

NPTEL

NPTEL ONLINE COURSE

CH5230: SYSTEM IDENTIFICATION

SAMPLED: DATA SYSTEMS 8

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Okay and it's a good day to just introduce now you to the world of probability, at least now wake up to the reality and know that now it's just not deterministic world alone that I have to look at, I have to actually look at the stochastic world as well.

So now we go back to our original schematic to recall the schematic,
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Sampled-Data Systems

Recap: General framework for linear identification

- ▶ Additive noise
- ▶ Quasi-stationary input
- ▶ Stationary stochastic noise

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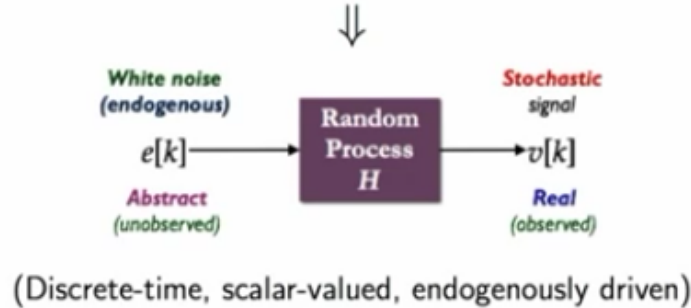
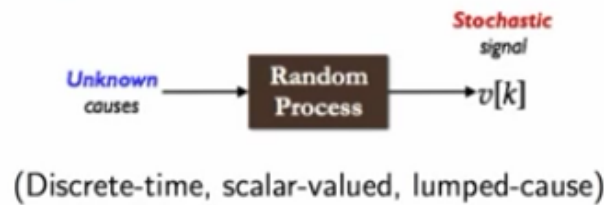
this is the general frame work for linear system identification at least for discrete time systems. And we have spent considerable time in understanding the mathematical descriptions of G , right, and that this G receives a user defined input or maybe a known input and produce us a response by star which I don't show here, but what we have in hand is Y and the difference between Y and Y^* is what we call us V which we do not know it's a lump effect, it's a lump signal and it contains effects of noise, unmeasured disturbances and in practice also modeling errors, all of it gets, all of those get lumped into V .

And up front in this course we have said V can be modeled as a white noise passing through an LTI process, and we will review the concepts underlying that major assumption that we are making that this stochastic signal V can be thought of as a fictitious signal driving, no passing through this LTI, typically we refer to that as a filter or a noise model.

Now in order to understand that,
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Sampled-Data Systems

Random process



in order to understand that you have to keep in mind that we are first of all lumped in everything that is one, and secondly we are assuming so called stationarity which we will also review a concept that will review shortly,
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Framework

1. Univariate / bivariate
2. Linear random process
3. Stationary and non-stationarities (of certain types)
4. Discrete-time
5. Time- and frequency-domain analysis

The cornerstone of theory of random processes is the concept of a random variable and the associated probability theory.

but the most important thing that you have to remember is at the heart of all of this random signal analysis is this concept of random variable, and probability, so one has to be well versed with the basics of random variables and probability theorem, it doesn't take to, I mean it's not so difficult at all, because you don't need advanced concepts, right, and what we are going to do is initially we'll review this concepts of random variables and probability very quickly, as I have told you already detailed lectures on this are available online in the applied time series analysis course that's being currently run, you can always go back to those videos and spent time, today is Maha Shivaratri you can actually spend your entire night listening to that and then also chanting the name of Shiva, both so that you get maximum benefits.

Fine, so the frame work that we are going to look at is univariate or bivariate, and we are going to look at linear random processes only, most of the times we'll stick to stationary signals that is your, you don't assume we get to the stationary, occasionally they maybe non-stationarities, and when it comes to that we'll talk about that, of course we are looking at discrete time and we are going to learn how to view this random process both in time and frequency domain just as the way we did for the deterministic process, it is a frequency domain that can be a bit intimidating, but it's okay we'll review in a way that hopefully we'll make it look easier for you.

And the standard notations for understanding the theory, random variables are always going to be denoted by an upper case,
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Notation

- Random variable: UPPERCASE e.g., X ; Outcomes: lowercase e.g., x .
- Probability distribution and density functions: $F(x)$ and $f(x)$, respectively.
- Expectation operator: $E(\cdot)$
- Discrete-time random signal and process: $v[k]$ (or $\{v[k]\}$) (scalar-valued)
- White-noise: $e[k]$
- Angular and cyclic frequencies: ω and f , respectively.
- ...

the values that they assume will be denoted by lower case, and probability distribution and density again upper case and lower case respectively, expectation operator you should be really comfortable with this expectation operator, okay, as I say I always expect you to have very good friendship with this expectation operator, and we will refer to this discrete time random signal as VK and random process VK in curly braces, sometimes we will use this interchange W also. White noise by default in this course will be denoted by EK , unless otherwise specified and angular and cyclic frequencies will be denoted by ω and small f , right.

