NPTEL

NPTEL ONLINE COURSE

CH5230: SYSTEM IDENTIFICATION

SAMPLED: DATA SYSTEMS 5

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Good morning, we'll continue our discussion on sampling and discretization. Mainly we'll focus on sampling today, we have discussed quite a bit on discretization, and the point where we ended our discussion yesterday in the previous lecture is on the discretization in presence of delays.

As I had mentioned yesterday if you have integer delays, delay that we are referring to is a delay in the continuous time process, so that D, that you see on the slide is the delay in the continuous time process,
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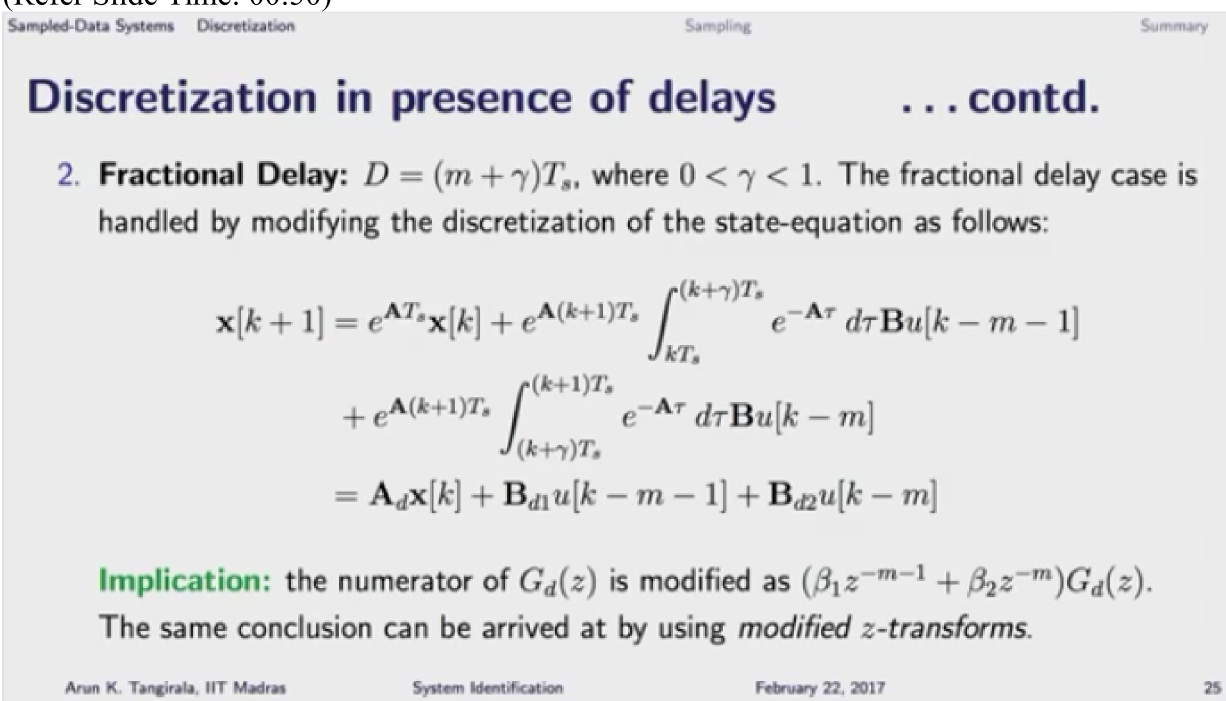

and all that we are saying is if the delay is an integer multiple of the sampling interval,
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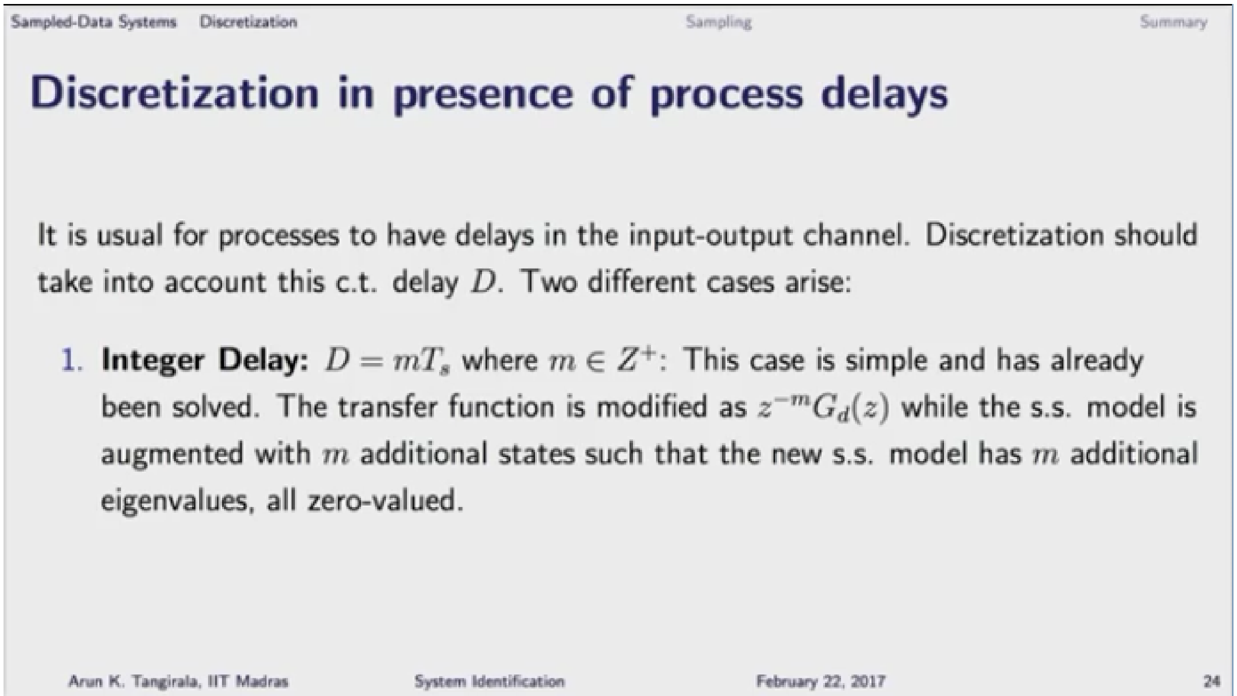

then the solution is straight forward, all you have to do is compute the transfer function of the delay free part if you already have it, simply multiply it with Z to the –M, where M is the number of delays in terms of sampling interval.

