CH5230: System Identification

Journey into Identification

(Case Studies) 5

All right. So now we turn to a different kind of situation where we're going to discuss effects of randomness.

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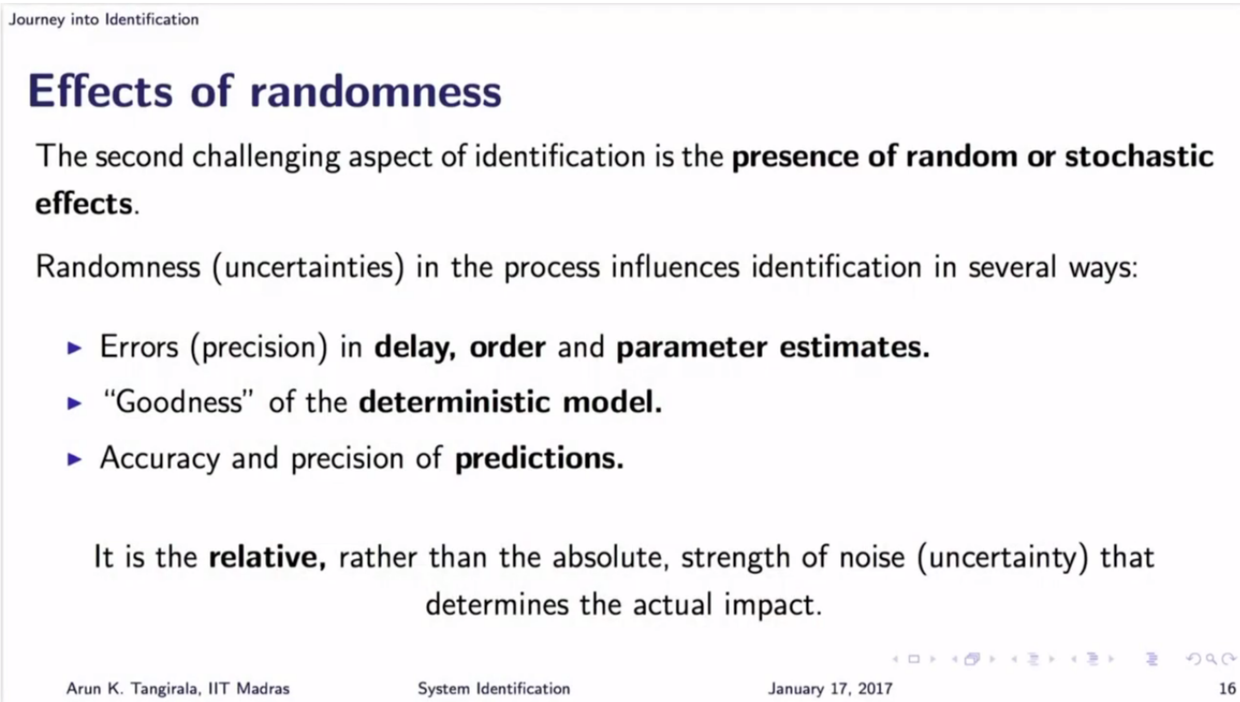

I will go through a couple of examples and then we'll take up the case study, liquid level case study in tomorrow's class. So until now we have discussed about identifiability and closed that discussion with a mention of the effects of noise. And as I said, one of the main effects of noise is that from finite data points, we will never be able to estimate parameters accurately and precisely. Therefore it becomes important to get a feel of what this noise can do to our identification. Right? This is actually the crux of the challenge in identification, when you get to estimating the parameters. So what can this randomness in my data do? Remember, the randomness as we discussed in last week is not just due to measurement error, it can be due to effects of unmeasured disturbances as well.

And there maybe process noises, process randomness as well, deterministic model alone may not able to explain the process dynamics. So there are various sources of randomness, but we lump all of them and whether we lump or not, this randomness in the data that we have and in the process that we have affects at least three important aspects of identification. One is that it results in errors in estimation of time delays, estimation of order of the model, estimates of the parameters themselves. That means, there's another way of saying, we'll never get accurate estimates.

