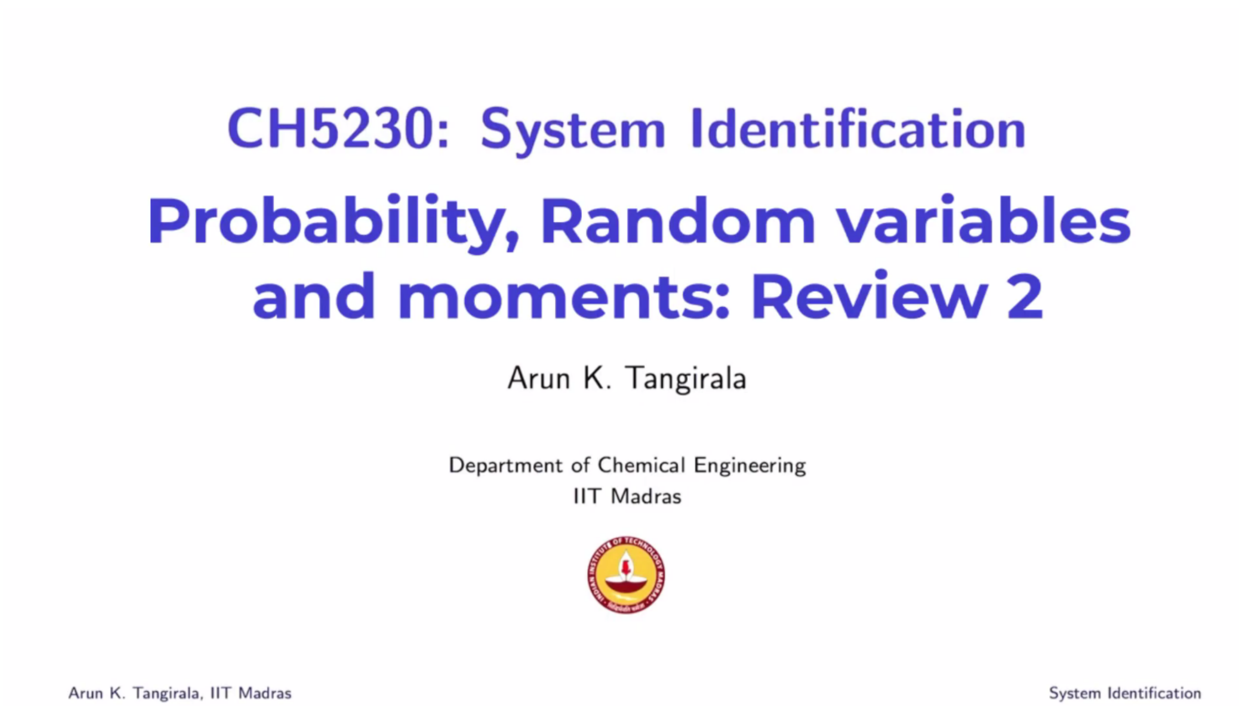
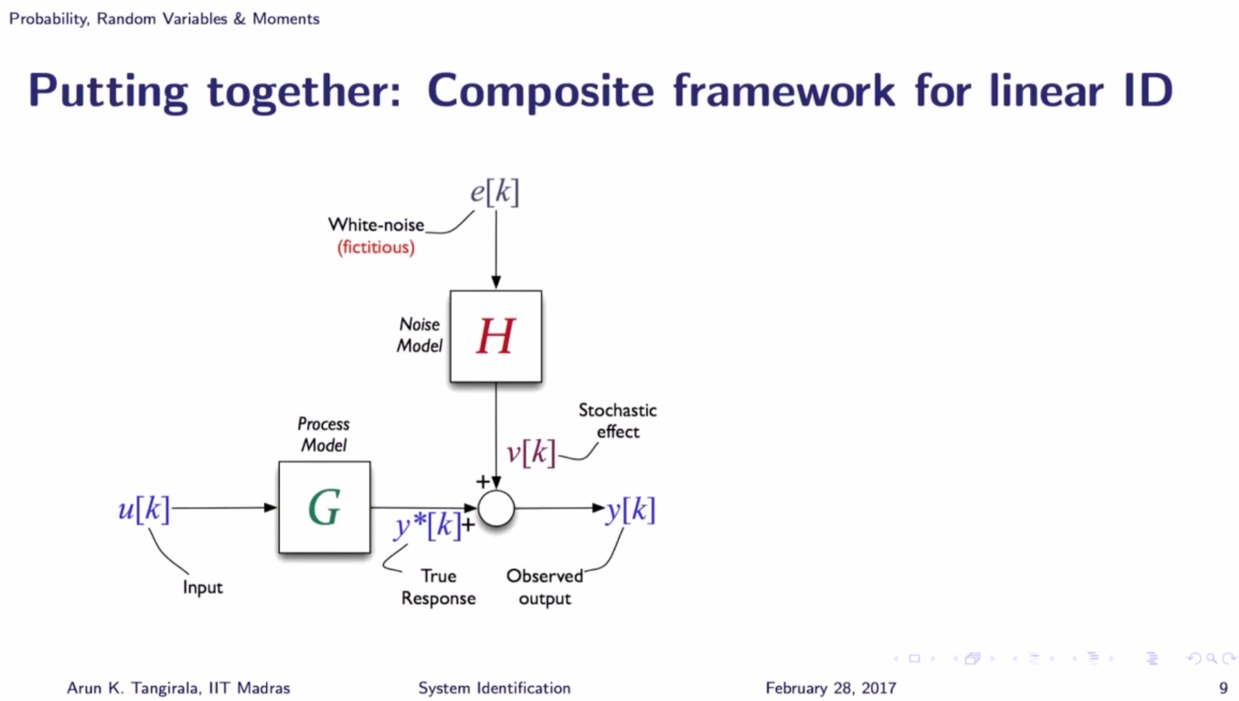