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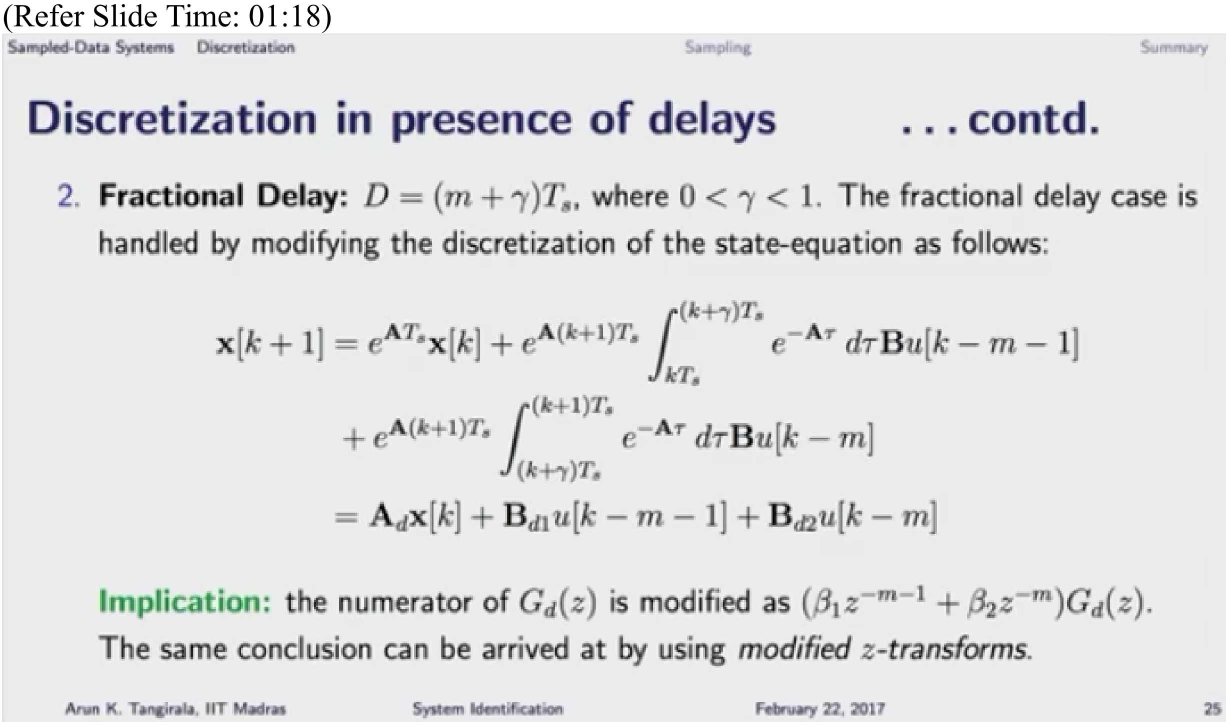

And the situation gets a bit trickier when you have a fractional delay, but then some amount of imagination will help us resolve this matter, here we are saying the delay is M + gamma times TS where gamma is this fractional quantity between 0 and 1, of course when gamma is 0 it falls back to the previous case.

Here what happens is suppose the sampling interval is 1 second let's say for just, for the purpose of discussion and delay let's say happens to the 1.3 seconds,

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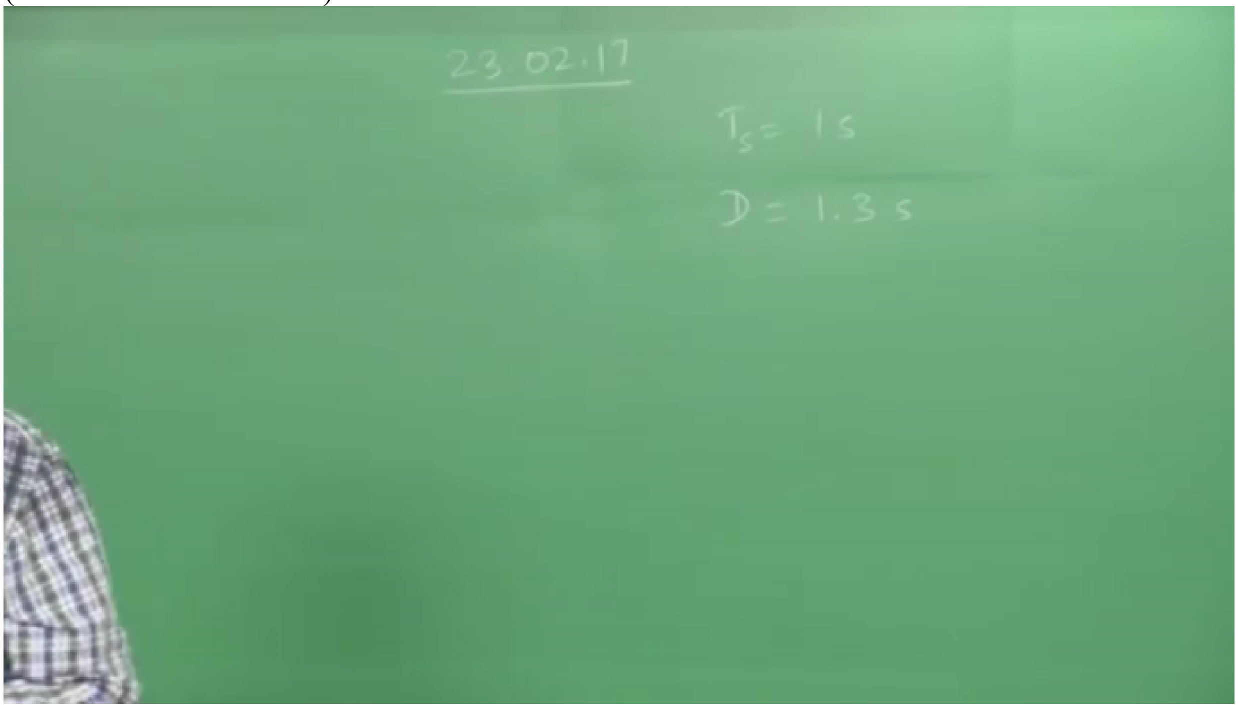