Secondly, it affects the goodness of the deterministic model, naturally. This is now how good your deterministic model is going to be. Whether it's a poor one or, you know, a high quality one and so on. And consequently it has an effect on the quality of predictions. So all the way from your estimation to the model to the predictions, it has a significant role to play. Now, often we tend to think that it's absolute levels of noise that matters but fortunately, it's a relative aspect that matters more. What we mean by relative is how much of noise is present relative to the input.

Now, you hear the AC sound, right? When you walk in, as you are sitting, you know that this AC is going to make some mild amounts of noise. In this room some noise, another room another level of noise. But it doesn't matter to you as long as I speak loud enough, because the purpose of sitting here is to learn something about the subject. And you say, well, I'm able to hear clearly, so doesn't matter. Let the noise be present. You'll never understand what I just said. Or maybe I speak just loud enough. It's not a correct way to say, but loud enough so that only the first-benchers can listen.

Then, there is a problem. But as long as I speak loud enough, what we mean by this, relative to the noise there, then you are okay. You say, that's okay, that noise can be there, right? This is what is quantified using a term called signal-to-noise ratio.

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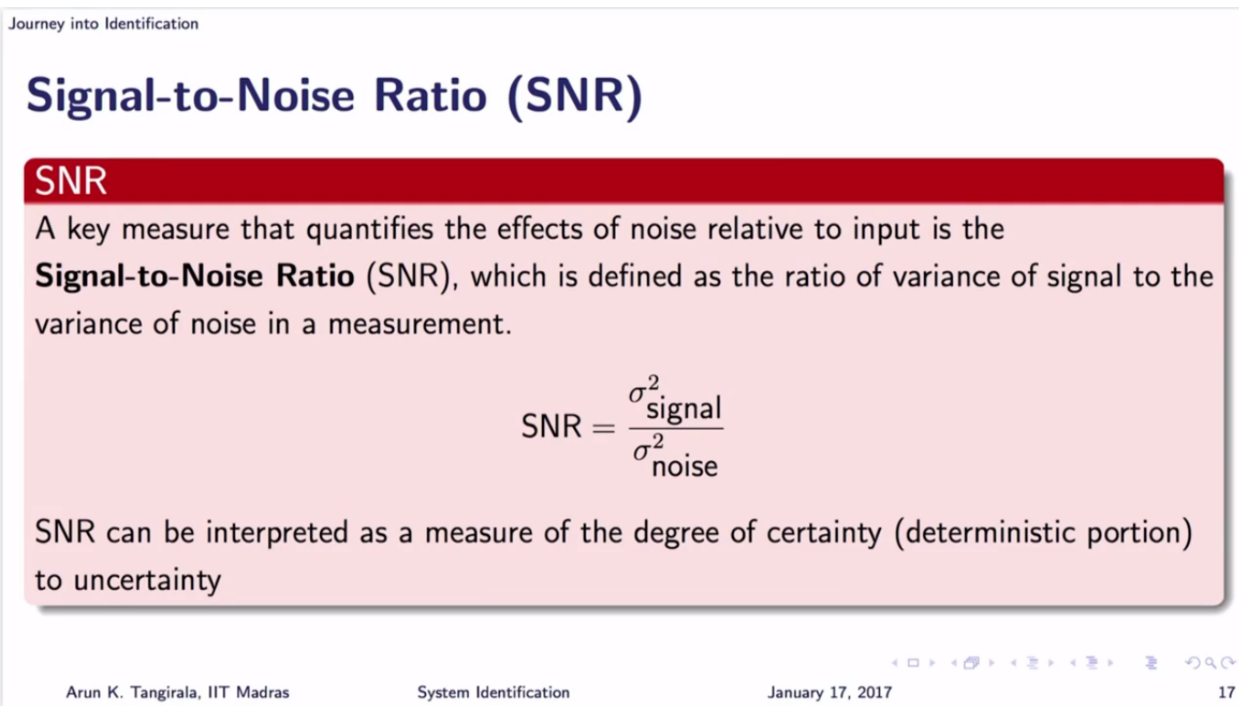

There is a factor called signal-to-noise ratio, which appears everywhere in the entire estimation world, you'll run into this factor called signal-to-noise ratio. And again, I'm giving you the definition here but I won't go deep into the definition, but this definition will give you a feel of what signal-to-noise ratio is. It is the ration of the variability in the signal to the variability in the noise.

