Fluid Mechanics

Prof. Sumesh P. Thampi
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 11
Continuity equation in cylindrical coordinates

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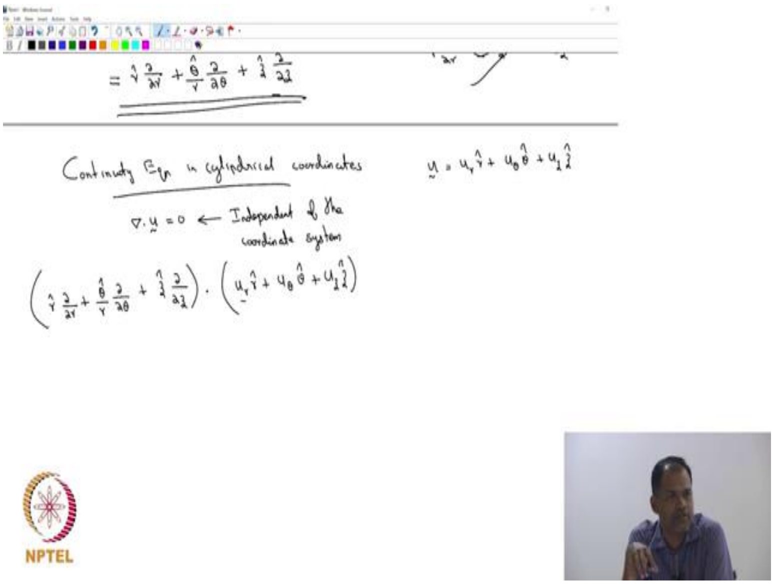

So next is Continuity equation in cylindrical coordinates ok. So, we have what is continuity equation in general? Is that ok. So, what we have done now we just derived the gradient operator in cylindrical coordinates ok.

And, you can see that it came out to be slightly different than what you would have expected. You would have thought that you would first write r cap del by del r plus theta cap by delta theta plus z cap del by del z, that is the instinct that we would have, but that is not correct you have an extra r that should come and that will come because if you look at actually the units this theta is basically an angle it is in radians ok. It has no dimensions of length ok. So, that is why you have an extra r that is coming in to make this whole thing dimensionally consistent ok.

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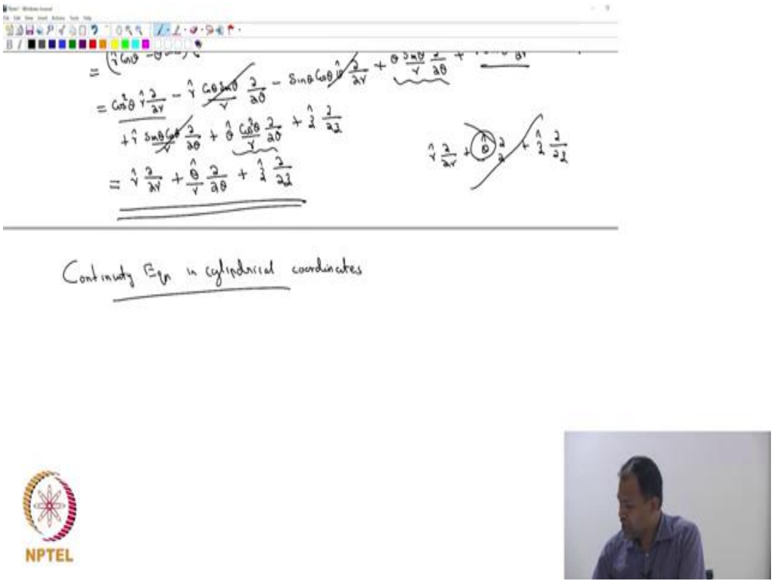

Both r and z have dimensions of distance ok. They basically talk about some length while theta is in terms of radiance and r theta is the one that basically makes it a length that is why you have that extra r coming in cylindrical coordinates. If you go to spherical coordinates then you would have more problems because you have two angles there and each of them will do in a different way you know you will have r combined with theta in different forms you might see there some r sin theta might come, r cos theta might come and so on. So that is not right.

So, considering that we have gotten this it is very easy to actually do it in cylindrical coordinates now because the velocity; so, what is velocity now? So, you have any point velocity you can write as u r r cap plus u theta theta cap plus u z z cap where u_r, u_ theta u_z are the three components in the cylindrical coordinate systems.

So, what do I mean by that?