What are the Ts and Cs here? One that VK should be stationary. That's the first requirement. We will briefly review what is stationarity and secondly the spectral density of VK should be factorizable. Now this of course may go over your head for lot of you because you don't know what is spectral density and what we mean by factorizable. But in due course you will understand.

Most important thing is to remember that there are terms and conditions to this representation and these are the two terms and conditions. In fact there is another condition also but I am not stated that here. It's okay. these are the two primary conditions that apply to this kind of representation.



Okay. Now with these in mind we now put together, we now wire everything together and say now I have a nice unified framework. Both G and H LTI which is good news. That is the main reason why I said it's good news because I don't have to now break my head on understanding G and H separately. So and the wiring looks extremely similar for both Y star and V but with a huge difference the input that is driving V is white noise and the input driving Y star is U. and we have just now discussed what difference it makes. The main difference that it makes is identifybility of X. So we have to fix that. And secondly unlike in this here, here the goal is only to estimate G given Y star and U but here the goal is to estimate H and also sigma square e. never ever forget that. Your time series, your noise modeling is incomplete without reporting an estimate of sigma square e. You have to give an idea of what is the variability in EK.

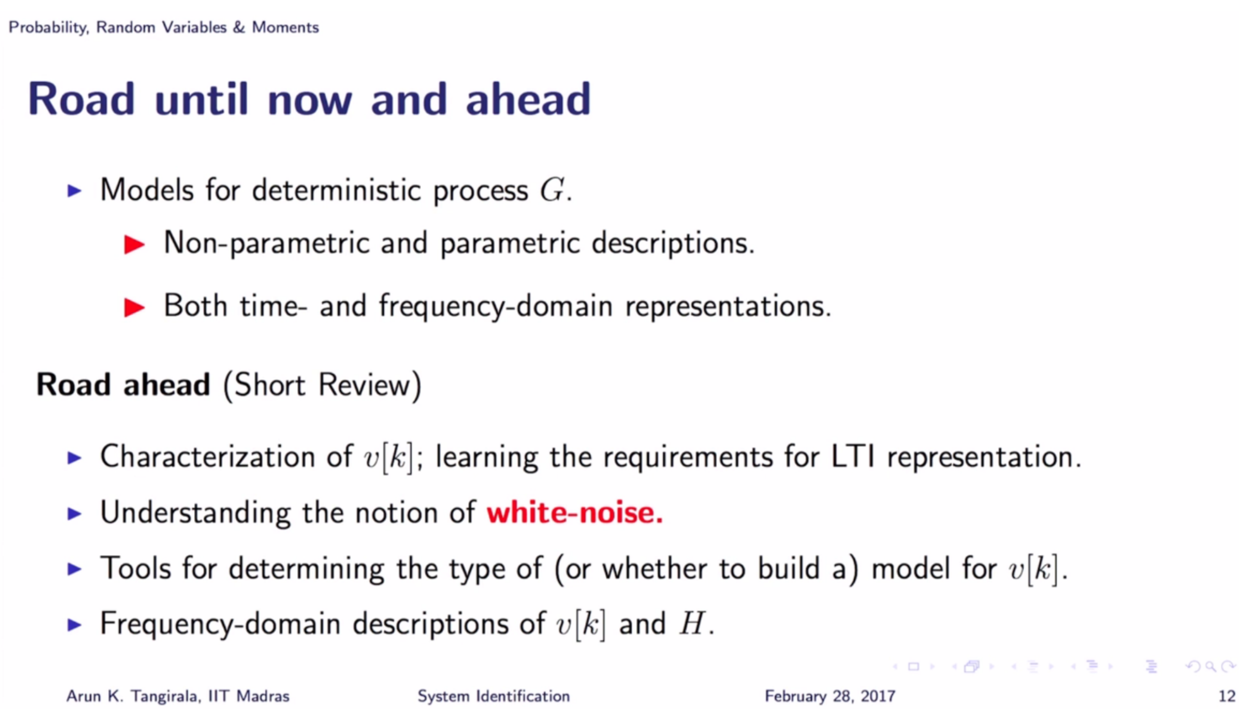
Okay. So the assumptions now for this general framework is that we have additive noise and we have quasi-stationary input. We have not talked of quasi-stationarity yet. Now as simply as I have done that I have just taken U and G and E and H and I have put them together mathematically it is not so straight forward at all. Why? Because ultimately I am fusing signals coming from two different worlds. Y star is coming from the deterministic world. V is coming from some other planet. And I am just marrying them. It's not so easy. There are going to be challenges, cultural challenges. So many problems are going to take place. Between the same planet itself we have numerous problems. So here these signals are coming from two different planets. And for those of you who are newly married you know.

