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NPTEL ONLINE COURSE

CH5230: SYSTEM IDENTIFICATION

SAMPLED: DATA SYSTEMS 7

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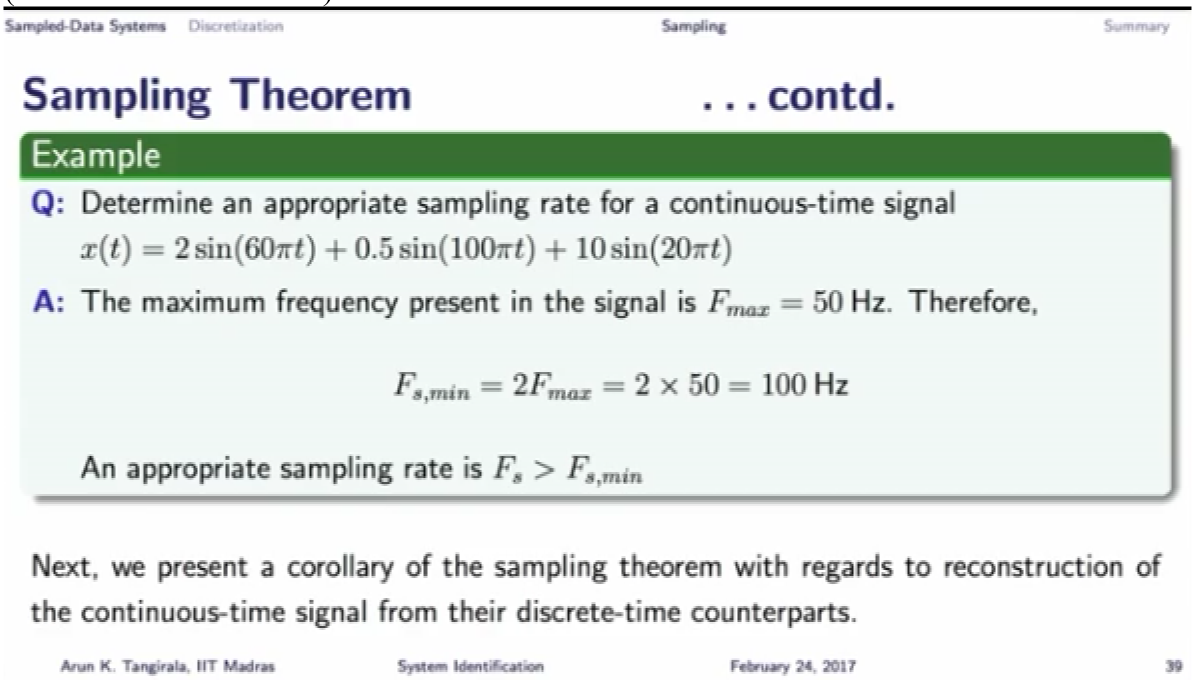

Continue on discussion on sampling and then review, begin our review of probability theory, right, okay, so let's recap what we learnt yesterday, I think the main message that we learnt yesterday is the on the sampling theorem which essentially states that if a continuous time signal has a maximum frequency of F maths or if you have sinusoid of single frequency F, then we should choose our minimum sampling frequency as a two times happen, right.

So if you recall yesterday what we discussed is the main reason for the sampling theorem is, I mean is that the recommendation of the sampling theorem is based on this phenomenon of aliasing, right, and why does aliasing occurs? So you have to ask this question one after the other, so why is there a sampling theorem? What happens if I do not follow the sampling theorem? If I sample lower than what is recommended, then I run into aliasing, and what is aliasing? Well aliasing is essentially this phenomenon where a high frequency discrete time sine wave maps back to a low frequency sine wave as we went through the example yesterday, suppose I have a sine wave of frequency 1.25 cycles per sample, then we know that mathematically this would be indistinguishable from this other frequency, there is no way you would be ever able to figure out whether it is sine 2 pi, whether it's a frequency 1.25 or 0.25, that's again because of the periodic nature of the sine waves, and the fact that you are looking at discrete time signal, that's the most important thing these are the two things.

So when you sample a signal, right, like the one example that we looked at yesterday, when you sample a signal let's say of 10 hertz at a frequency of 8 hertz,

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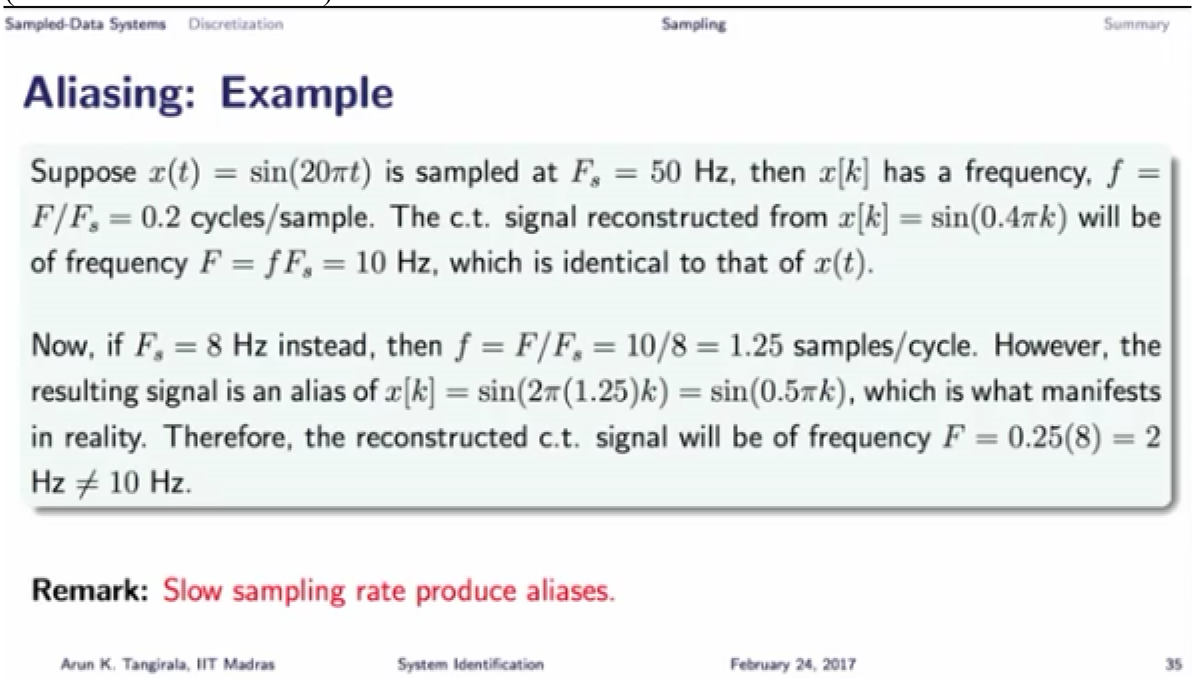
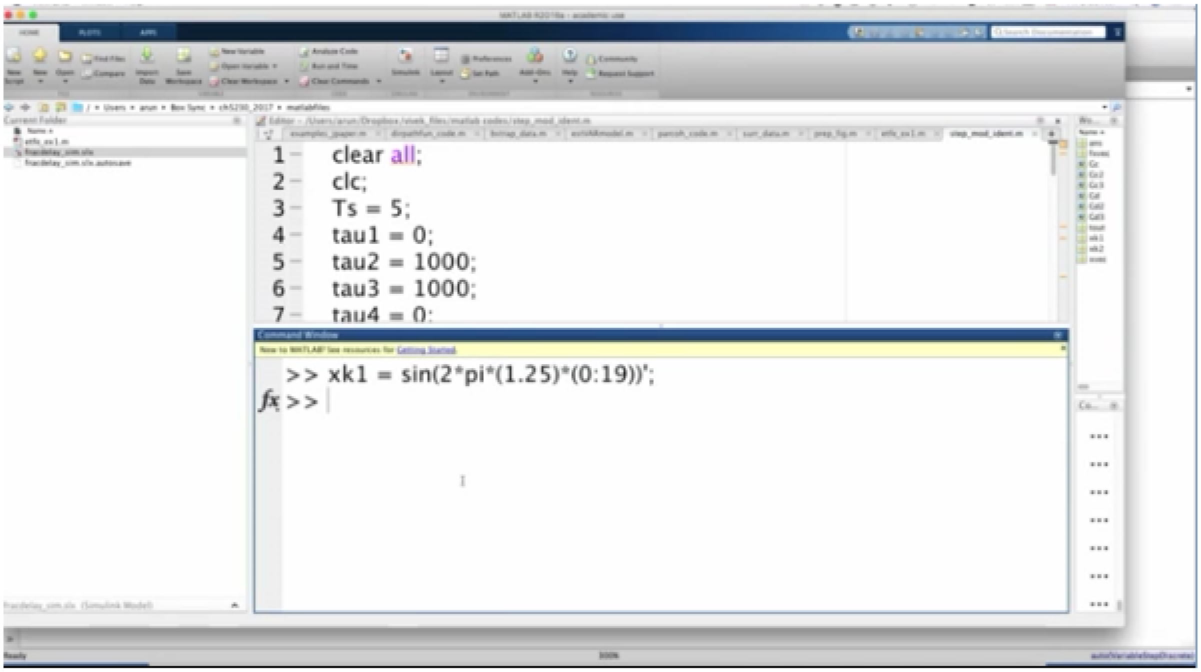