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Random Variable

Definition

A **random variable** (RV) is one whose value set contains at least two elements, i.e., it draws one value from at least two possibilities. The space of possible values is known as the **outcome space or sample space**.

Examples: Toss of a coin, roll of a dice, outcome of a game, atmospheric temperature.

So we'll just conclude the class with a quick understanding of what a random variable is and probably distribution just maybe 2 or 3 minutes and will be done. So random variable is defers from a deterministic variable in the sense that there are multiple values that it can take, in the sense of possibilities.

In the end when it comes to an event it will assume only one value, but the possibility set contains more than one element, and there are many examples you can think of a toss of a coin or you can think of the roll of a dice, rainfall or the value of reading at any instant in time from a sensor and so on.

And I will talk about this imagination of randomness in the next class, but at the moment understand that random variable is that variable for which the possibilities are many, but eventually it will take only one value, whereas in deterministic variable there is only one possibility, it cannot, for example if I ask you what is the value of sine at some K ? $K = 2$, there can be only one value right, that's a deterministic value, so when we discuss random variables by the way we freeze time, you have to understand, at this moment we are letting go the notion of time, why are we discussing random variables, because this is at the heart of understanding random signals, your random signal is an infinitely long signal and it's a collection of random variables, that's why we are looking at random variables, we are trying to understand the element of random signal, then we'll put together, you know tie them together and then understand the random signal, and that is why I'm saying you have to let go the notion of time when it comes to understanding random variables.

When we come back to the random signal world we'll bring back the notion of time or whatever the independent domain is, alright,
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Formal definition

Outcomes of random phenomena can be either qualitative and/or quantitative. In order to have a unified mathematical treatment, RVs are defined to be quantitative.

Definition (Priestley (1981))

A random variable X is a mapping from the sample space \mathcal{S} onto the real line s.t. to each element $s \in \mathcal{S}$ there corresponds a unique real number.

- ▶ In the study of RVs, the time (or space) dimension does not come into picture. Instead they are analysed only in the outcome space.

so there is a formal definition of a random variable given you know long ago which is that it is the mapping from the outcomes space, first of all you've to understand what a random phenomenon is, what is the random phenomenon? It's a phenomenon that when you repeatedly observe everything else held fixed, whatever you can control you've controlled, and despite that you repeatedly observe you will get, you will observe a different outcome, that is the difference between a deterministic phenomenon and a random phenomenon, many people think oh it randomly occurs, no the occurrence is not random, random phenomenon exists always, it is the outcome that can change the moment you again, again if you repeat the experiment you will find the different value, why has this occurred we do not know, I have held to the best possible extent all the controllable factors fixed, despite that I see a difference, okay, and that's when, there is one way of thinking of a phenomenon as random phenomenon, and every random phenomenon you can think of outcomes, right, because it's going to give you several different outcomes over a period of, over the entire possible set, we call this entire possible set as sample space, okay, whatever outcomes are possible, so if you take rainfall if you observe now there may be no rainfall, if you observe later there maybe, it maybe raining, so if you look at all possibilities there are only two, so sample space contains only two elements.

For the dice there are 6, but for a sensor reading there are infinite because it's a continuing, correct. Now what is random variable does is it allows you to unify all the random phenomena, analysis of random phenomena under a single umbrella by mapping all the outcomes if they are categorical for example through a numerical space, because not all random phenomena will give you numerical outcomes, now if you take a game there are only victory, lost or draw and so on, there is no number attached to them, how do you analyze such processes? You may assign some numbers and that is what is done in many games, in many leagues and so on, this random variable is the formalization of that, the reason for introducing random variable is to unify all such random phenomena including those that give you categorical outcomes as well as numerical outcomes put together everything, obviously this means that for processes that give

you categorical outcomes you have to do a mapping by yourself, and the mapping choice is yours, the freedom is yours, but for processes that give you a numerical outcomes like sensor reading and so on there is no need to make any choice, already whatever value you get is your outcome, right.

So what we'll do in the next class is look at the notion of probable, review the notion of probability distribution, probability density, why does probability come in now all of a sudden, right, and what is its role in understanding random variables and in particular the moments of the density function in describing the signals. Always remember that the reason for a separate theory for random signals is simply because they can take multiple, there multiple possibilities for this random signal or random variable, unlike a deterministic, okay, so we'll meet next Tuesday, thank you.

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