So what are these challenges? How do we actually deal with these and so on we will discuss a bit later. But one of the requirements for Y star to be fused with V so that I can have a nice framework for identifying G and H is that input has to be quasi-stationary which means not all people in the planet of deterministic world are eligible to be married with the people from the stochastic world. A minimum qualification is that the input should be quasi-stationary. What is quasi-stationary we will learn a bit later.

For the first time we are imposing restrictions on input. Until now we have not talked about any restriction. Now that Y star and V are going to come in contact with each other and that we want nice framework for identifying G and H it becomes necessary to impose some restrictions on the kind of inputs that are admissible. And the kind of inputs that are admissible are quasi-stationary inputs.

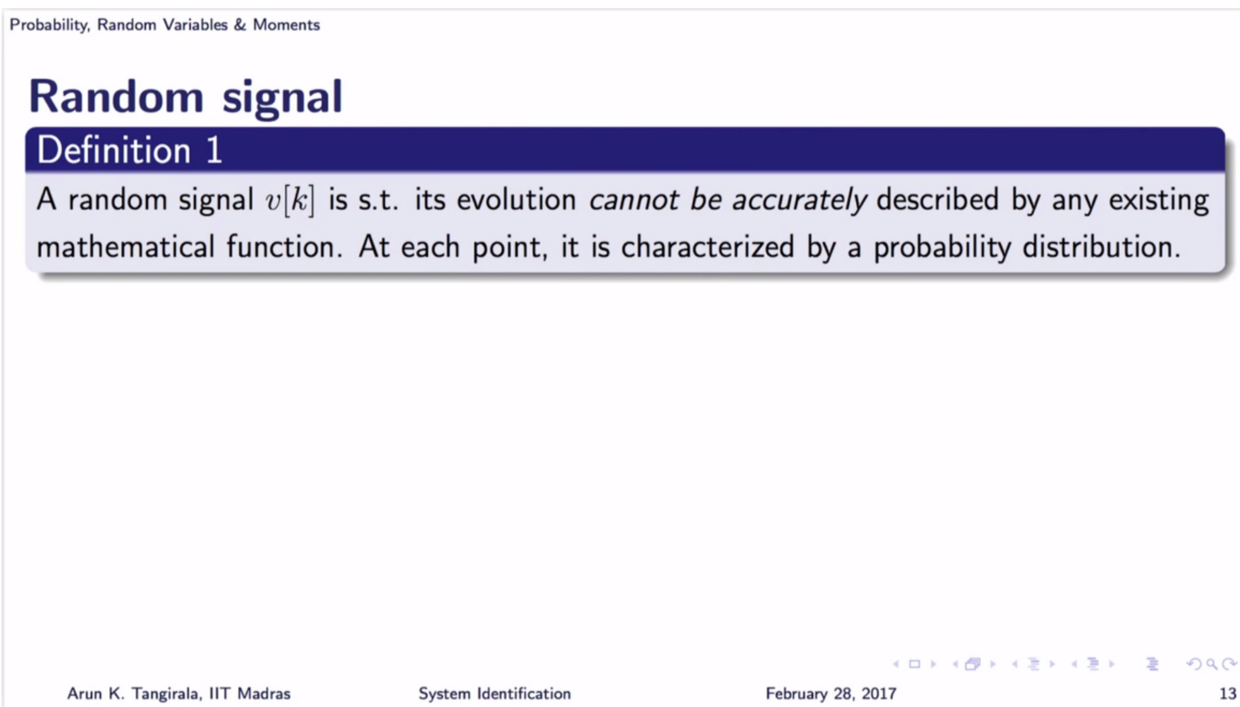
What about VK? So it is not sufficient now only the bride should have satisfy has some qualifications, groom should also have some qualification. VK should be stationary. So not all people from the random world are going to be allowed to be having an alliance here. So on one hand we are saying input should be quasi-stationary. On the other hand we are saying VK should be stationary. Only when all of these qualifications are met the situation is right for identification.