Now many times people think that signal-to-noise ratio should be the ratio of amplitudes. Why can't it be amplitude of the signal to the amplitude of the noise? Why is the signal-to-noise ratio defined this way? It again comes from parameter estimation theory. Estimation theory tells us that the errors in estimates actually depend on this ratio, not on the amplitudes. And from an electrical engineering viewpoint, you can think of this variance as being the power of the signal. Okay? So signal-to-noise ratio is a measure of the power in the signal to the power in the noise. There are different reasons why you can give but the strongest reason for defining signal-to-noise ratio this way is this quantity that we can show theoretically influences the errors of the parameter estimates, not the amplitudes of the signal to the amplitudes of the noise.

Although, amplitudes do have a say here. It's not that the signal-to-noise ratio is not a function of the amplitudes at all, it is but it's not defined as a ratio of amplitudes. It's defined as a ratio of the variances. And that's again because as we will show theoretically later on, the errors in parameter estimates depends on the ratio of variances rather than ratio of amplitudes.

Qualitatively you can interpret this as the amount of certainty that you have, the signal, now the signal is a very vague thing. What is signal? Depends on what you are looking at. If you are looking at a signal processing application, signal may mean something. If you are looking at system identification, the signal may mean something. For example, there is something called signal-to-noise ratio at the input, signal-to-noise ratio at the output. What is the difference in the SNR at the input, the signal is the input. And the SNR at the output, the signal is the output.

As far as system identification is concerned, remember, we drew this schematic, we said that there is this input which excites a deterministic process and generates a truth y star and here I have v. I don't know y star v, but I know the input and I know the output. Typically, it is the output signal to noise ratio that plays a role, of course, that's also function of the input signal-to-noise ratio. But the signal-to-noise ratio that we are looking at is how much, how loud is the response with respect to the noise.

If y star, if your input – definitely y star is generated by input, there is no doubt about it. But if you have used an input such that your y star is pretty low compared to v, it's not loud enough, then it gets buried in noise. And estimation algorithm, we'll have a tough time discovering what is y star, what is v, it's in this big heap of noise that it is searching for a needle.Right? You don't want that. And estimation algorithm has its own limitations. So here, the SNR typically is at the output side. And this sigma square y star over sigma square v.

Right? The signal that's of interest here is y star. Now when I'm doing this, those of you who are quite, you know, fond of rigor and so on should recognize that I'm assuming y star to be a random signal, when I write there as sigma square y star. Because typically we use sigma square or variance, notions of variance are defined for random signals but there are also notions of variance defined for deterministic signals.

So you should not really think that the sigma square y star is only valid if if y star is a random segment and so on. It is not necessarily true. It can also be defined for deterministic signals. Anyway qualitatively it means that the power of the output, the true output to the power of the noise. Do I know it upfront? When I perform the experiment do I know this quantity? Most of times, no. Sometimes, yes. Depends on what access you have to the experimental setup. At least you can get an idea of what the sigma square v. How do you get an idea of what is sigma square v. Any idea?

How do I know how much noise is present in the data. I mean, sigma square v is a measure of that. It tells you, it gives you an idea of a range of fluctuations and so on. Very simple you perform the experiment at steady state. Right? Typically although we write here u, y starand so on. As we will understand shortly these variables are called deviation variables, deviation from steady state. So when you are performing the experiment at a steady state, y star is 0.That means with respect to a steady state it's not changing. All you have in the measurement is noise only.

That's it. So you have the noise with you from the noise you can obtain an estimate of sigma square, I mean, from the readings. So you can get a feel of what [09:47 inaudible] depends as I said, on how much access you have to the experiment. Suppose somebody has conducted some experiment during some process operation where it was not had steady state at all, then you'll have to discover this from the data. But suppose I'm looking at the experiment in my laboratory then I have the privilege of getting an idea of sigma square.

Okay? Anyway, so here the purpose is not to know how to find out SNR but rather to figure out what iseffect of SNR on parameter estimates. Okay? So for this I take a very simple example. It's a very straightforward one.