I made the corrections now, you end up with a discrete time sine wave of frequency 1.25, which then manifest as 0.25, so I asked you to generate this sine wave right in MATLAB as a simple homework, we can do that in fact although it's mathematically obvious, let's actually do this as a very simple example so here 2 into pi times 1.25, that's generate the 20 observations of the sine wave.



What's the periodicity of the sine wave? What happened, what is the periodicity? It's so tough? What is F here? It takes this long to written on a period, one million now, 1/10 now what is that? This is exactly what I've been saying that you should never do for a discrete time sine wave, how can you say it is 1 over of F? We've gone through this discussion if you recall a few weeks ago that for discrete time sine waves the period cannot be determine by 1 over F, but if you want to do it that way I cannot help it, how do you determine the period? Why is it so intriguing to you this morning? Oh come on, representation is the fraction first of all you are not paying attention to the units, for a discrete time signal you are looking at the period if you call it as NP it is the number of samples taken to complete, you know,

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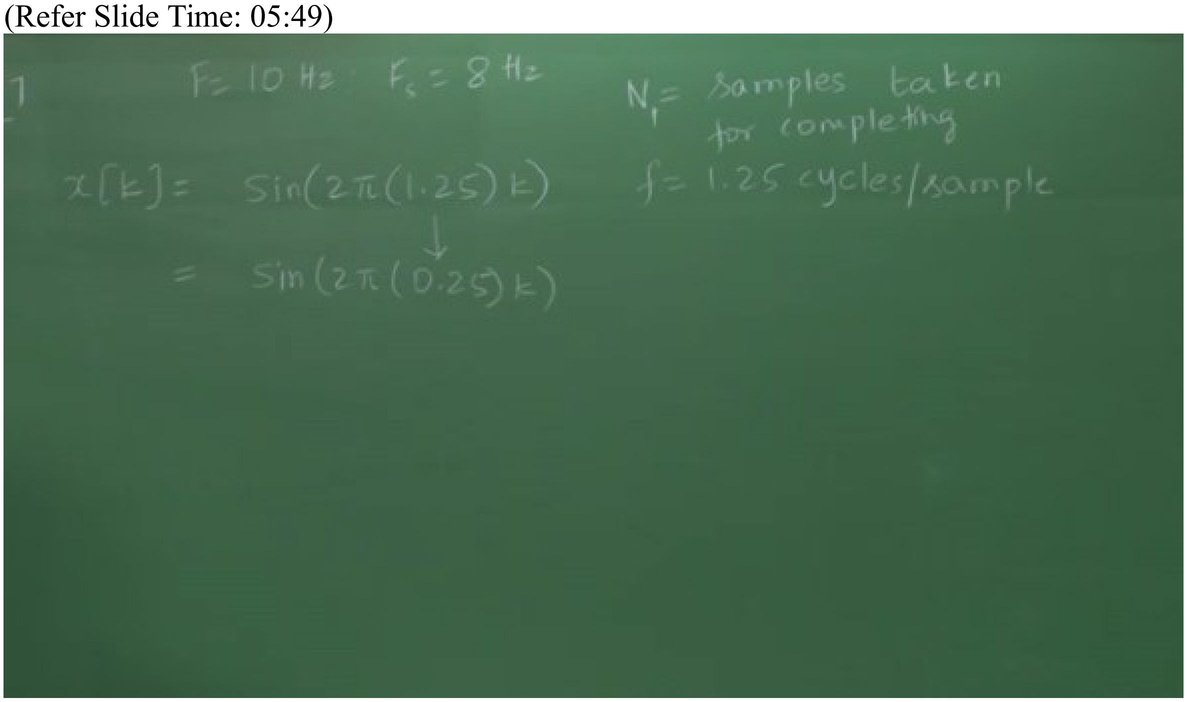

taken for completing not ideally one cycle, right, ideally one cycle but then you have to find the nearest maybe that's what you meant by integral multiple for completing a few cycles if necessarily, the first few cycles, ideally it is the number of samples taken for completing one cycle, and in this case the number of samples taken to complete one cycle that's where your 1/F thing comes in, but there is nothing like 0.8 samples.

But instead of going over that kind of calculation, you just write it as 5/4,
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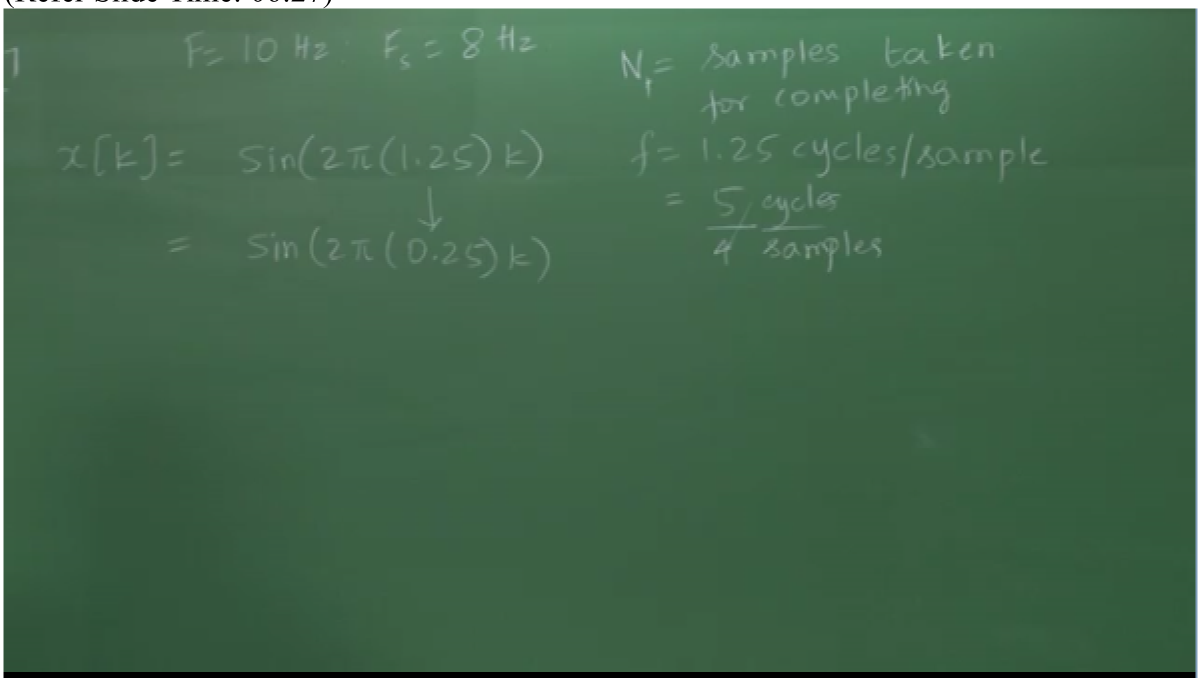

so 5 cycles are 4 completed in 4 samples, that's why you read the units, we have gone through this discussion again I'm saying, and the point is you express F in the most simplest rational form such that the denominator and numerator are co-primes, so that now you know as far as an observer, I mean from the observers view point you will see the first reputation of the signal after 4 samples, you will not be able to say how many cycles are incomplete, that's the difference between a continuous time sine wave and the discrete time sine wave.