That is the important thing to remember. So you should remember this schematic throughout the course. These three important restrictions. There are additional restrictions on input that we will talk about persistent excitation. We have talked about that. That doesn't go away. You still need input to be persistently exciting otherwise you will have a problem in identifying itself. You don't have enough information.



Okay. So the road until now it has been that we have understood what are the models for G. we have looked at non-parameteric and parametric description. We have looked at time and frequency representation. So we have only understood a part of the story. We have to understand very quickly now how to model G, sorry H, or V rather. And then go on to study the estimation algorithms. We are still in the modeling world. So the road ahead which in this short run, short term is to understand how VK is characterized, how this random signal is characterized and when this LTI representation is possible for VK, and understand the concept of white noise. For now you can see white noise has been introduced as an unpredictable signal, uncorrelated signal but there are other interpretations to white noise as well.

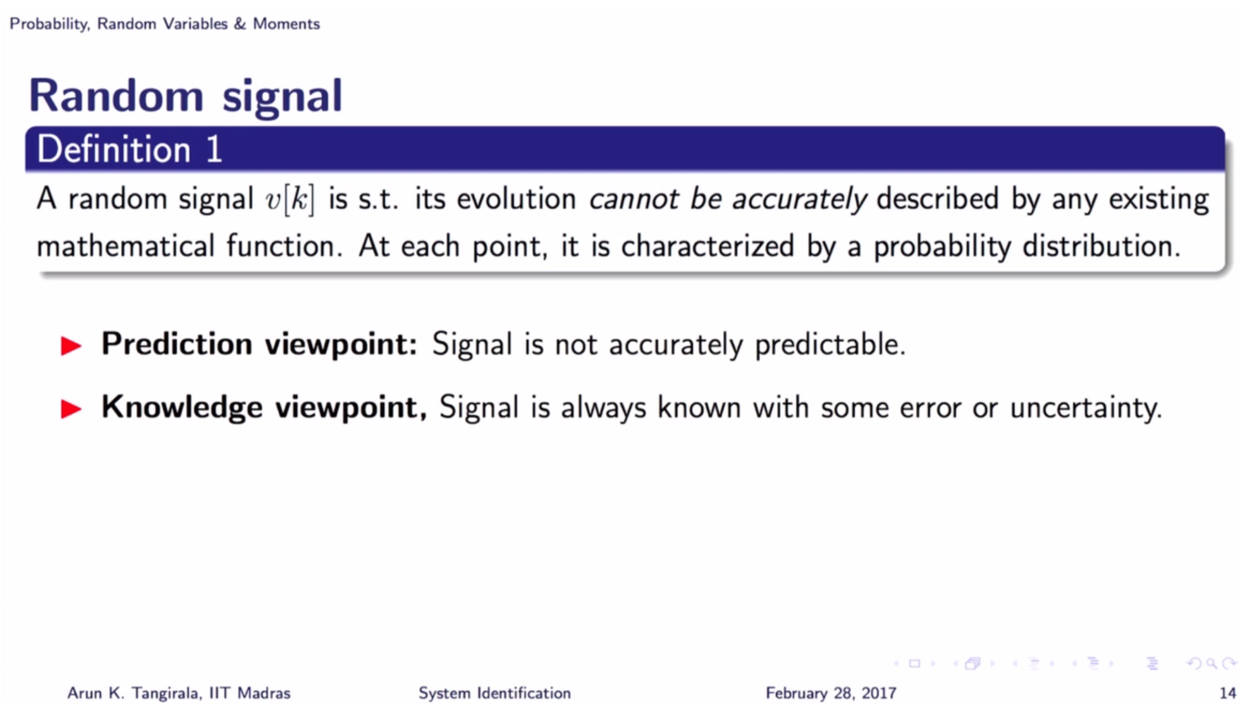
The second interpretation that we must have derived by now is white noise is this indogeneous signal, force that is driving VK. That is second interpretation and so on. So there are a couple of more interpretations. We will talk about that later. Practically what I want to understand is to learn is what are the tools for determining whether a model for VK can be built or not. That means whether I should identify, build a model for H or not we call the liquid level case study. We made some assumptions and then we explored. Is it possible for me to know through data analysis whether I should build a model for V or not without making too many assumptions. Okay. those will involve correlations. Those are the most important tools. And the fourth thing that I have to understand is is it possible to have a frequency domain representation of VK. Usually that is the most dreaded topics in time series analysis. It is called spectral representations and usually people run away after that. I am not saying you should drop out of the course but I am saying that typically that part is something that people find difficult to understand but we will make it simple for you.