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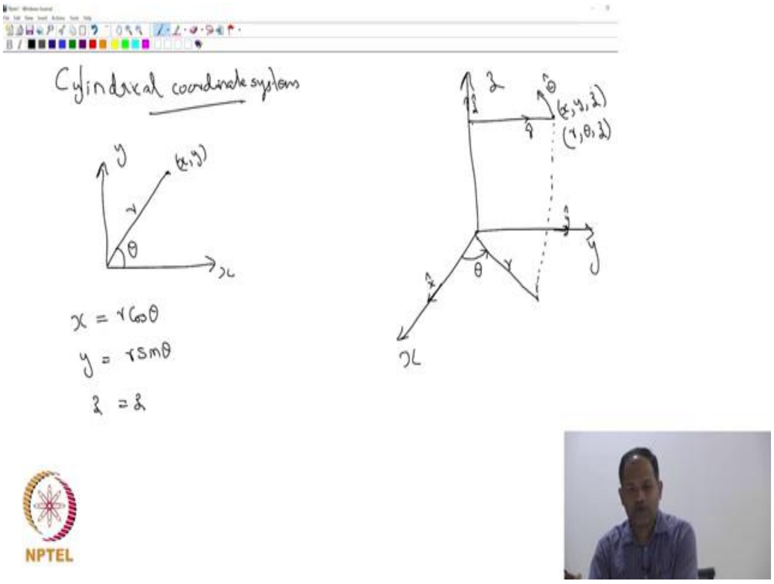

Like if you go back to the original coordinate system that we started with x, y, z if I am looking at a point there and if I am defining my velocity I would have resolved it in x, y, z if I were doing the Cartesian coordinate system. Instead of that now I will do the u r u theta that is perpendicular to the plane and a u z that is you know along the z-axis. So, if I have a velocity here which is flowing in this direction I will resolve it in r direction, I will resolve it in theta direction and I will resolve it in z direction, that is all and each component is what I wrote it as u r u theta u z ok.

So, now, we can easily do continuity equation, if you start by the general form which is divergence of u is equal to 0 right, that is the continuity equation and when it is written in this form this does not care about what coordinate system you have used actually right because there is no information about the coordinate system in this way when you write it. So, that is the advantage of writing this equations in this particular form ok. In this form it is independent of the coordinate system.

So, we can now use our expression for divergence and u in cylindrical coordinate system. So, we just said this is the gradient operator remember it is the gradient operator in cylindrical coordinate system. So, that is r cap del by del r plus theta cap divided by r del by del theta plus z cap del by del z dotted with u r r cap plus u theta theta cap plus u z z cap ok.

So, we have this is really two vectors and dot product ok, but in you have to remember that you cannot just do this times that because this is actually differential operator ok. So, you have to operate each of these terms.

So, for example, this term has to operate on this, this and this and so on and you should be writing down all such operations. In Cartesian coordinates when you do it only few of them remain which will be just you know the simpler du x by dx du y by dy and so on, but that will not be the case when you do it here. So, let us do it and see that it is different. So, that is what we want to do.

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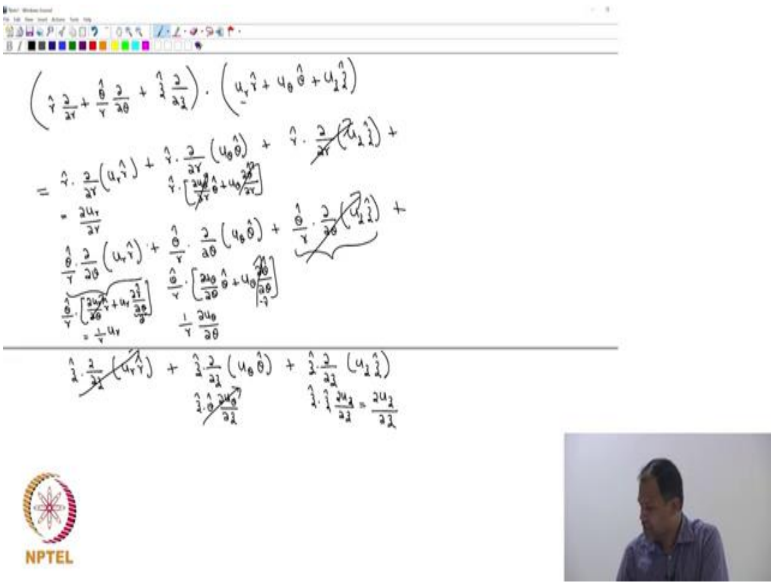

Is equal to let us start this I am going to operate on all of them r cap ok. Now, what is this? r cap del by del r dotted with this thing now. What is the dot operation between? The dot operation is between this r cap and this r cap ok. So, r cap dotted with del by del r of u r r cap plus let us do each of them r cap del by del r cap dotted with del by del r of u theta theta cap plus r cap dotted with del by del r of u z z cap plus theta cap divided by r del by del theta. So, theta cap dotted with del by del theta of u r r cap plus theta cap by r dotted with del by del theta of u theta theta cap plus theta cap by r dotted with del by del theta of u z z cap plus z cap del by del z dotted with u r r cap plus z cap del by del z is dotted with u theta theta cap plus z cap del by del z dotted with u z z cap right.

I just expand it wrote down all the terms and let us look at each of the terms now. Let us start with the last term, z cap del by del z of u z is z cap. Now, I am doing derivative of this quantity with respect to z. So, as far as this quantity is concerned only u z can depend on z z cap is a unit vector. So, this is really z cap dotted with z cap del u z divided by del z, but z dot dot z cap dot z cap is unity and therefore, what you basically have is del u z divided by del z that is only term that is coming from the last term.