The fundamental period is only going to tell you after how many samples a reputation will occur, but within the period how many cycles are actually occurred you will not know unless you do a Fourier analysis, if you do a Fourier analysis then you will start with the fundamental frequency 1/4, then 2/4, 3/4 you will include all harmonics and then you'll figure out that yes this is the frequency, right, any questions on this? I'm going to ask this question again at a later time and hopefully at that time all of you should be clear.

The concept is extremely simple, only if you pay attention to the fact that the period of the discrete time sine wave is no longer a continuous valued quantity, is there number of samples? And it's not a monster that's going to gobble you up, it's just after all a very simple concept you don't have to be scared of, okay be bold, be brave, nothing can eat you up, fine, so let's get back to the discussion.

Now what I wanted to show here is that although this is a very high frequency, I mean 1.25 right, now high frequency in the context of discrete time signals, let's draw, I mean let's generate sorry, another sine wave which has a frequency of 0.25, and plot 1 on top of the other, right, so let's do that what we have then let's plot a stem, so I'll maximize the plot,

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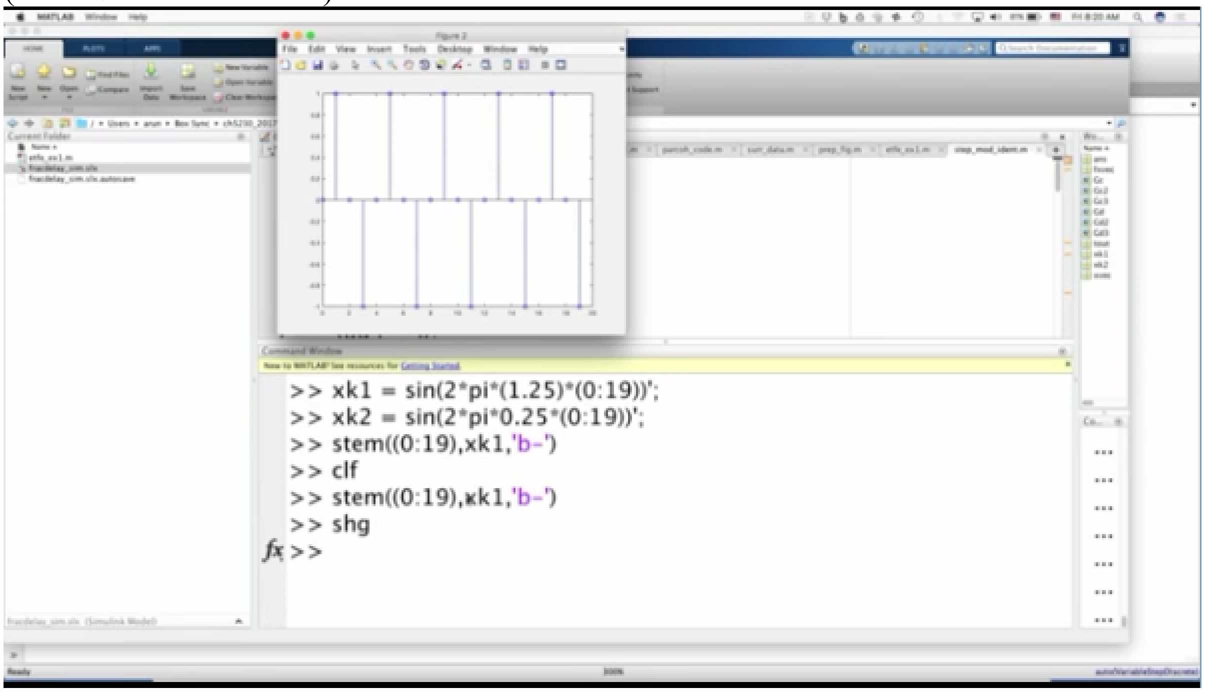

so here is your first sine wave and then let's plot the second stem with a red colour and dashed line style, oops, okay, so this is how these two signals look like.

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If I had used the same colour, would you be able to distinguish? This is a very easy question, you don't have to think much, I don't know, are you all able to see or you want me to increase the line width of this? Is it figure clear to the, look at the back, what you can see is that these two sine waves are indistinguishable, as I said had I use the same colour for both sine waves, there is no way you would have known that there are two signals, right, you want me to zoom in and show? I'm sorry, you're not able to see two colours, actually that is the problem I will show you here, let me see if I can zoom and then show you,
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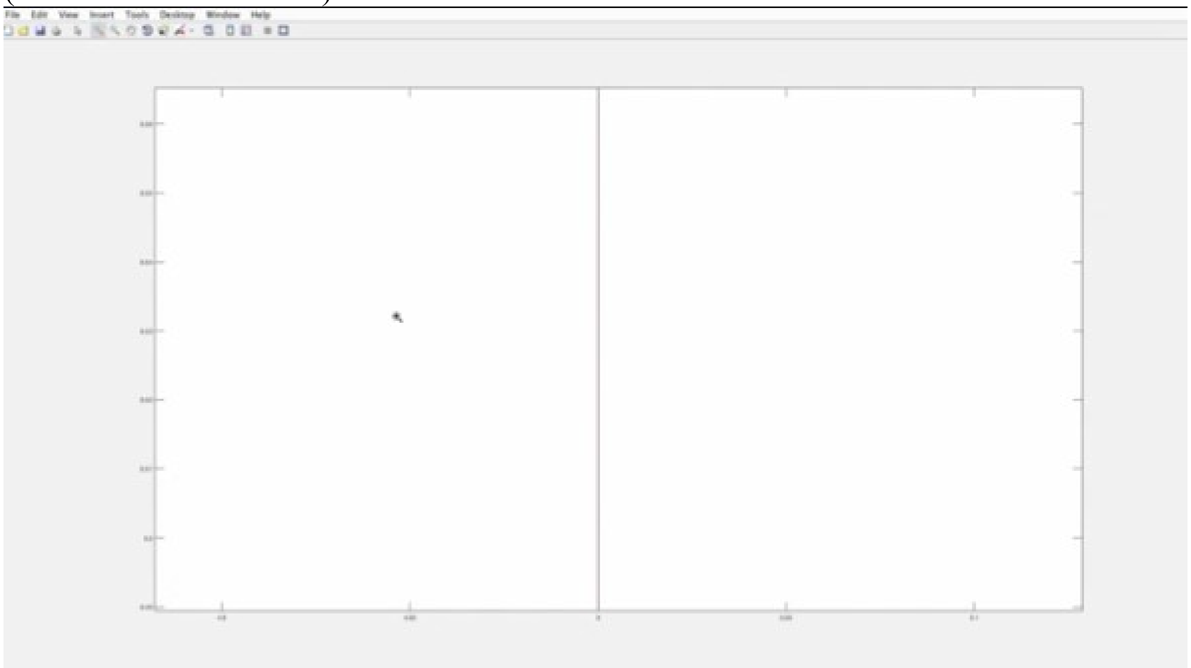

there is the red and the blue which I can see on my screen that, no way, so that itself goes to show that they are indistinguishable, okay.