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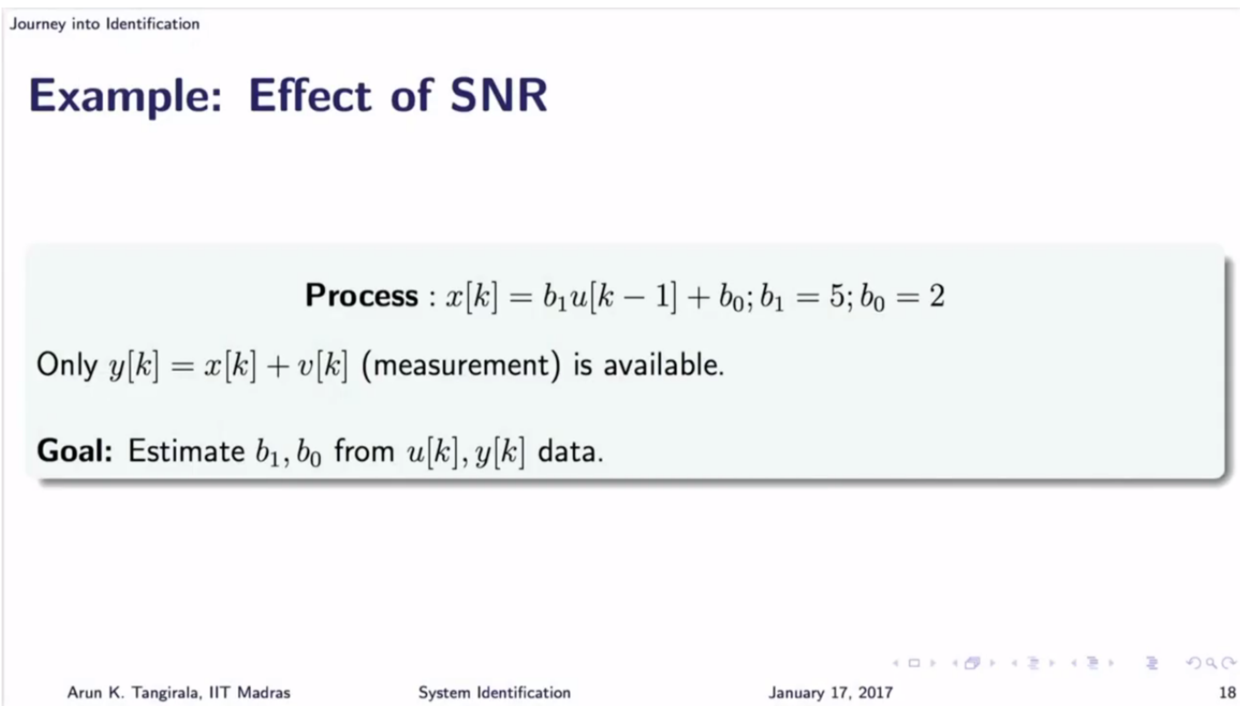

Now, we distinguish between-- although I have used x here. Please replace x in your mind with y star. So I have a process, that is very simple it's just b1uk minus 1 plus b0 a linear model with a unit delay. When I say x[K] as I said it's y star. So the process is responding to the input as b1uk minus 1 plus b0, I do not have access to x. I have access to a corrupted version of x which is a measured one.Now the goal is to estimate b1 and b0 from y and u. Correct? If y was noise free, then why would I be discussing this, right? It's very easy. All you have to do is pick two points, get your [11:23 inaudible]. But now, I'm going to use all the data that they have with the hope that using all the data points will improve the estimate. I then just picking two points.

So let's look at the estimates now, obtain from a dataset where the signal to noise ratio was maintain at hundred because it's a simulation. I have control over how much noise I'm adding. So I've added the noise in such a way that SNR is 100.