so this is a fractional case. In this case what happens is the process receives UK – 2 for a part of the interval, remember that you're looking at some time between KTS and K+1 TS, if you have no delay then UT would be UK, right, one unit delay, when I say unit delay here unit is in sampling interval, then what would be U(t) during this interval? UK-1, right, that's kind of obvious.

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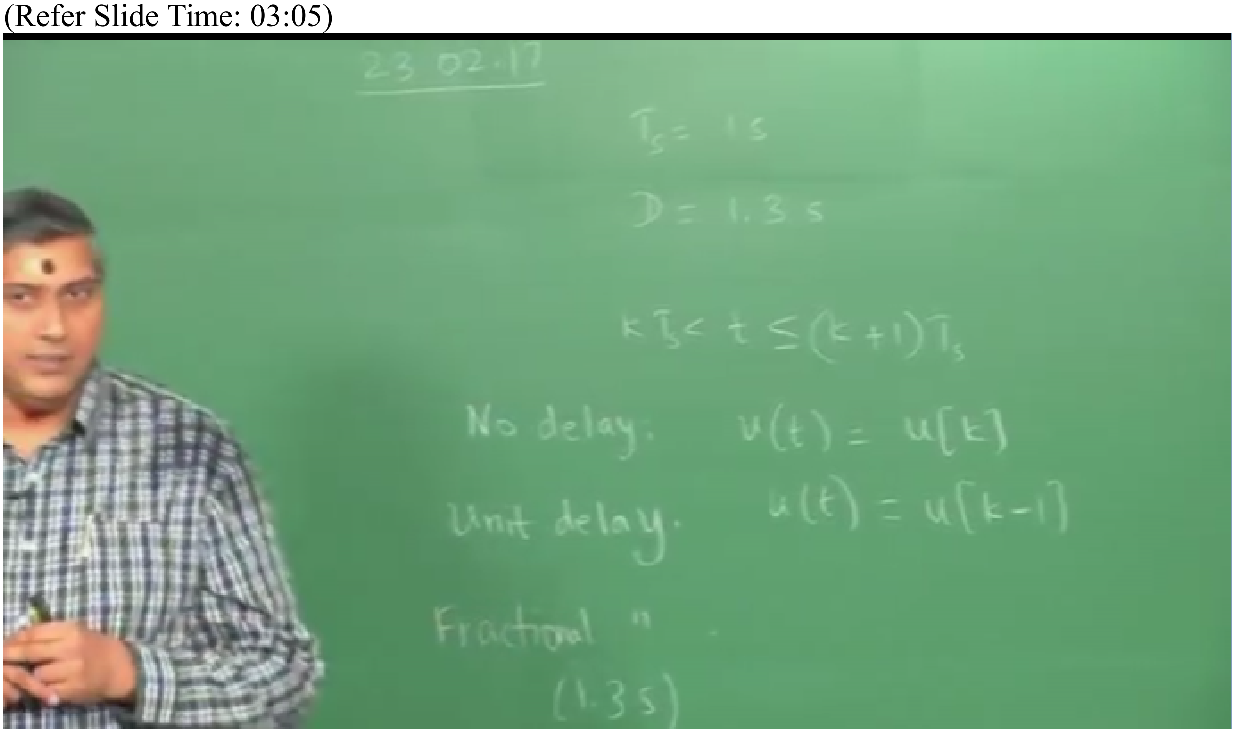

Now I have fractional delay, and especially we are looking at 1.3 seconds, in this case what happens is for a part of the interval that is between KTS and K + gamma TS your input would be UK-2, because it is still receiving the previous, that is the two delayed input and once the point 3 has elapsed then you would see UK-1 coming in, so it's receiving a combination of the inputs.