But I can see a blue tincture there to the red, alright, maybe what I could do now you can see, no, so the best way to do this, alright, let us try drawing a dot block, dotted line and,
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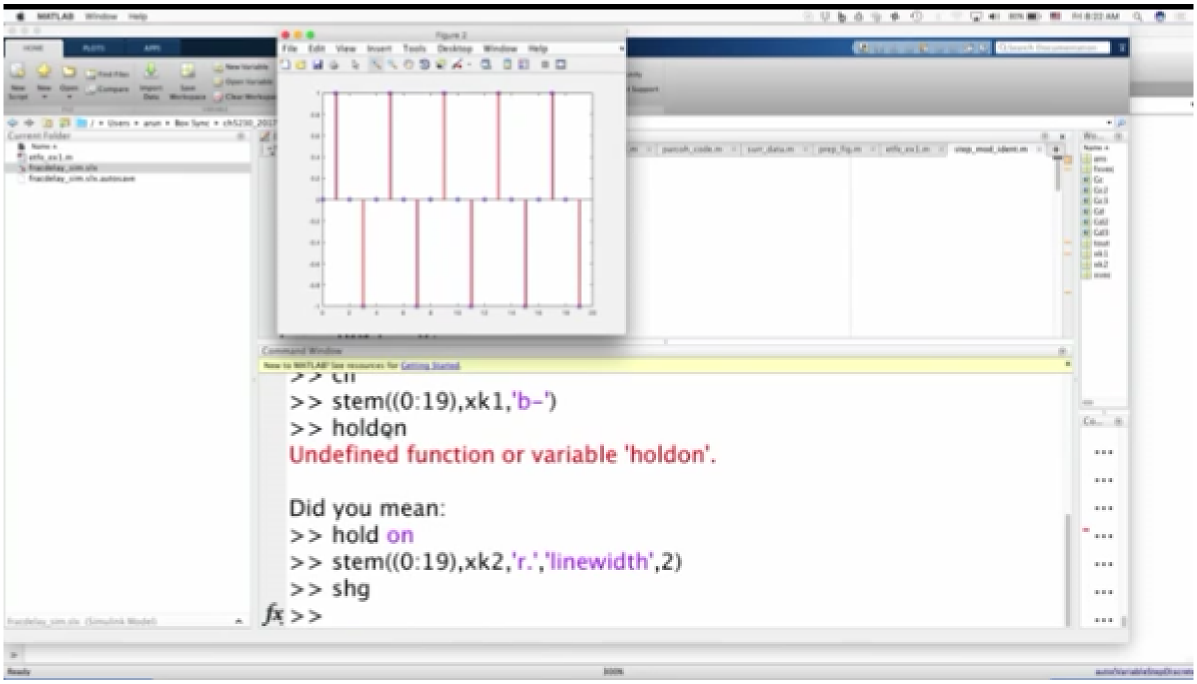

oh oh again you can't see, that itself shows that these two are aliases of each other, okay.

Well of course what I could have done is actually have a line width of 4 for the other plot maybe as a last ditch effort, now the blue has downloaded, right, the red is gone missing, so you can play around with this but the fact is they're indistinguishable we know it mathematically I just want to show you, so what happens is although you have a 10 hertz signal being sampled at 8 hertz which results in 1.25 cycles per sample, you as an observer we'd always see 0.25, okay, in fact I'm not proving through this plots that you will see 0.25, what you should actually do is to prove to yourself you will have to generate the signal, do a Fourier analysis and see for yourself that Fourier analysis tells you it's actually, there is only 0.25 frequency there. What I have shown you instead is that these two are indistinguishable, so it's an indirect way of proving that you will see 0.25, okay.

Now going back to our discussion on sampling theorem because of this alias in phenomenon what we have is that when we want to reconstruct the continuous time signal I have an issue, because what this signal will manifest as is the frequency of 0.25, so for all working purposes X(k) has this frequency, which means the corresponding, when I try to reconstruct I'll end up with a 2 hertz continuous time signal which is not the signal that I actually sampled, right, so this has occurred because I have not sampled fast enough, that is one way of looking at it, but that's only half of the story, the main culprit or the main reason is the fact that discrete time sine waves are unique only in the interval -0.5 to 0.5, this is called a fundamental frequency range as I keep saying.

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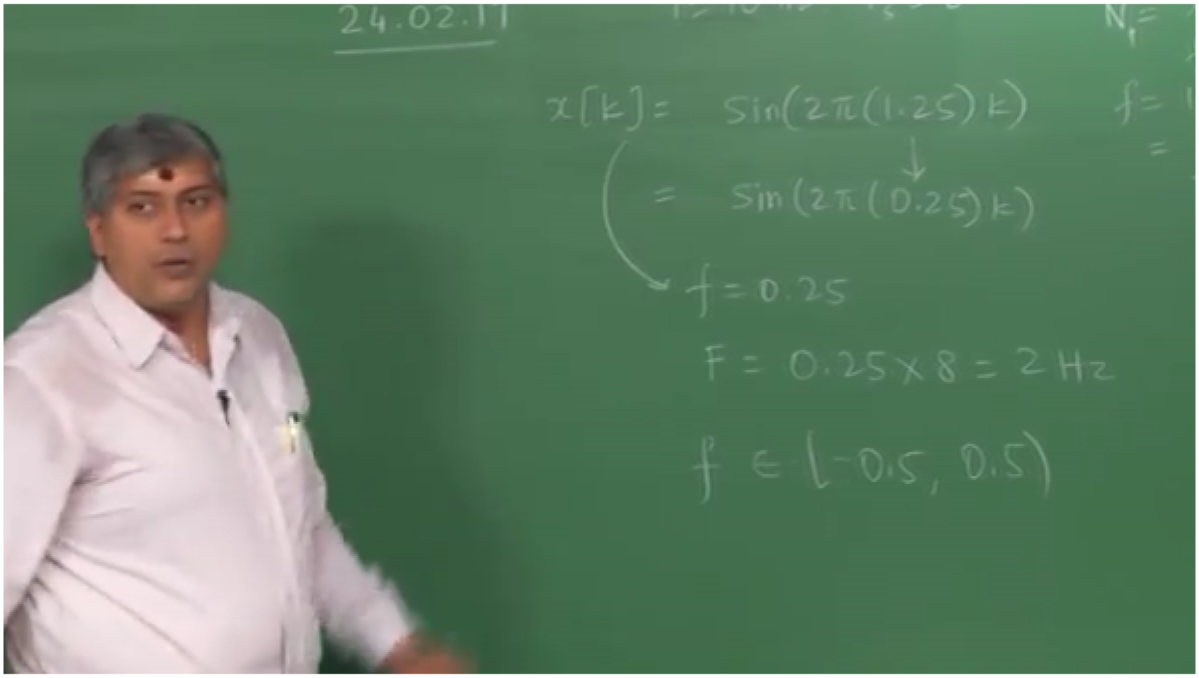

So as long as you sample your signal such that the frequency of the discrete time sine wave or cosine doesn't matter falls within this fundamental frequency range, you will be able to recover the signal correctly. If you sample in such a way that you have a situation like this, then there is a map outside that is all the frequencies outside, there is an alias here, so it maps back and this is also called folding, there is another technical term in a sampling called folding, so it's just folds to a low frequency signal and that is what you'll be able to recover, that means you have lost the information, okay. So you have to remember that the main reason behind having the sampling theorem is because of the nature of the discrete time sine waves, okay.