Alright. So let's quickly get into characterizing the random signal and spend maybe a few minutes and then we will meet again tomorrow.

Remember the first task I had for us is characterizing VK. That means I have to understand what is meant by random signal. Earlier I said a random signal is one that is unpredictable. But that is not a statement I mean I said it is not predictable accurately. You should not take it as being unpredictable. That means there is always going to be this unpredictable component in VK. So the first definition of a random signal that you will see in the literature is its evolution cannot be accurately described by any existing mathematical functions. By the way, the random signal by definition is assumed to exist always. Existence is not random or it may exist or it may not exist. No that is not correct. By definition and in theory a random signal is assumed to exist forever. It existed before you. It will exit after you.

The randomness is something that we give a name and I will talk about it very soon. But the only thing that you have to remember is random signal is such that it's evolution that means it's generating equation you do not have a mathematical form like the ones that you see for some smooth curves and functions and so on. And the second characteristic of a random signal is at each point so you have this random signal, that may look like this; one realization. I am connecting the points although it's discrete time signal. So you may have this kind of a random signal and so on. At each instance, so let's say some K0 here, the random signal is actually a random variable. And since random variables are described by means of probability distributions, random signal at each point is characterized by a probably distribution. I have only shown one realization. There are many possibilities. Why there are many possibilities? Again that comes from predictability view point. So the prediction view point of a random signal is that you cannot predict it accurately.



Whereas the knowledge from a knowledge view point so there are two view points of a random signal. One is from prediction theory view point, other is just from knowing view point. When I look at it from that perspective a signal, random signal is always known with some error. That means there is some uncertainty shrouding the value of signal at each time. And I don't have any mathematical means of describing that uncertainty. I have to reach out to some probabilistic ways.

So there are two different view points usually it's a prediction view point that helps in understanding. Now the moment we have said at each point the random signal is a random variable and since a random variable by definition has at least more than one possibility or at least two possibilities, possible values, straight away it means that at this point there are many values for this signal of which I have observed only one. Is this all true? I mean do actually random phenomena exists. What do you think? What we are saying is whatever I call as random signal at each point there were multiple possibilities of which I have observed one. But is this really true that for a signal there are multiple possibilities and if I am observing only one or is there a reason for giving having that perspective? Right and therefore correct why is this multi-possibilities business is coming up?