Now if it is hard for you to imagine, construct a semi link block diagram like I show you here, so here is the block diagram I don't know, let me actually maximize this but, will it allow me to zoom, yeah, are you able to see? All of you are able to see, right,
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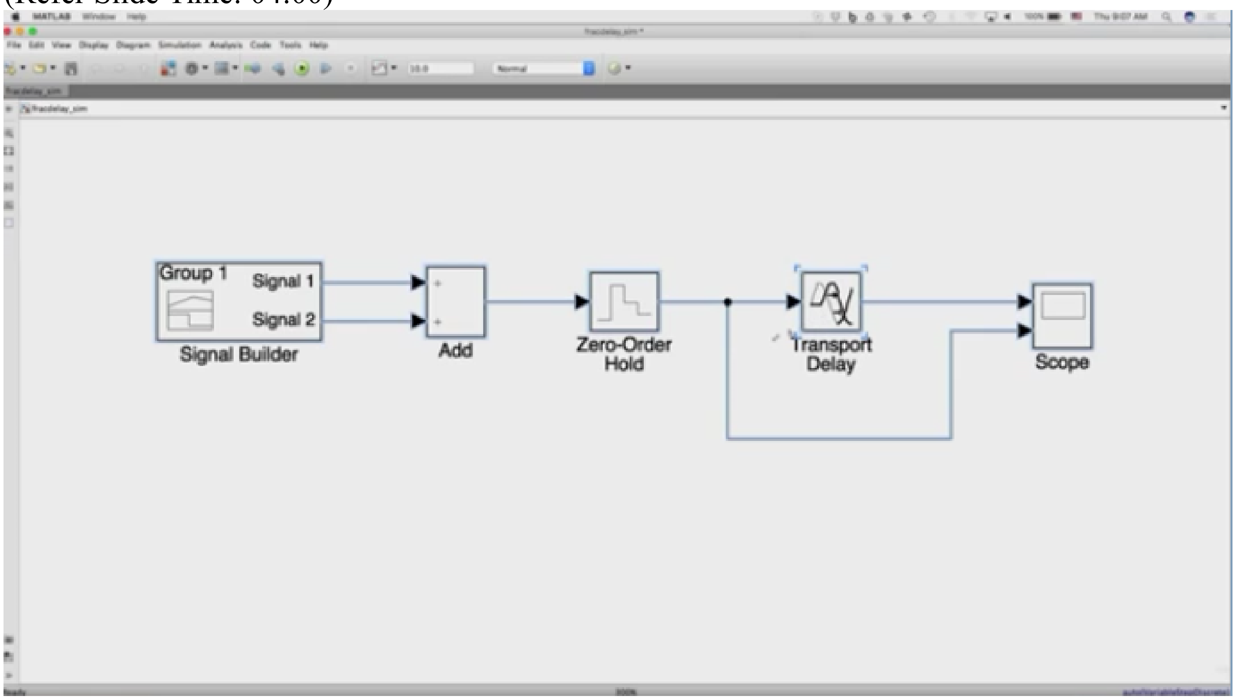

so now what I have done here is I have a signal builder, I'm not going to simulate this for you here, but I'm showing you how you can construct the block diagram in semi link, you can put in any signal here I have just drag the signal builder from the sources library, and I am just adding it up so that I get a composite signal, at this point you can inject any signal that you want which allows you to interpret things easily.

Let it pass through ZOH, and then the output of ZOH here goes through a delay, alright. Now actually what you should be doing here is a better way is to have a signal here, continuous time signal here, sample it pass it through ZOH and then compare what you get out of the delayed one, so the transport delay there are many types of delays that you see, transport delay is an easy one but you can perhaps look at other delays, I found this to be the most appropriate one, and you can compare, now you can see what happens when the delay is just one unit, I have se the sampling interval here to 1, so if you open a block, ZOH block, I have set it to one, and this I have set it to 1.3, but I can set it to 1, say 1,
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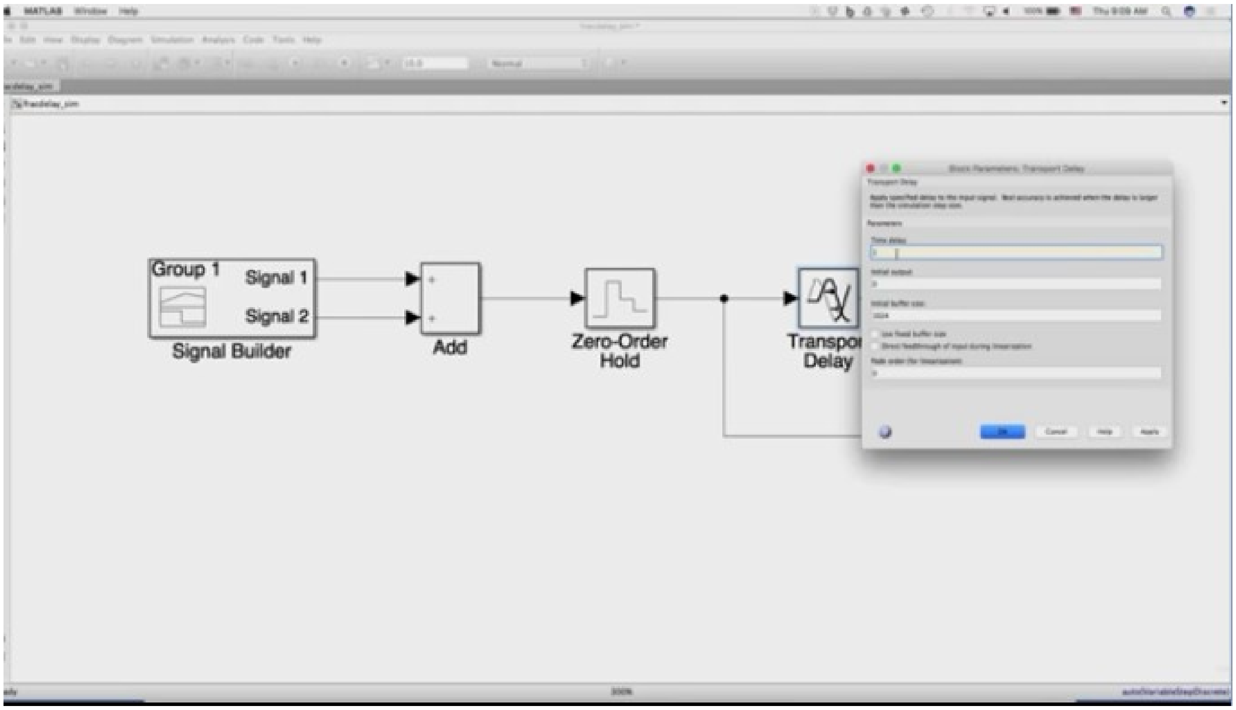