Now we'll have about 5 or 7 minutes discussion on some practical considerations and then move on, as I said in reality you are not going to have signals with a single frequency, you are going to have signals with mixed frequencies, in which case you are sampling frequency is going to be based on the maximum frequency obviously, because if you have fulfilled the condition for the maximum frequency you're fulfilled the condition for the, all the other frequencies lower than that, right, so as an example here this is the example that we looked at yesterday,
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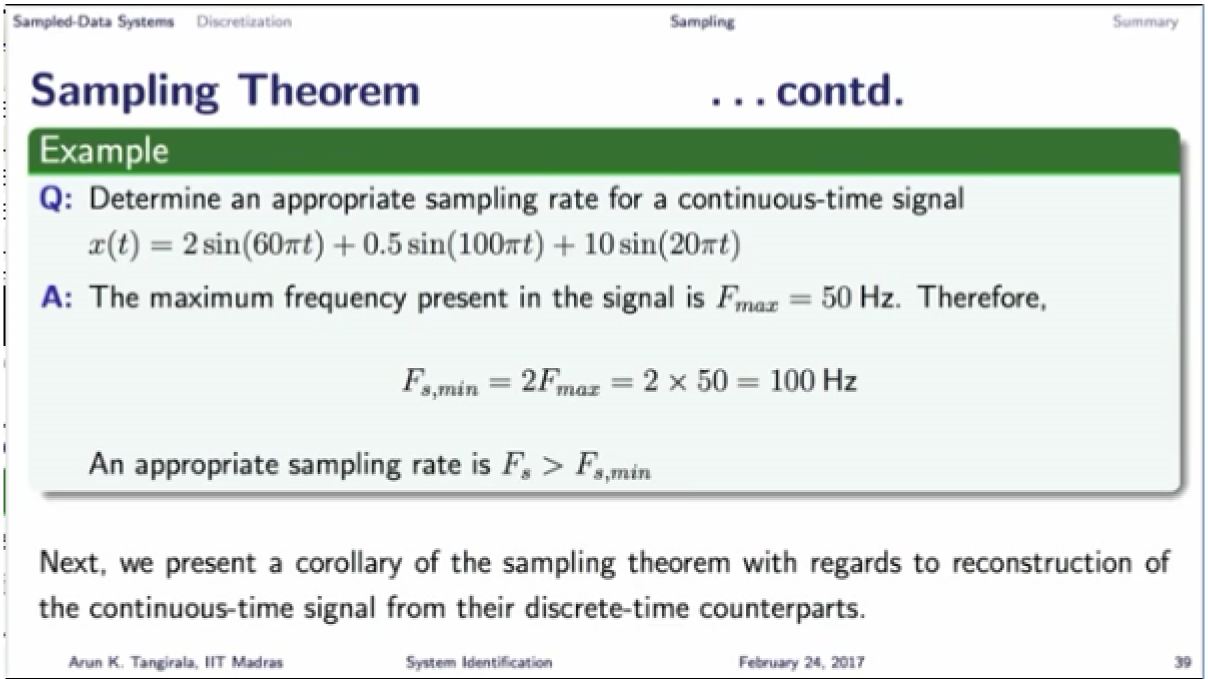

here I have 3 frequencies in XT, and the maximum is 50 hertz therefore the minimum sampling frequency is 100 hertz

Now the question that should come to your mind is in practice how do I know what's the maximum frequency, before even I sample who will come and tell me what is the maximum frequency, because that is what I do not know, that's what my experiment is going to tell me perhaps, so how do I decide my sampling frequency, it's a catch friend to problem, correct?

Now there are at least two different remedies to this, one is you choose a very high sampling frequency hoping that you have taken care of the maximum frequency in the same way, okay, and of course your choice of sampling frequency is also limited by the hardware, but today you know many good hardware give you quite a large bandwidth for sampling frequencies, that's not an issue, but that can also run into the risk of aliasing if your choice of sampling frequency is not meeting the highest frequency that you expect to see.

In industry, in the many practical applications what you do is you force the continuous time signal to have a maximum frequency of what you want, and this is achieve by using what are known as antialiasing filters, so what you do is you let X(t) pass through a filter, analog filter, what is the role of this analog filter? It's going to click the maximum frequency it's going to make sure that the filtered signal will not have a frequency beyond what you specify.

Now having assured that you can choose your sampling frequency, how do you decide what should be the cut off frequency in the analog filter, well that varies from process to process, so if you know that Apriori you will never expect to see frequencies beyond 500 hertz, right, or anything above 500 hertz will be noise only, there is nothing that will come from the process then you choose 500 hertz just as an example, and then you choose your sampling frequency to be minimum, to the 1000 hertz.

Now practically you will choose more than the minimum, and that's for an obvious reason, suppose you are not using antialiasing filter, you are just using a sampling frequency and you are basing it with respect to some maximum frequency that you think you expect to see, then you may be in for a surprise if you just stick to the minimum, so practically you set much higher than the minimum but how high can I actually go to million hertz is that good, is that recommended is the next question.

Should I choose as high as possible because intuitively what we feel is that higher the sampling frequency better, closer I am to the continuous time signal, but is it a good thing to do, now unfortunately that is not great thing to do, because there is always going to be some noise, and when your signal is not changing much with respect to the sampling frequency and you just sample very fast, yes in the sample signal you will see some variation but most of that variations is going to be due to noise, and your algorithm may not be able to distinguish necessarily between the process variation and the noise, so you're unnecessarily over loading your signal processing, signal processor or your estimator, we don't want to overburden, the classic example that I always gave is news channels when they broadcast news, typically news does not occur I mean what we call us news, what we think of news you don't I mean, interesting news doesn't get generated as at the rate at which the channels want, sometimes there can be sponsored news that's different, but what the channels ideally like to have is news being generated every second, so that they can keep the viewers hooked to the television channel.

So unfortunately, I mean fortunately news doesn't get generated as fast as they wanted, so what do you see? Some event happens, so they are all hungry for some event and some event happens and then a lot of noise is made out of that event, so you invite some 10 people, and then you ask what do you think, you know, somebody would like to know or you may say well what do you think, do you think that this cockroach died because of heavy winds or and so on, I mean anything is an event and then noise is made out of it.

If you are listening to the television and you're a first timer, you think well there is a lot of news happening in the world, only later you realize most of it was nuisance not news, okay, so if you are unable to, if it's a gruesome event that they are discussing that, anything is discussed and if you happen to be heart patient, then the consequences can be disastrous, okay, in fact many people do died because they keep watching this emotional things, and gruesome things time again and again and again, because the screen is split into two halves and well been showing the same event again and again. So the moral of the story is don't sampled unnecessarily fast, because they're going to bring in noise than anything else, and that is where there are some practical guidelines when it comes to choosing the sampling rate.

Before I go to that I just want to remind you that for any choice of sampling frequency as I said yesterday,
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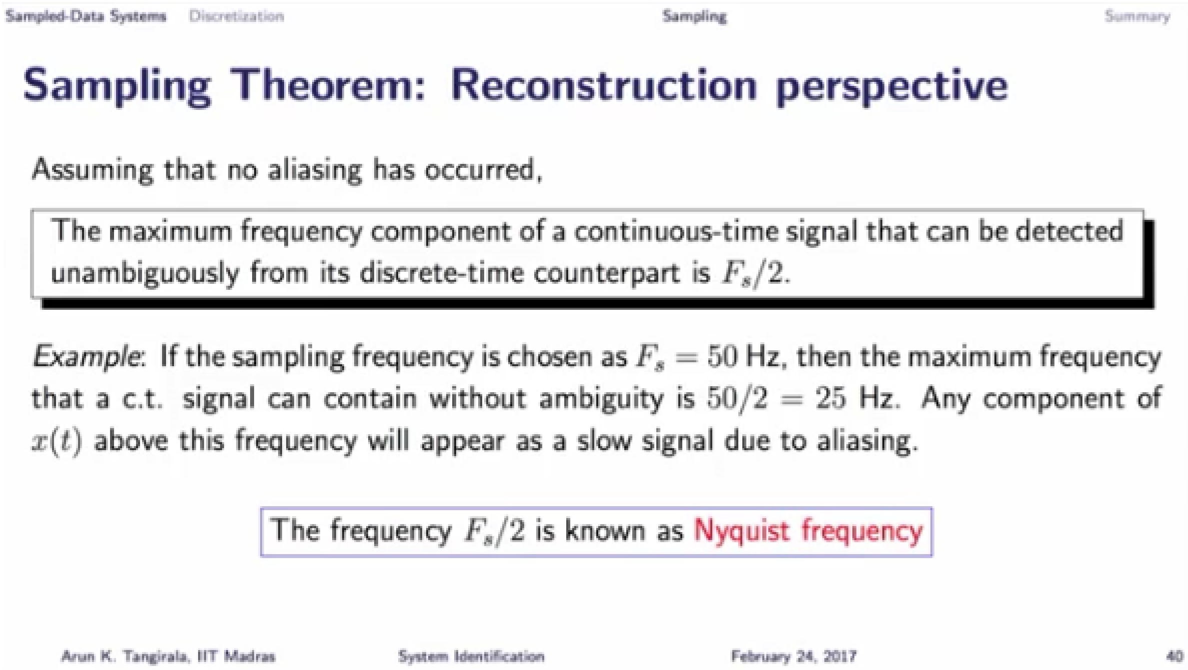

if you were to ask what is the maximum frequency that you can recover an ambiguously that is obviously FS/2 because then that meets the sampling theorem criteria, and this frequency FS/2 is known as the Nyquist frequency.