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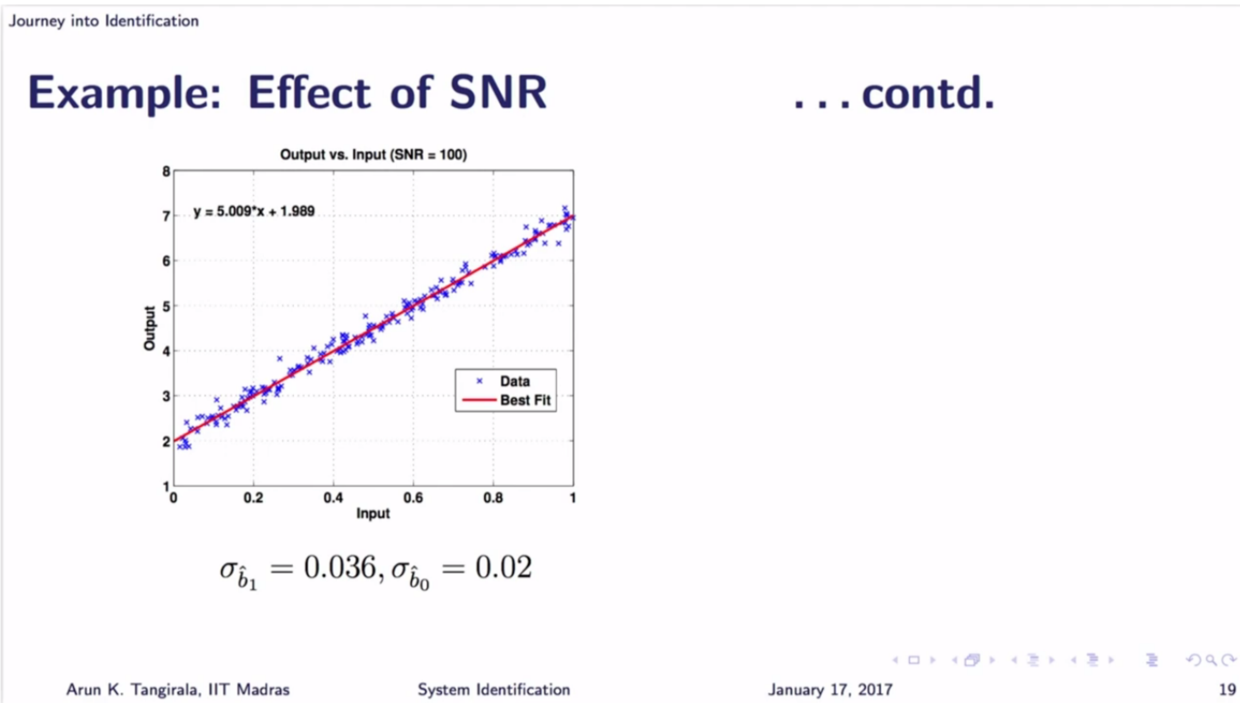

And I don't know how well you can read the parameter estimate. Let me zoom in for you, simply plotted y versus u. Use MATLAB'S basic data feeding tools from the plot. All of you can lead the parameter estimates. Let's see. People are nodding.Let's see if you're able to read. What are the values that you see? Many people are silent. Now 5.009 and 1.989. What are the true values that we used? Five and two, so really close. What you see here are called point estimates. Okay? It is just we have obtained one point in the possibilities.

Now, also below I have given this so-called standard errors in the parameter estimates. These are denoted by Sigma b1 hat and Sigma b0hat. Let us not going to how thisstandard errors calculated but some interpretation of what the standard error is. This is called standard error, not the error. If it is the error then I will simply add it to the and get the truth. What the standard error denotes is, if you repeat this experiment many, many times you will get different estimates because each experiment will have a different realization of noise. SNR will be maintain the same.

So each experiment will generate slightly different set of estimates or maybe more. We don't know how much they vary. So this standard error is a measure of how these estimates vary with respect to the truth across multiple experiments. The process is the same, SNR is held the same. I repeat the experiment. I'll get a different set of data. Right? That is what we mean by randomness because I'm going to see a different realization of noise. So the standard error is telling you how much of variability we expect to see with respect to the truth. And lower the error is better as the situation.

All right. Now we conduct another experiment where the SNR is maintained at 10. So have I lowered the SNR? Yes.Now this new data set, will it contain more noise or less noise. More noise, more noise should get me poorer estimates. But do we actually get really very poor estimates, point estimates are almost the same. So what do you mean by SNR seems to be impacting. Go to the bottom and then you will see. The average error in the estimate has gone up.

What does it mean, if I-- Again as I said,just average error is a measure of the variability that you see in your parameter estimates. In this case if I were to repeat the experiment many times I will see a lot more variability in the parameter estimates than in the previous case.That because I have more noise. So the noise is dictating terms now slowly. Of course don't expect SNR 100 and so on. SNR 100 is only good for publishing papers. SNR 10is somewhat practical to expect. Anything below SNR 10 is what you will, below or equal to 10 is what you should expect to see in reality.