so that’s a time delay, and we can simply simulate this, and bring up the scope,

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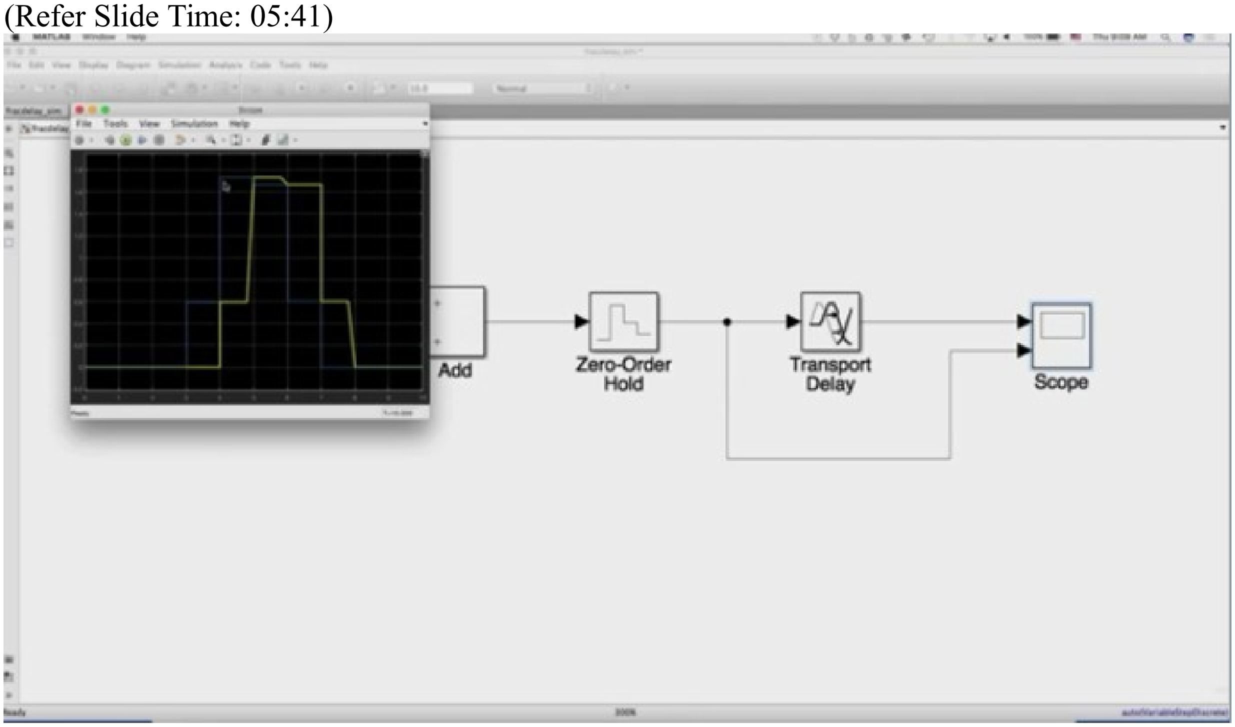

well because of the way I have constructed the signal, actually an easier signal would allow you to interpret things easily, but what you can see is the signal is exactly delayed by one instant and everything is shifted, don’t worry about the signal between the, here you have a ramp here don’t worry about that, actually that is because of the nature of the original signal.

So if you were to use a better signal then what I have used, you will see exactly the signal is shifted by 1, what I would like you to do is go back and play around with this, in this case the signal that I have built consist of a mixture of these two, okay,

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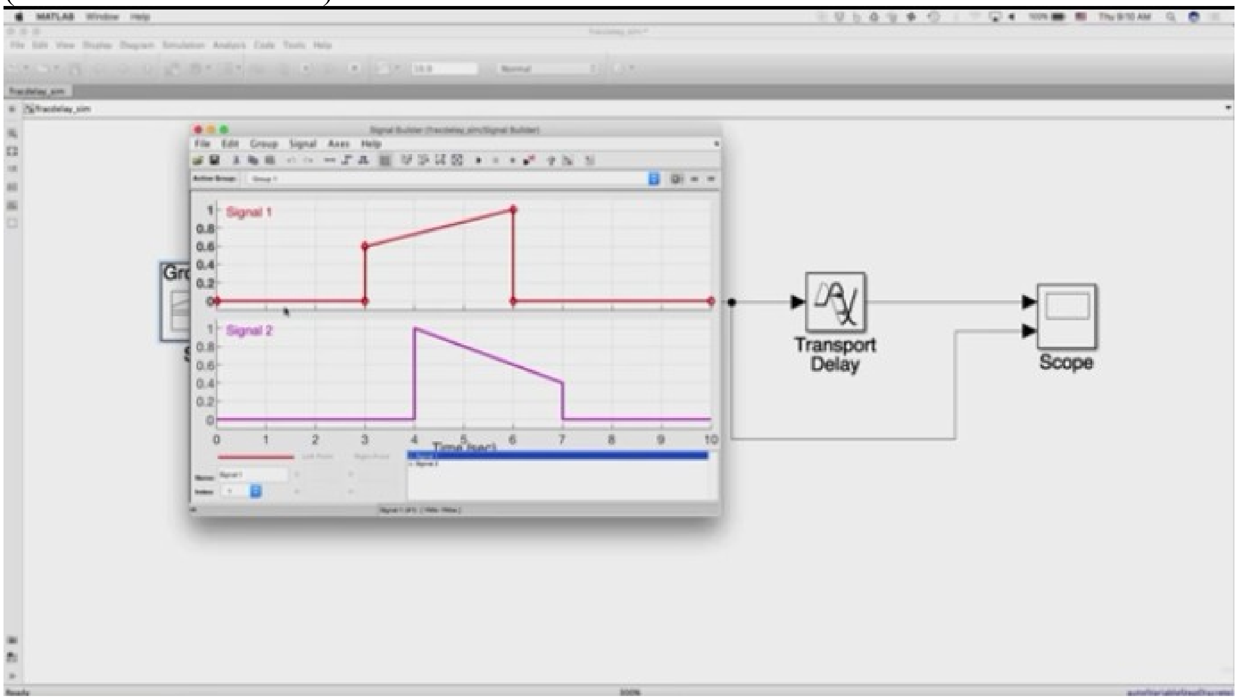

maybe it is better to use just a pulse followed by another pulse, then it makes it easier, and the way you do it here is in the signal will there, I don't know if it's allowing me to do here, because semi link does present me some challenges when I connected it to display, so you can actually drag the thing here and you can adjust the height which it was showing earlier, I'll try if I can do it, else you will have to do it offline, yeah, it just won't allow me to do it, then I'm connected to a display, there is some issue with the display.