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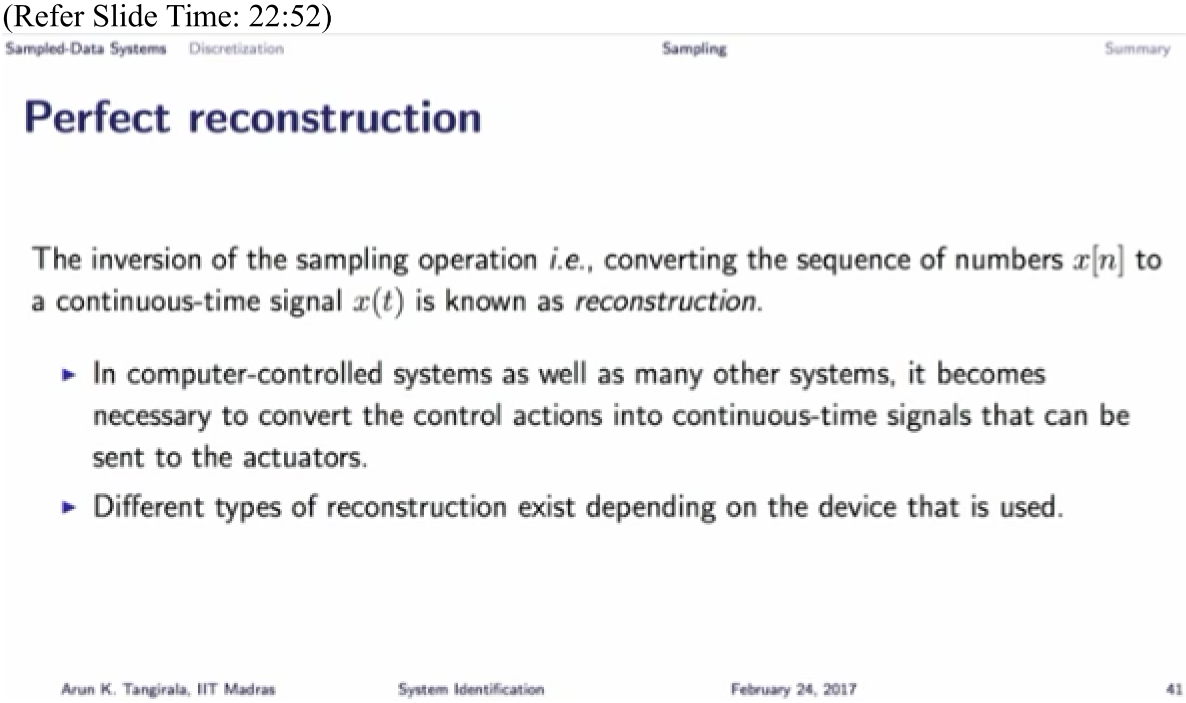

And also if you want to have a perfect reconstruction which we won't use this in the course but just for your own information if you want to ideally recover the continuous time signal from examples, then Shannon gave this reconstruction formula which as you can see requires infinitely long samples, the summation runs from –infinity to infinity, and it also requires therefore future values, so therefore it's a non-causal reconstructed but nevertheless it says if you give me the full sampled signal I will be able to recover X(t) exactly, provided the sampling theorem conditions is met.

Whereas the zero order hold how does it reconstruct? Very simple, no complicated formula, any signal that you give it says the continuous time signal is simply a piecewise constant signal, piecewise constant only sampling interval and we know that it is exact for continuous time signals that are piecewise constant over the sampling interval.

So let's now talk of practical considerations, when you sample fast two things can occur one that you can bring in a lot of noise, right, and two recall the eigenvalue mapping theorem that we had, we said lambda of the continuous time system maps to E to the lambda I TS, right, so if TS,
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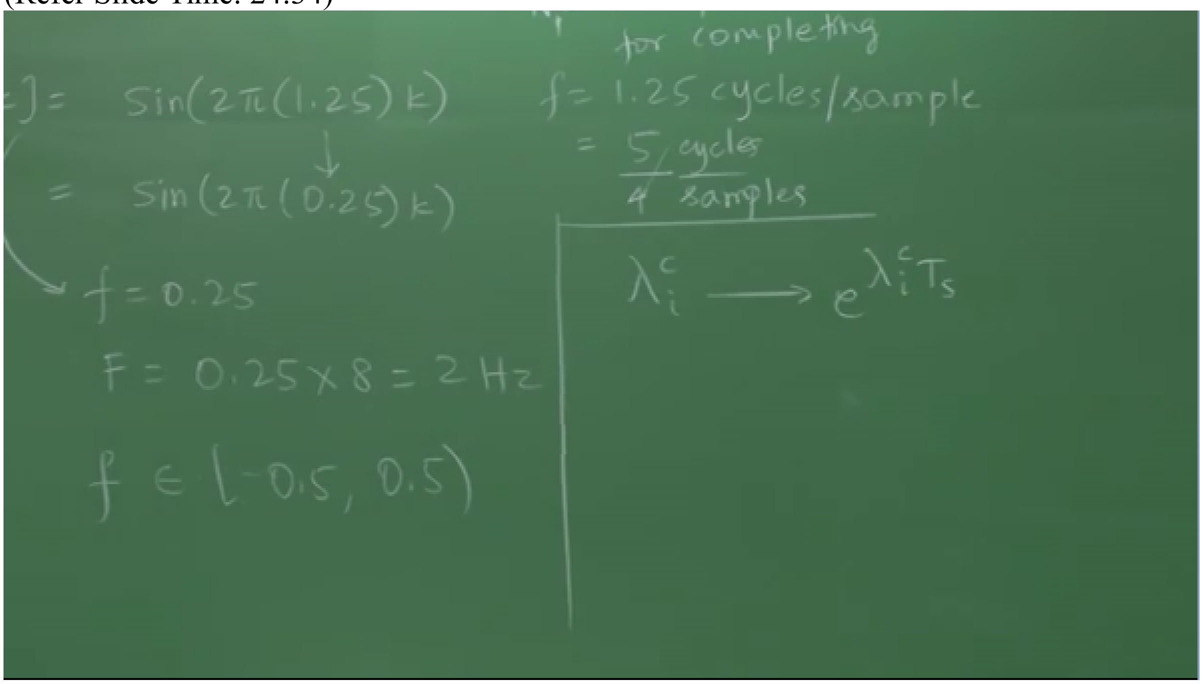