Typically when people present come up with estimation algorithms, if the author is intending to show some realistic settings, you know, show the performance on realistic settings, then SNRs above 10 are not so great to be considered. So here we have-- the purpose is to show that as you lower theSNR the errors increase, the point estimates may not take a beating. It's important. Precision of the estimates takes a beating.

And more importantly you should see something that decreases in SNR increases the error in the parameter estimates approximately proportional to one over square root of SNR. What we mean by one or square root of SNR is, if you look at the ratio of errors in these two different SNR settings, the SNRdiffers by a factor of 10 which means a error should increase by a factor of square root of 10. Right? Because a error is inversely proportional to square root of SNR. Higher the SNRerror is lower the error is.

So when I'm comparing two settings with two different SNRs, as you can see, if you look at Sigma b0 hat, you have point 0.36 when SNR is 100. And you have point 114w when SNR is 10.Right? Theyroughly differ by a factor of three-point something. And that is, that it is not coincident, so that's a square root of 10.You can show theoretically that the error is proportion to the square root of one over SNR.

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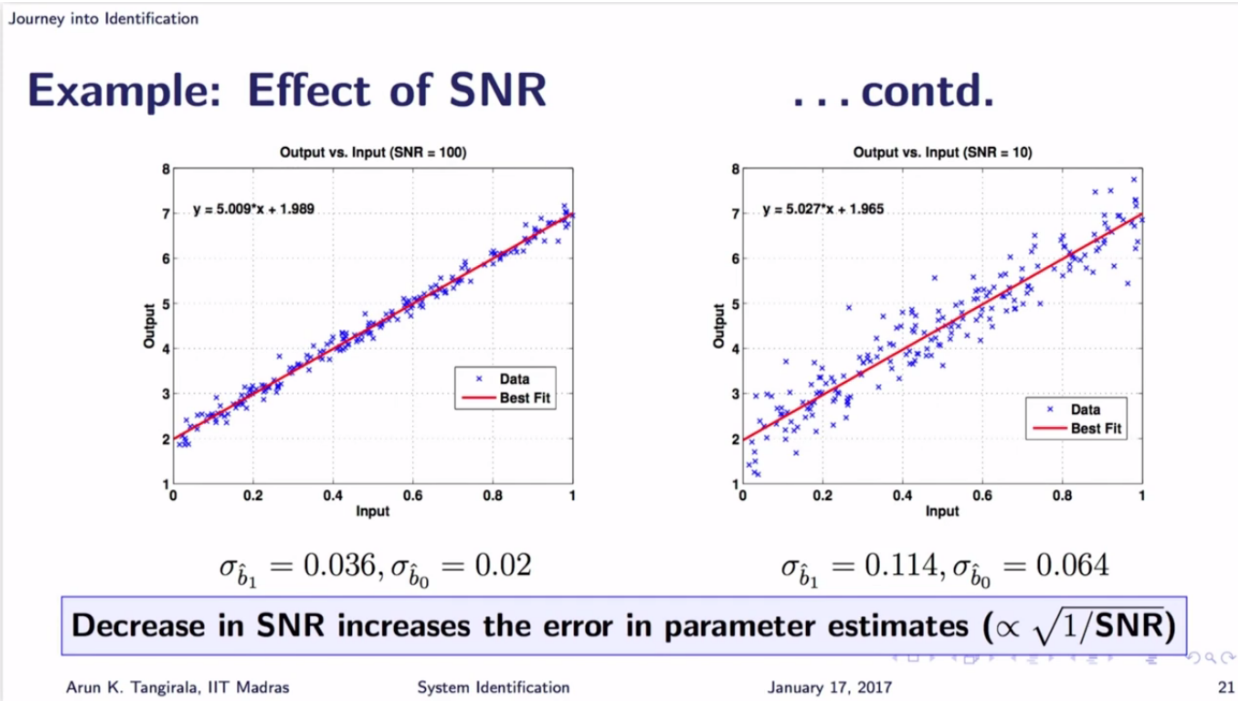

So clearly telling you that SNR plays a big role. The other thing that I wanted to talk about are--