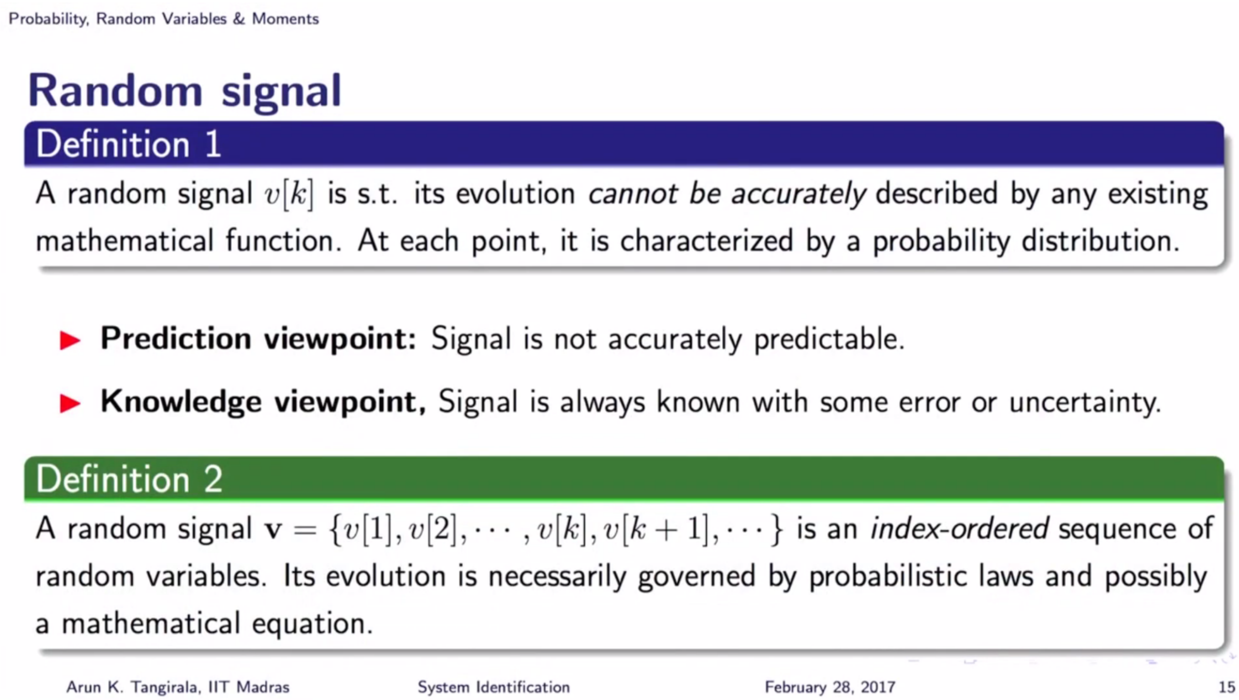
Student: [00:12:51]

Male Speaker: Okay. But maybe something must have changed from experiment to experiment if you look at it at a very subtle level even at a quantum level, something must have changed. You can't guarantee that the conditions for one experiment is identical at all scales from quantum to macro. For the second experiment, can you guarantee? No. something changes because we say this standard statement, change is the only constant thing and you see this roaming around in Whatsapp and so on. So we know that something changes from time to time. So I cannot guarantee strictly speaking that two experiments are identical. And you can say that is the reason why the signal has changed. I can always argue that way. It is not because if you read carefully the definition of random phenomena is all controllable factors are held fixed. And then at repeat experiment I see a different value which means there were some uncontrollable factors I couldn't fix them. Those have resulted in a different value. It is not that this random signal there are multiple possibilities and it looked at your face and said now I am going to give you this value. It's not going to do that. It is our own ignorance, it's our inability to understand fully the processes, the responsible factors for the generation of the signal that I am unable to say exactly what value will show up. Or you can put it this way, it is my inability to control all the factors either know or control all the factors that are responsible for VK that actually is causing a different value. So you throw away all of these and you say from a prediction view point the problem is this signal is not accurately predictable? So what is the natural recourse you say this is, suppose you take rainfall for example, standard example. I do not know whether it will rain or not. Maybe it can rain in the next one hour. It can rain anything can happen. Of course you have a certain probability in mind. But in your mind you will not rule out a rain even you may say yeah 0.5% but still that means you are giving a chance to that. So you are unable to predict accurately. What are you doing in your mind? You are listing all the possibilities. So that natural thought process that goes on in mind for any unpredictable signal is list out all the possibilities. And then imagine that nature is going to pick one of them. Do you think truly it must be happening that way? What do you think? Do you think truly that nature is waiting for meteorological department to predict and do the other way? Let's play around with this guy? I don't think so. Nature must be evolving on its own term. Nothing to do well if it has to play around then it has to play around several meteorological departments which one will we clear on it? It's not going to work that way right. It's evolving on its own. It's our inability to understand that we think there are multiple possibilities of which it is going to pick one, this, this, had a 0.5% chance, that had 99.5% chance and so on. There is no other recourse for us. There is no other choice for us. If we have to discard the fact that there is no mathematical way of predicting it is next recourse is probability. It's a natural recourse. But the fact maybe that the actual process is actually deterministic. There is no other possibility. Whatever was suppose to happen has happened. There was no like multiple possibilities of which nature has randomly picked one. There is nothing like that maybe. We did not understand fully the situations that have let to this and therefore we think this is all our imagination. We think after a while, after joining one IIT you think maybe I should join other IIT that was also possibility or you join one company and say no, no, I think I wish I had joined the other company. That was also possibility. Maybe not. In your path that life has for you maybe these were the only possibilities. The imagination that you have that I could have had that food item, I could have ordered that dish or I could have joined this company. I would have sat there not in the front bench, back bench and so on maybe not. Maybe all of it is actually fixed. We don't know. It's only our ignorance that has led to this world of randomness. So you can think of randomness as a [00:17:47] term to hide the human ignorance. Instead of saying constantly I am ignorant, I am not capable of predicting accurately and so on I have coined a new term called random. And I keep using that word random which also I don't understand properly. That is another fact of life. So that is something at a philosophical level you should remember because it's a grad level course. I wanted to just trigger your thinking that in practice there may not be a random process. The process maybe deterministic. So in the eyes of the creator this probably this world is deterministic. Now that's exactly what's going to happen. But we are given the joy of thinking that there were multiple possibilities and that with that joy comes also the sorrow because you always weep for the other one.