sorry if FS is very high then TS is very low, which means E to the lambda I TS is pushed to 1, right, when we say low relative to lambda, and that means you are pushing the pole close to unit circle, and that's not a good thing in system identification because if you have poles close to unit circle, let's say you have a pole at 0.995,
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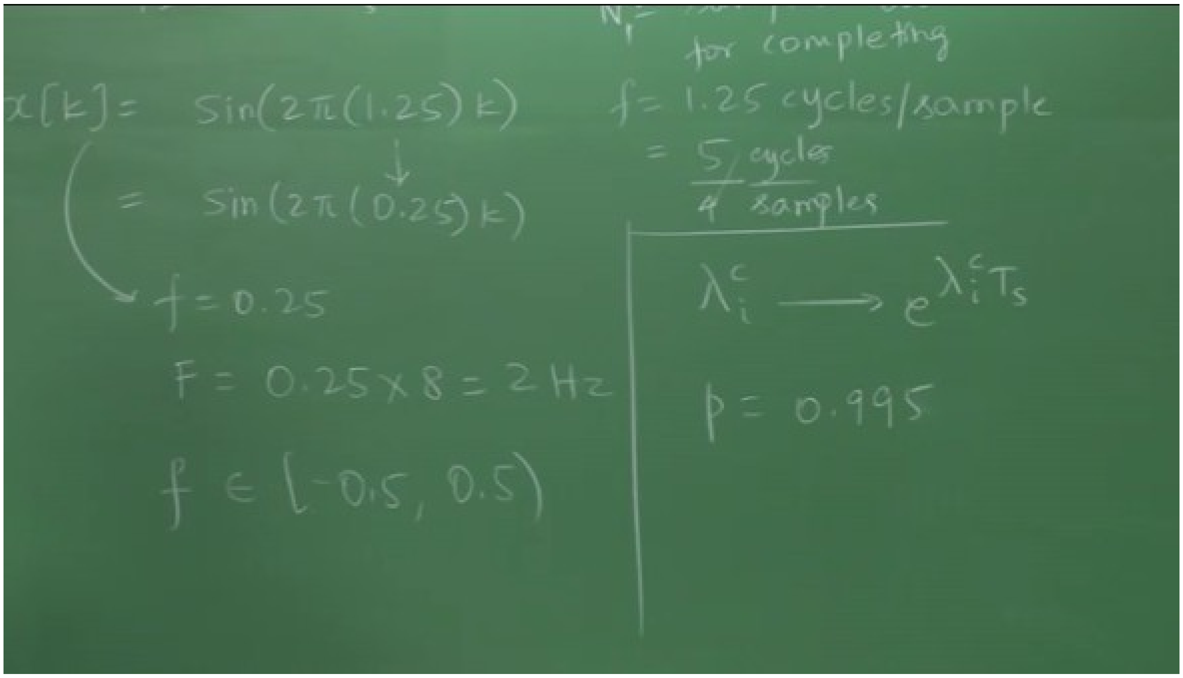

because you have sample very fast, how do you explain this pole being pushed to unit circle and the fast sampling rate, one way is to use this formula and show that the pole goes to 1.

What is the other way of looking at it? Intuitively if you had to explain without using the pole concept, what does the pole at 1 mean with respect to difference equation? For the difference equation suppose I have a difference equation description and I say that this system has a pole at unit circle, what does that tell you about the difference equation? That's one way of looking at it, but correct, so how do you write the difference equation for the integral? Suppose I have a single pole and let's say exactly at the unit circle, how does the difference equation look like? Plus or minus? Correct, it was UK okay some input, right, so that's good, so if there is a pole at 1 or even at 0.995 you can say approximately Y(K) is Y(K-1) + U(K),
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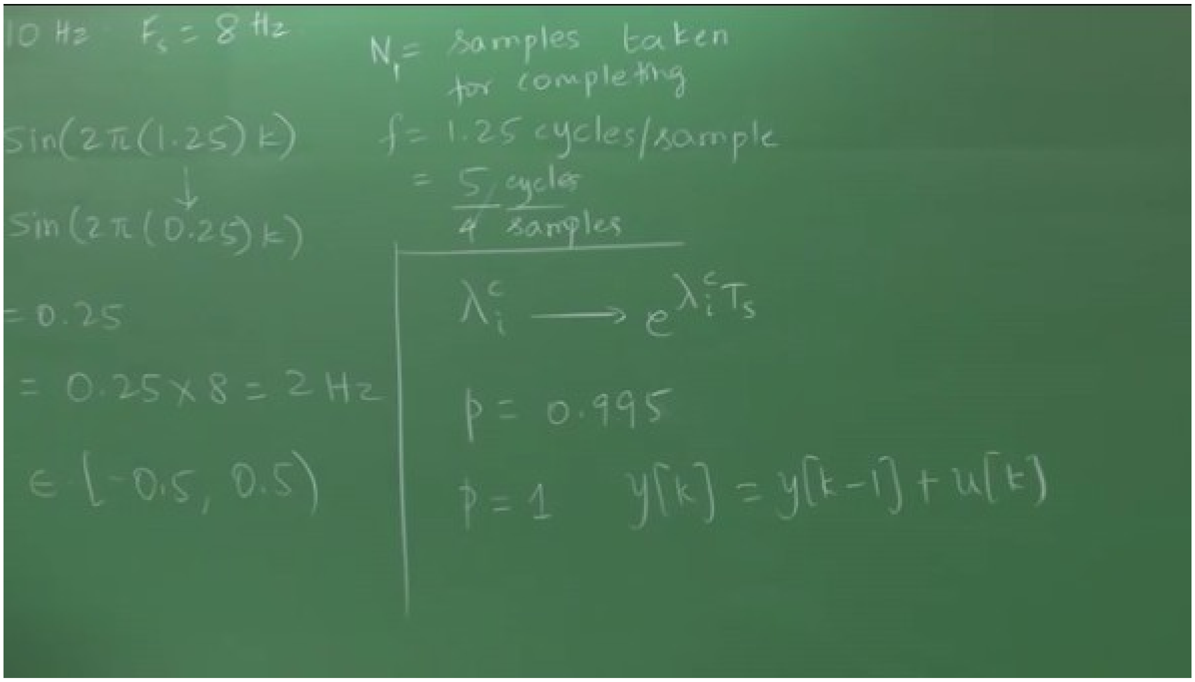

it tells me that two successive values are identical, suppose I take away the input, suppose the input is in impulse or it stops acting after a while, two successive values are going to be identical or nearly identical, but that is what fast sampling rate would mean for a very slow process, suppose I'm watching the movement of a snail and I'm watching every millisecond, what do you observe of the position? Between two successive instance it won't change much, correct, that amounts to saying that you have a pole close to unit circle, which means over, for a large number of data points you will have a constant signal, yet this noise will actually make it look like as if it is changing, but the fact is that the underline truth has not changed, and that can confuse your estimation algorithm and you can prove that these are called ill condition systems when it comes to estimation, that very small errors in your algorithm can result in estimates of poles outside the unit circle, which means although the system is stable theoretically you may end up with an unstable bond which you don't want, so these are the things that you have to watch out for when you are sampling very fast.

This is unfortunately a fact that you one has to live with when it comes to sampling, when you have a system and in fact it becomes even more pronounced when you have what is known as a multi scale system or a stiff system, you must have heard of stiff systems right, what is stiff system? When you have, do you remember what is stiff system is? Do you recall? In ODE's numerical integration anyone? What is the stiff system? It looks like most of you are in Shiva Dhyana today being Maha Shivaratri you don't want to be perturbed, there are many who are already preparing for the night meditation, Lord Shiva won't mind you answering, I can assure you that, so what's the stiff system? What is the stiff system? Oh come on you can answer, let's hand pick Prem, this is not a sponsored question, what is the stiff system? Rashmi, sorry, that's all, because you're very stiff, no, that's not a way to describe the stiff system, come on. What about you man? What is the stiff system that you encountered in numerical integration and so on, do you recall what, you have heard this name stiff system in the first place? Got to do with the bunch of ODE's right, which have really differing eigenvalues, right, there is one ODE which has an eigenvalue, that means time constant that's very high, and there is another ODE of the same system which has very low eigenvalue, and these two are a part of the same system, it’s like having a snail and a rocket in the same system, right, a snail and a rat.

Why is it so difficult to deal with this, set of ODE’s? I have to integrate them jointly, what is the difficultly that you in research, the main issue is step size, very good, right, if I choose a fast step size, then I will find the slow moving one very annoying, because it just doesn’t move, the solution doesn’t change at all.

On the other hand if I choose small step size what happens? I would have missed out the dynamics of the fast sub system, I would have irritated that guy, so one of this is going to be irritated, right, and that is the major challenge.