Anyway, so you can actually adjust those and create a signal which has only a mixture of pulses, not like triangular once like this idle ones, and that will allow you to see that when you have fractional delays for a part of the time in this interval you would receive, so between KTS and K + gamma TS, U(t) would be UK in this case UK – 2,

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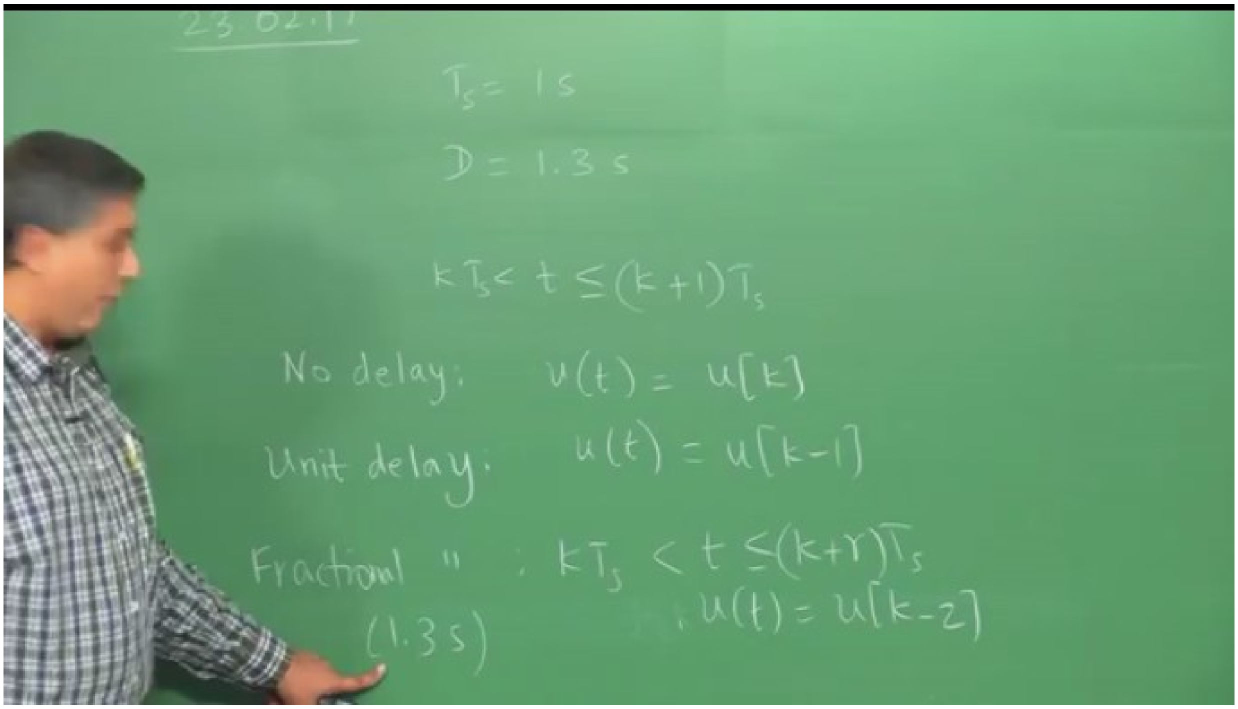

in fact if it is M + gamma, then it would be UK – M-1, okay and for the rest of the interval K + gamma TS, you would see U(t) taking on UK -1, so that is the story,

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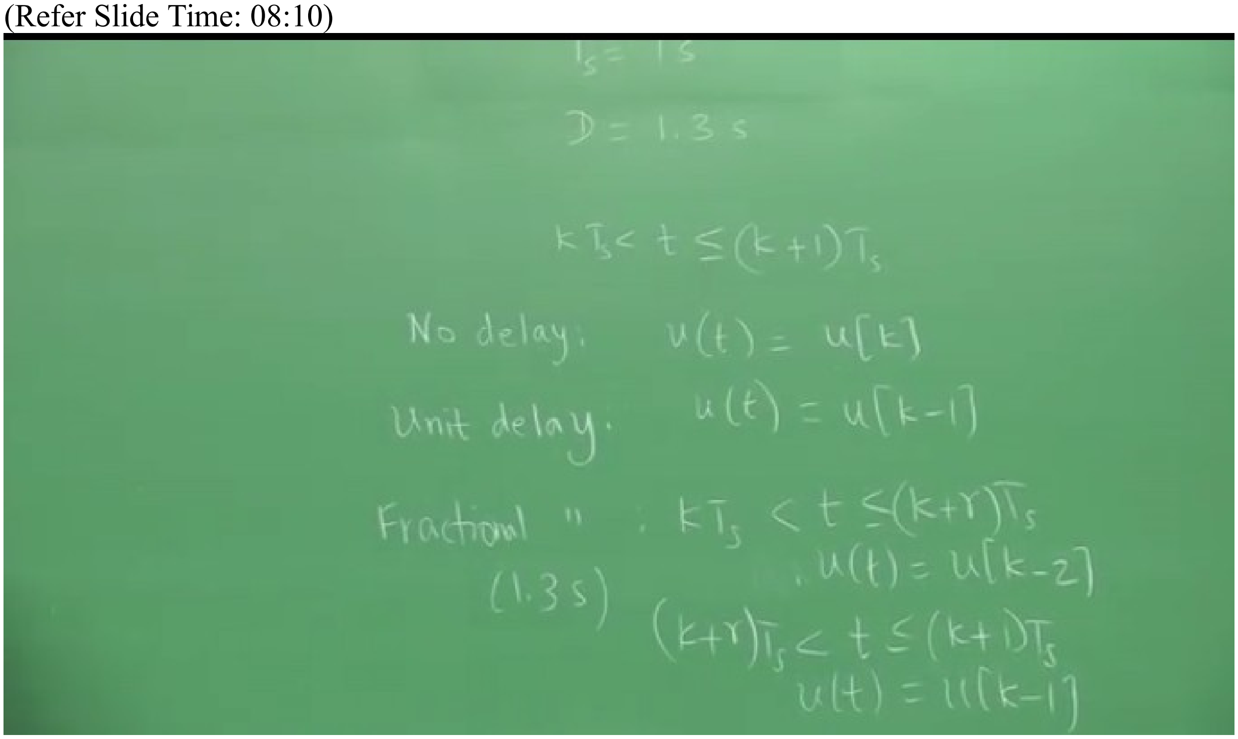