So keep that in mind that this randomness maybe only an artifact of our ignorance and not the process at all. In fact most likely it is that. As we keep understanding the process better and better and better we come to a conclusion yes that's it. I mean very often I feel that whatever government we have today is right for the citizens of this country. We always think you know if that party had come to power, this party – everything is going as per some grand plan. So as the citizens get better we will get better governance.

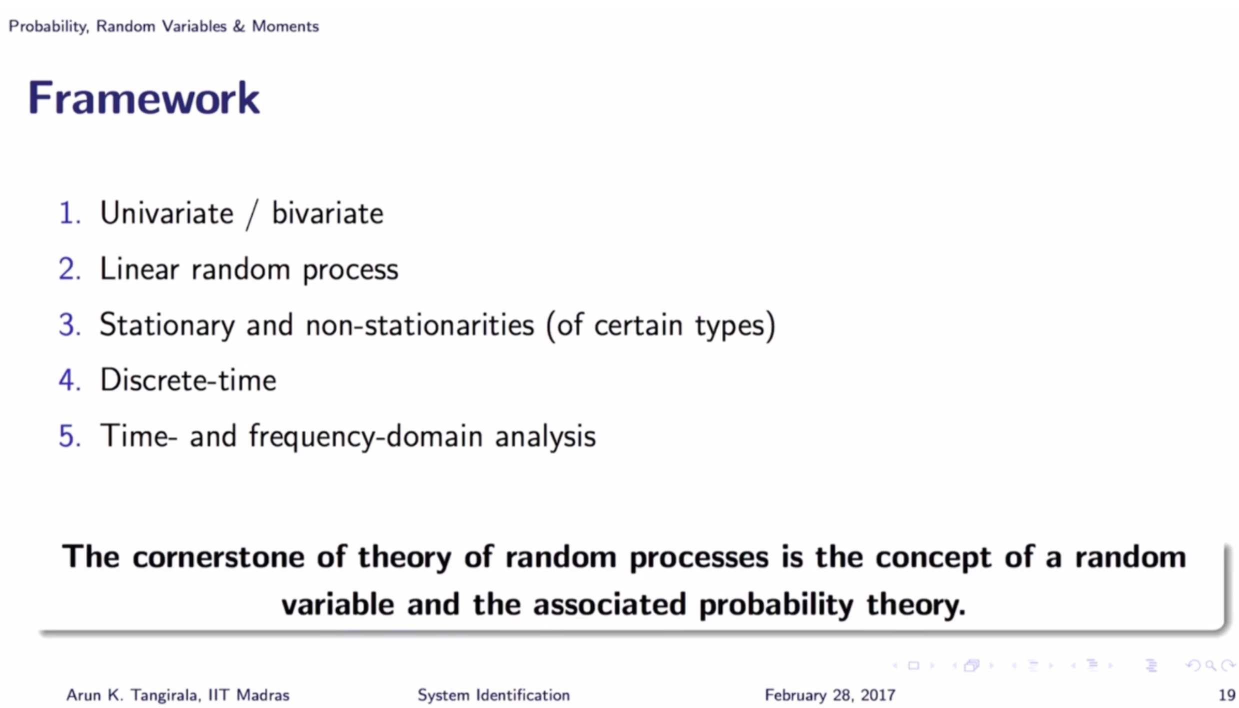
So to think that wish we had another form of governance and so on maybe it's wrong. This is the only solution that is available.

