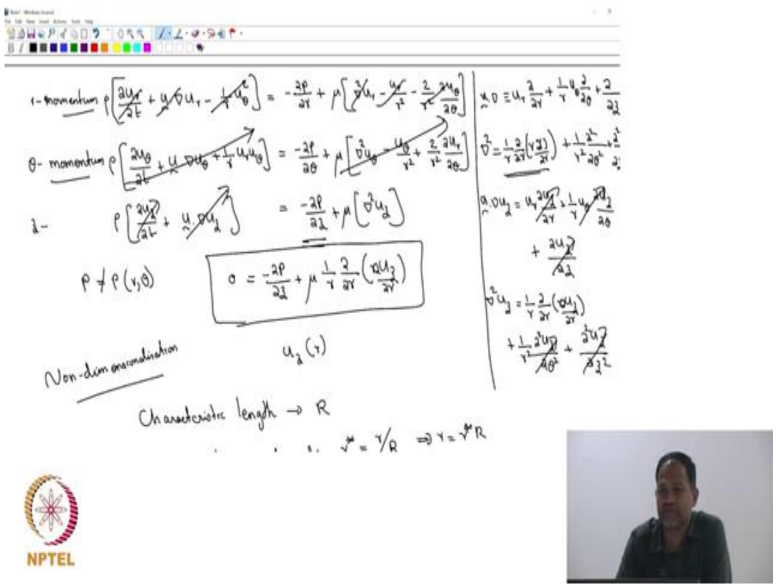

So, here if you look at the original equation that was our original equation we had a pressure gradient, we had a viscosity and that is it and each of them had it is own units and so on ok, but now we have gotten rid of all that and we said oh look at our entire equation, the entire equation has become non dimensional.

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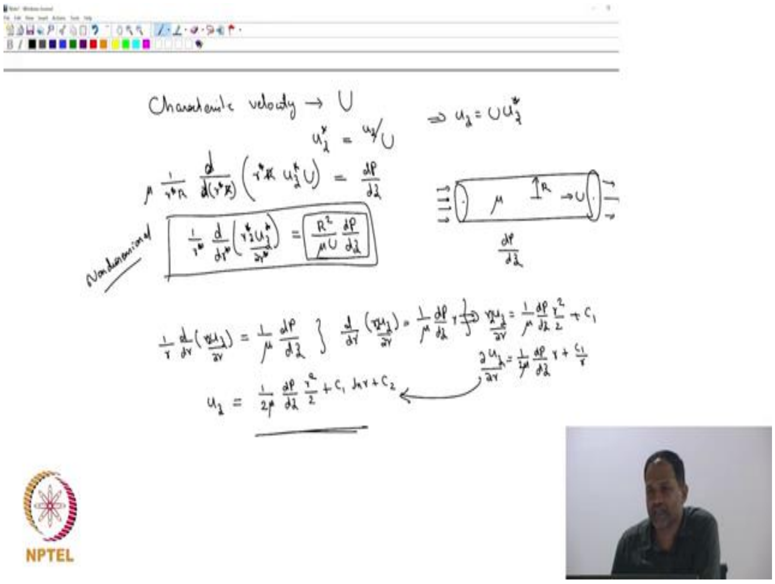

And we have also figured out, if you want to change flow in some fashion this is the quantity that we should look at nothing else we do not have to really care about anything else because we have. So, it is so, in fact, this is going to be related to Reynolds number, we will see that at a later point maybe we will see. So, we you get a little more idea about this when we substitute for u which we will do at a later point. So, this is the non dimensional version of the equation. So, we can go ahead and solve our original equation, which is that can you solve it and tell me what is the solution? You have to do 2 integrations and the expression will be:

$$u_z = \frac{1}{2\mu}\frac{dP}{dz}\frac{r^2}{2} + c_1 \ln r + c_2$$

So, now we have 2 constants c 1 and c 2 and we need to calculate what is c 1 and c 2 right ok? So, what do we need for that boundary conditions. At the pipe wall:

$$At\ r = R, u_z = 0$$

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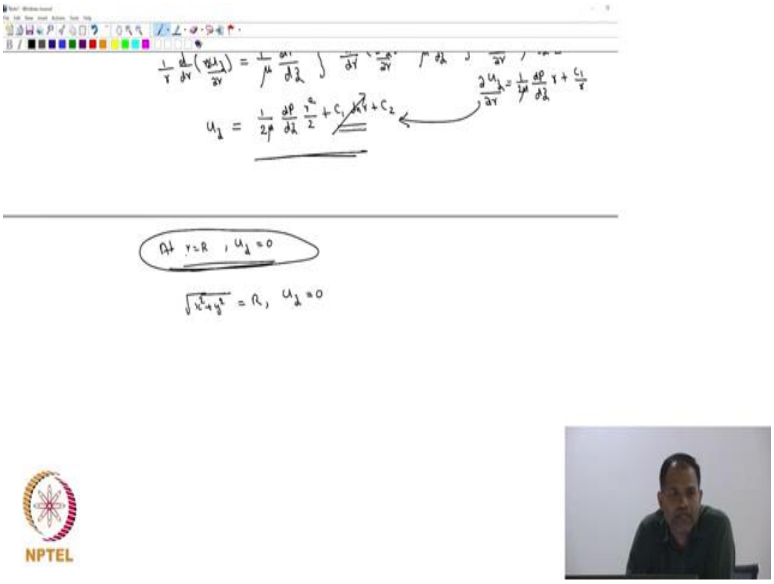

And I think, that is sufficient because we can already throw away one another term from this. So, how do we know anything about at r is equal to 0 what is u z.

So, but what about this term c 1 ln r, at r is equal to 0 what is log 0? C 1 must be 0, otherwise it will go to infinity at r is equal to 0 and you cannot have infinite velocity. So, that can immediately throw away and you will have only c 2. Now left out and therefore, u z will only be I mean you can use this boundary condition to determine what is c 2, agreed. Now this is where the choice of the coordinate system becomes important you are able to write r is equal to capital R u z is equal to 0 suppose, you had done Cartesian coordinate system what would you write instead?

$$For\ cartesian\ system, the\ BC\ would\ be: At\ \sqrt{x^2+y^2}=R; u_z=0$$

So, the boundary condition become complicated, if you actually had solved it in Cartesian coordinate system such a thing is much easily written, because now you have chosen a cylindrical coordinate system ok. So, it is really the shape of the boundary that determines the choice of the coordinate system, because application of boundary condition becomes simpler in that coordinate system there is no other reason to prefer any coordinate system over anything else ok, it is a application of boundary condition how simple it can be, clear?

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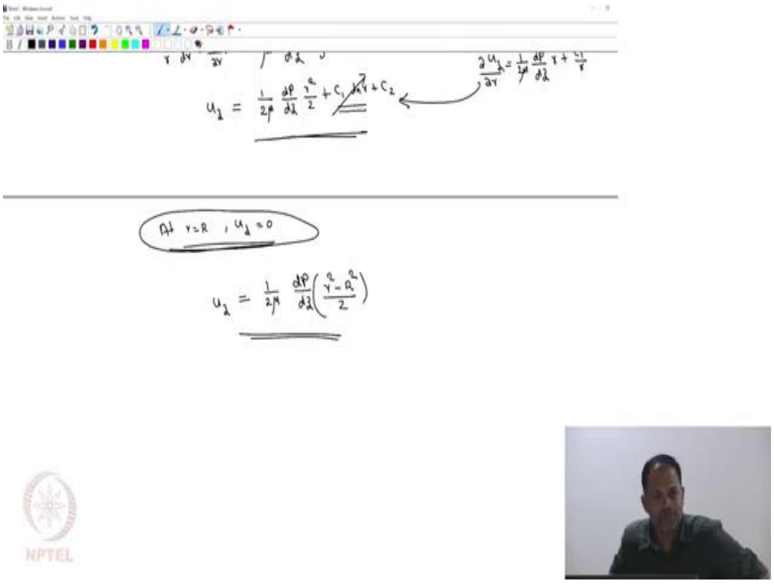

So, substituting that

$$u_z = \frac{1}{2\mu}\frac{dP}{dz}\left(\frac{r^2 - R^2}{2}\right)$$

So, we have gotten the velocity profile, now I think we will continue with the calculation in the next class now any doubts? Here, I will stop and we will meet tomorrow.