How about z cap dot del by del z of u theta theta cap? So, you have to do del by del z of u z u theta theta cap. Does theta cap depend on z? no. So, I can pull out theta cap outside which is going to give me z cap dot theta cap del u theta divided by del z, but what is z cap dot theta cap? They are orthogonal. So, they will go away ok. How about this term z cap dot del by del z of u r r cap? Does r cap depend on z? No. So, you can pull out r cap that is going to give you an z cap dot r cap which will be 0. So, that will go away.

So, you can see that out of all the z derivatives only dou u z by dou z remained. Agreed? Now, let us look at the next term. So, let us look at this. So, this is del by del theta of z cap. Does z cap depend on theta? No, you can pull that out, but that will make it 0. So, that term will go away. Does theta cap depend on theta? Yes? No? Yes. So, that means, you need to apply the product rule there to differentiate. So, we have theta cap divided by r dotted with del u theta divided by del theta times theta cap plus u theta times del theta cap divided by del theta.

What is del theta cap by del theta? Minus r cap ok; so this r cap so, if I take dot product of theta cap with r cap that is going to go away. Only this remains I have a theta cap dot theta cap that is 1 and therefore, what is left out is 1 by r dou u theta divided by dou theta, clear? How about this term? r cap is independent of theta. So, I can pull that out so I get theta cap dot r cap that will go away. This term I can pull out z cap. So, that will go away this I need to be careful because theta cap is dependent on r and I should do a product rule. So, I have to do r cap dotted with dou u theta divided by dou r into theta cap plus u theta into dou theta cap divided by dou r.

So, this one r cap dot theta cap that will go. What is dou theta cap by dou r? What is dou theta cap by dou r? Oh yeah it is with respect to r. So, that is again 0 k. So, there is nothing to worry about. This one what is here again r cap is r cap dependent on r? No, r cap is independent of r so, you can pulled out r. So, that means, this will just leave you dou u r by dou r. So, you get that term, you get that term I have made a mistake.

Where did I make the mistake?

The first term or the second term? Second term it should be. Here right? So, that cannot be right because r cap is dependent on theta, theta cap divided by r times del u r by del theta times r cap plus u r del r cap divided by del theta. So, this is 0, but del r by del theta is theta cap. So, that is theta cap and therefore, that gives me theta cap dot theta cap basically gives you 1 by r u r. So, I will write down all the terms together that I have gotten.

I have gotten that del u r divided by del r plus I have this term 1 by r u r because I have that term 1 by r dou u theta by dou theta plus dou u z by dou z is equal to 0. So, that is what we get as the continuity equation in cylindrical coordinates.

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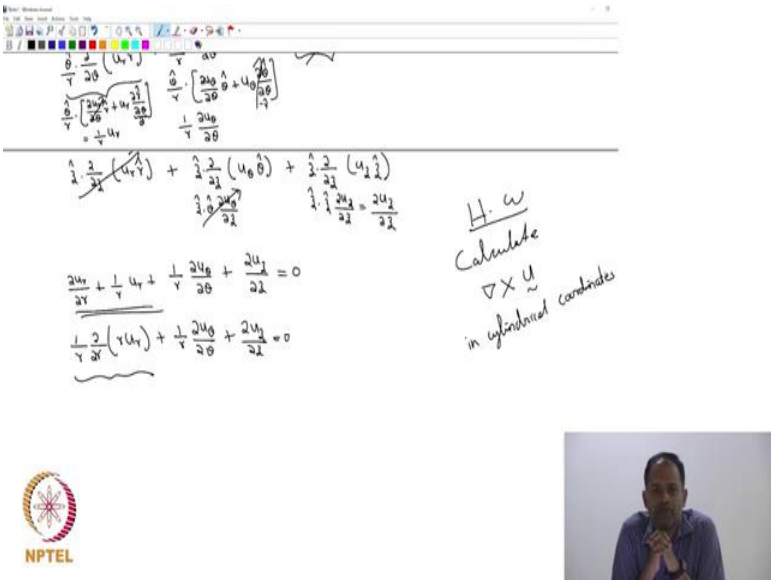

Often it is written as 1 by r del by del r of r u r plus 1 by r dou u theta by dou theta plus dou u z by dou z is equal to 0, because if you differentiate this you are going to get these two terms ok.

So, that is where that extra you know r factor and r dependence comes from ok. This will be useful exercise and once you do that then you will be clear what when you get the next expression how to transform between different coordinate systems.

So, well, this is one way of doing it, there are multiple ways of doing it, but this is something probably that something that you are familiar with because you are familiar with equations in Cartesian coordinate system and given that you will be able to convert it into any coordinate system that you want. So, you can imagine how it would get a bit lengthier if you want to do the momentum conservation equation. So, let us just take it for granted that we will be able to do it because the procedure is simpler, it is only the algebra that is lengthier ok. So, we will just take those equations and try to go ahead with the solving pipe flow problem in the next class that is tomorrow.