Anyway, so you have to remember this when it comes to a random process that it may not – randomness is not necessarily and a characteristic of the process we are blaming it on the process. The fact is that we are ignorant.



The other definition of a random signal is that it is an ordered sequence of random variables. And this helps in developing the theory for modeling random processes.

It's evolution once again is necessarily governed by probabilistic laws because we don't know any other rate as of now. Maybe several years later some other framework can evolve and possibly by some mathematical equation where definitely there are going to be probabilistic laws.



And we have already talked about random phenomena whether they exist or not. So the framework as I said in the last class we are going to look at univariate or perhaps bivariate and we are going to restrict ourselves to linear random processes and generally stationary processes but we may look at some non-stationary VK, discrete time of course and time and frequency domain analysis. So the cornerstone of understanding the theory of random process or cornerstone of theory of random processes is this notion of a random theory and the probability. Those are at the heart. You should understand the first thing that goes into understanding what a random signal is understanding the random signal at a point. Although a right time there it could be space frequency and so on. Don't keep thinking it's always time that's independent dimension.

So the first step in understanding random signals is understanding how it behaves at point in time. So it frees triangulate [00:21:25] the notion of time quickly understand how random – how is it defined at a point then you have to tie them together. The difference between a collection of random variables and a random signal is that random signal is an ordered collection of random signals. There is a thread that is tieing them together. The only time that you can actually disregard this order for what class of signals you can disregard the order. Why is this ordering important for a random signal? Why is this ordering in time important? Why can't it just collect a bunch of random variables and call it as random signal? Why? Because there maybe a correlation. There maybe a time dependence. There maybe so if you are probably looking at let's say spray, there are going to be large sized bubbles and then small sized bubbles and medium size bubbles and so on. There maybe a time dependence. A large bubble maybe attracting another large bubble or maybe small next to small by medium and so. So this is certain time ordering. That time order is what we want to see if we can exploit. That is what we mean by correlation. If there is no time I mean if there is no dependence at all if there is nothing in time that's going to help me construct this random signal at a later point in time then that means there is no hope for prediction. There is no point in looking at history. And what is the class of signals that we have been discussing that fall into that category white noise. With white noise it doesn't matter whether you look at the ordered signal or you jumble them up and put them up. It doesn't matter. The time ordering has no impact on its predictability. You do in any fashion possible I mean you arrange the values in any ways possible in time they will still remain white noise. Whereas for the correlated signal the moment I spoil the time structure they are on a thread. They are hanging on a thread. At each point in the thread there is some randomness but there is a thread. That thread we want to figure out. That is what we mean by model. That thread will give me some trend or you can say some global feature. Locally the random signal is still random. So we need to understand therefore random variables and then quickly move on to review tools that will allow me to figure out whether the thread is present which is what we will learn throughout the correlation and then quickly move on to looking at how this auto-correlation can be used to figure out what model is suitable. Whether there should be a numerator H or the denominator or both. The same questions. Like more or less the same questions we ask for G. So with a correlated signal if you destroy the time ordering what will happen? What is likely to happen? Suppose I give you a corrected signal, a realization of a correlated signal. And you don't worry about the time ordering at all. You just jumble them up randomly and then create a new signal. What will be the difference between the one that I gave you and the one that you create?

You would have simply broken the correlation structure and you would have ended up with the white noise because you have broken the correlation structure. There was a nice thread that was connecting them and it has like you tore them apart and then put all these observations in a bag and randomly pick them. The net result is a white noise, most likely white noise because you just randomly picked them. The thread that was connecting the observations that I gave you is lost. Why did I discuss this because this is a very common trick that is used in so called bootstrap algorithms that are used in determining estimation errors and so on. Errors and parameter estimates, hypothesis testing and so on. Bootstrapping algorithms use this trick or surrogate data analysis use this trick to generate a white noise sequence out of a correlated sequence. It becomes necessary to do so.

So when we meet tomorrow we will go through a sweeping review of random variables and hopefully get ourselves to the stage of discussing random processes. Alright and then we will move on to the frequency domain description.