okay and that's why going back to the integration here

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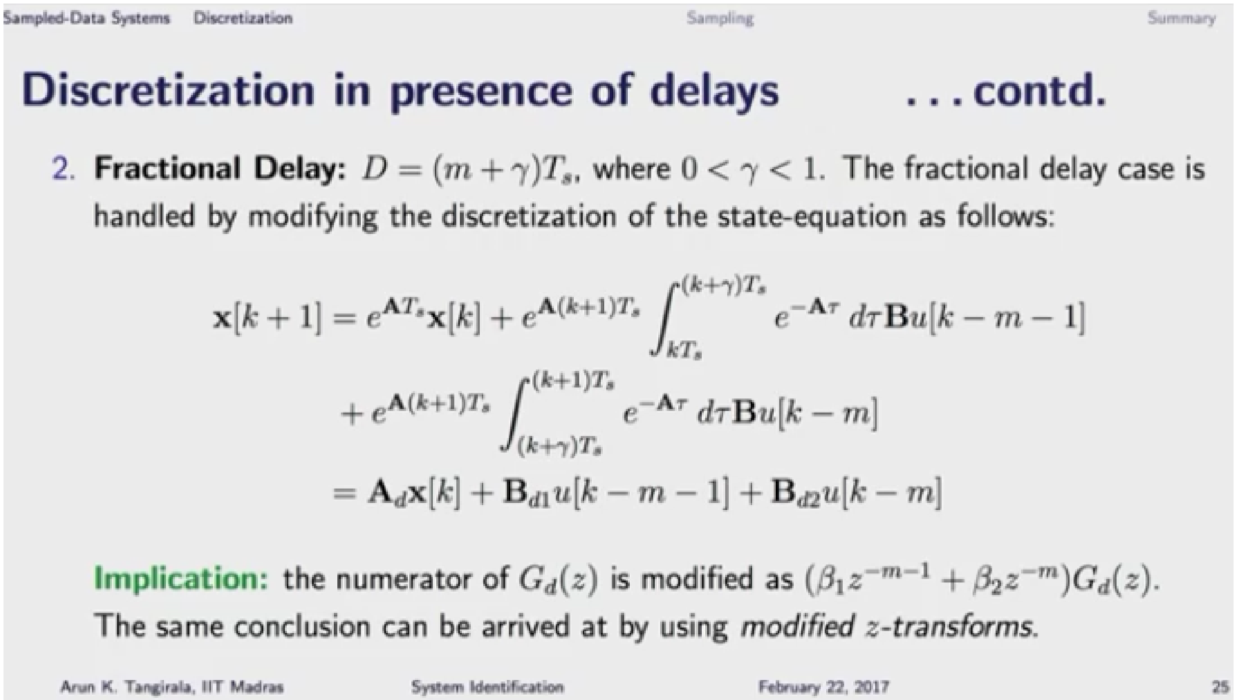

we have split the integral into two parts, one that runs from KTS to K + gamma TS over which the signal is piecewise is constant at UK –M -1, and the next integral runs from K + gamma to K+1TS over which the signal input is constant at UK-M, right.

And therefore you have a split integral and 2 B's here, of course to bring it to a standard state space form you will have to do additional work, that facts will remain so whether its integer delay or fractional delay, we know very well to bring it to standard space form, generally we may have to introduce additional states. And as I said yesterday when you convert the state space model to transfer function form then numerator will invariably have this factor here beta 1 times E to the –M -1 + beta, sorry Z to the –M-1 + beta 2 times Z to the –M, and that tells us whenever I have a fractional delay I will invariably introduce 1 zero into the transfer function.

And I have also said yesterday that you can arrive at this result which that is that you will have this factor directly using modified Z transforms, that mean you don't have to go through the state space route as well, remember we discussed the transfer function approach in that you will have to replace wherever you get Z transforms with what are known as modified Z transforms which we don't discuss in this course, in a pure system CD course, we would discuss that in detail, and as I said modified Z transforms are Z transforms that are introduced to handle shifts in discrete signals where the shifts are actually fractional multiples of the sampling interval, they are not integer multiples.

Okay so this kind of completes the discretization discussion that we need to have for system identification, sorry, and we also learnt briefly and we discussed briefly the rivers mapping that is going back to continuous time from discrete time, and at that point we did say that discretize systems always will have poles in the half of the semi-circle, that means the real part will always be positive, generally that is true only when you're looking at you know continuous time systems that have real poles, if continuous time systems have complex poles, then it's hard to say, but if you are looking at a continuous time system with real poles, then you should expect to have discretized systems to have poles only in the half of the semi-circle, and by the way there is a MATLAB command called D2C which allows you to go back from continuous time to, sorry discrete time to continuous time, and I just run one sample instance of a this morning, so I just want to show you, so I have just created a transfer function,