What is the relation between step size and sampling interval? They are more or less the same, when we talk of numerical integration we use the term step size that is the times at which I wish to observe the solution, calculate the solution. Sampling interval is no different, sampling interval is the time that I take to observe the next instant of the, next value of the continuous time system, so you should now grow beyond, I mean you should see the analogies between the step size and numerical integration and choice of sampling interval and signal processes, they are one and the same, the issues are the same, we may use a different terminology the stiff systems and step size, and here we may say multi scale systems and sampling interval, but the story is the same, the challenge is the same, in fact ideas from the solution to stiff systems are borrowed in multi scale analysis.

So if you look at semi link and if you ever open solver, if you want to set the solver for semi link you must first know that semi link uses numerical integration, do you know that, if you are simulating the continuous time process like the liquid level system that we simulated it uses a numerical integration method, and if you go to the solvers, when you go to the configuration it will bring up a list of solvers, ODE 23, ODE 45, ODE45S and so on, that ODE45S is a special algorithm meant to handle stiff systems, so you need to know all of this to be able to do a good job, anyway so bringing you back to the point of discussion choice of sampling rate is can be a big challenge
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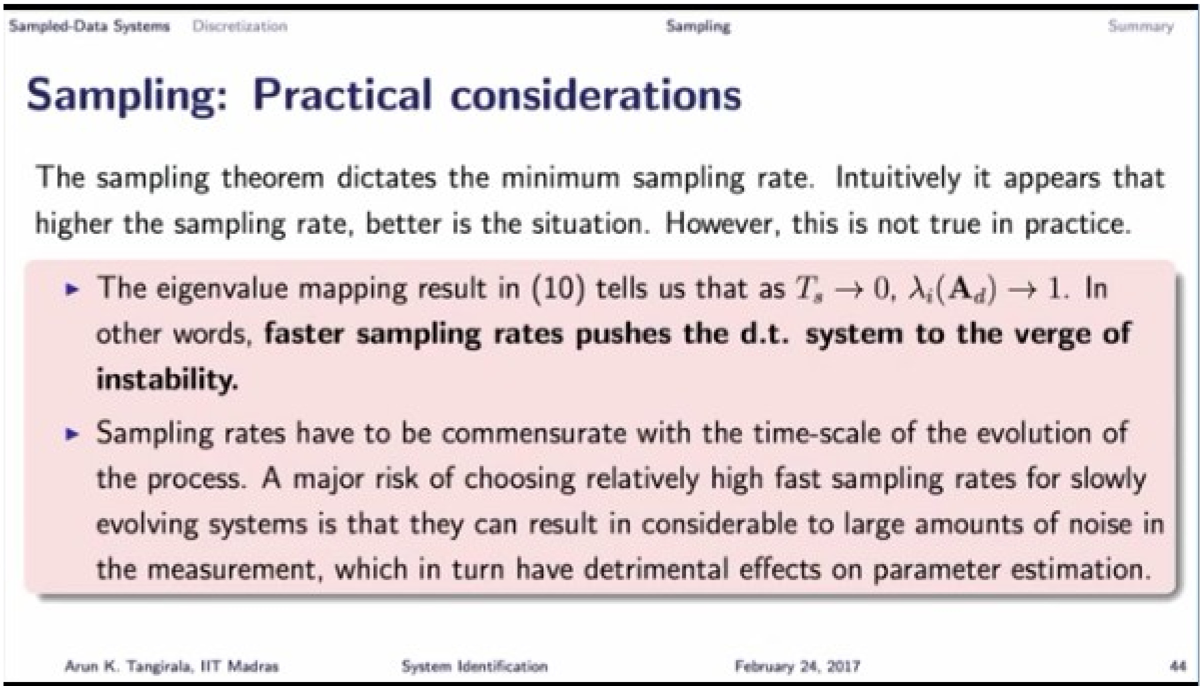

for many applications, but the guidelines are that if you choose very fast sampling rates, and fastest relative to the dynamics, then you can one bring in lot of noise unnecessarily to you may end up pushing the poles of the full system or the sub system depending on the system closed to the unit circle, this is unfortunately a fact that one has to live with when it comes to discrete time systems.

So let me close by giving you a rule of thumb that is generally followed when it comes to choosing the sampling interval,

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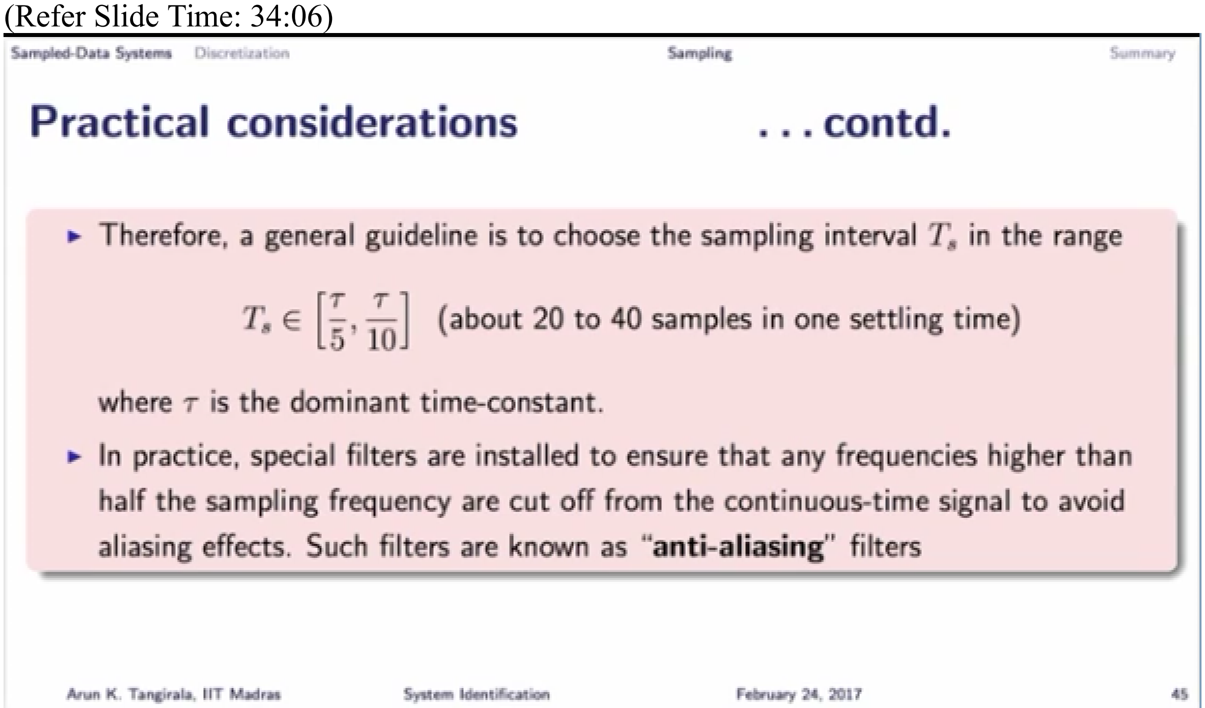

suppose I model the system as a first order just to give you as an example, suppose I think of the systems as a first order with time constant tau, then the general guideline for choosing sampling interval is that it should be anywhere between 1/5th to 1/10th of the time constant, which means that we want to have at least about 20 to 40 observations or samples in a single settling time. For those of you who are not familiar with linear systems theory settling time of a continuous time, first order continuous time process is roughly 4 tau to 5 tau, so when I say that I will choose 1/5th for example, 1/5th of time constant as a sampling interval that means in one time constant I'm obtaining 5 observations, right, and that means in one settling time I'm obtaining the roughly 25 observations, if you assume settling to be 5 time and likewise we have 40 or 45, so this is a rough guideline based on experience, you can choose higher than this, you can choose smaller than this, but then you should be prepared to face the difficulties that we may running, okay, and I've already spoken about the antialiasing filters.

So this brings a close to the discussion on sampling discretization where we have learnt some very important concepts as to how the continuous time and discrete time system that we look at or sample data system, how they map and what is the connection between these two, and the key parameter is the sampling interval. And then we have discussed how to choose this sampling interval, both one the theoretical and practical view point, and this not only brings us to the close of sampling and discretization, but also brings a closed to the discussion on the theory of LTI deterministic systems.

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