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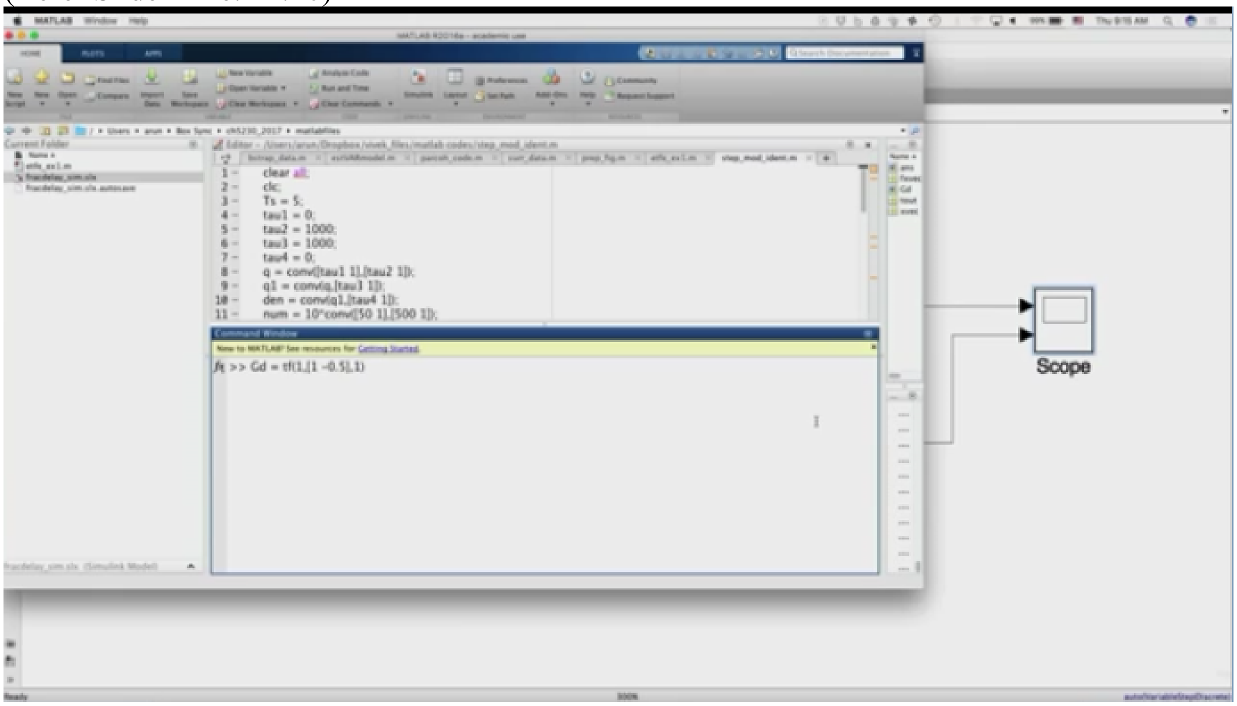

this is a, let me change the font size, okay, so I just ran one instance of this D2C,

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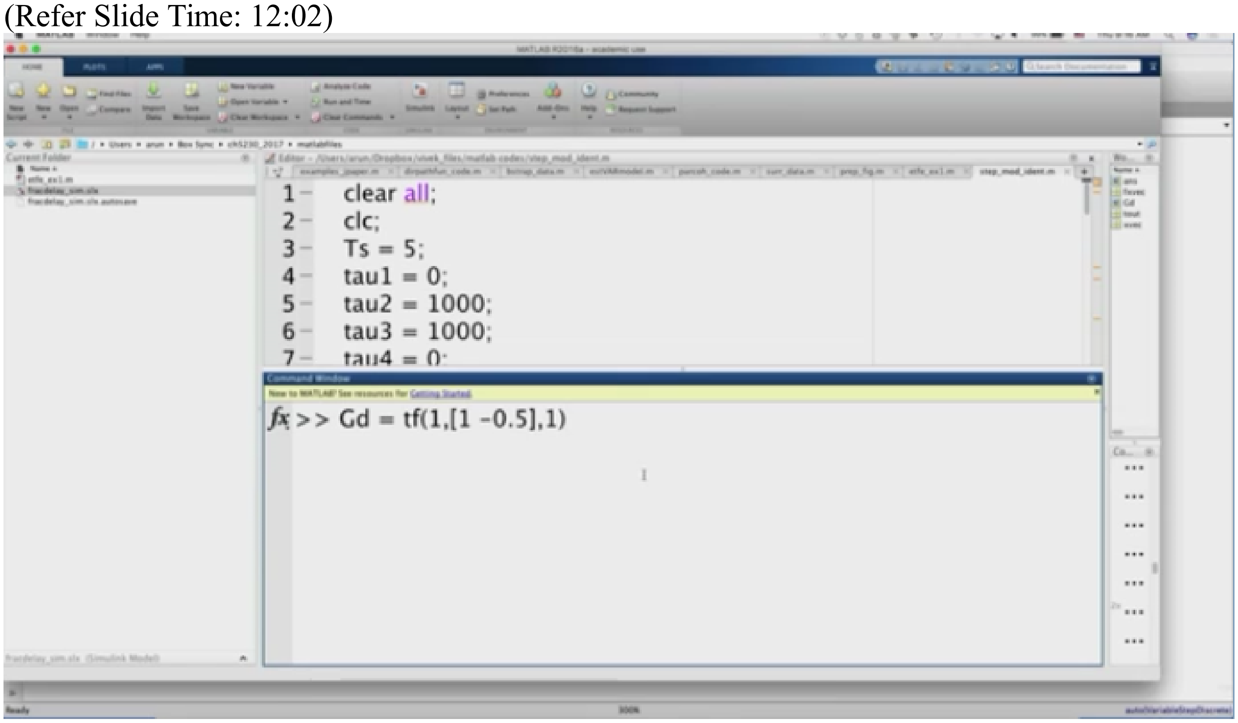

so here is the first order transfer function with pole at 0.5 and therefore I should expect a continuous time system equivalent to this, right.

And so this is my transfer function and I would like to know what is equivalent continuous time transfer function, and I have to mention the method again because I have to tell what was a mapping that was used to arrive at this discretize transfer function, obviously that can make a big difference, and here is the transfer function that you get,

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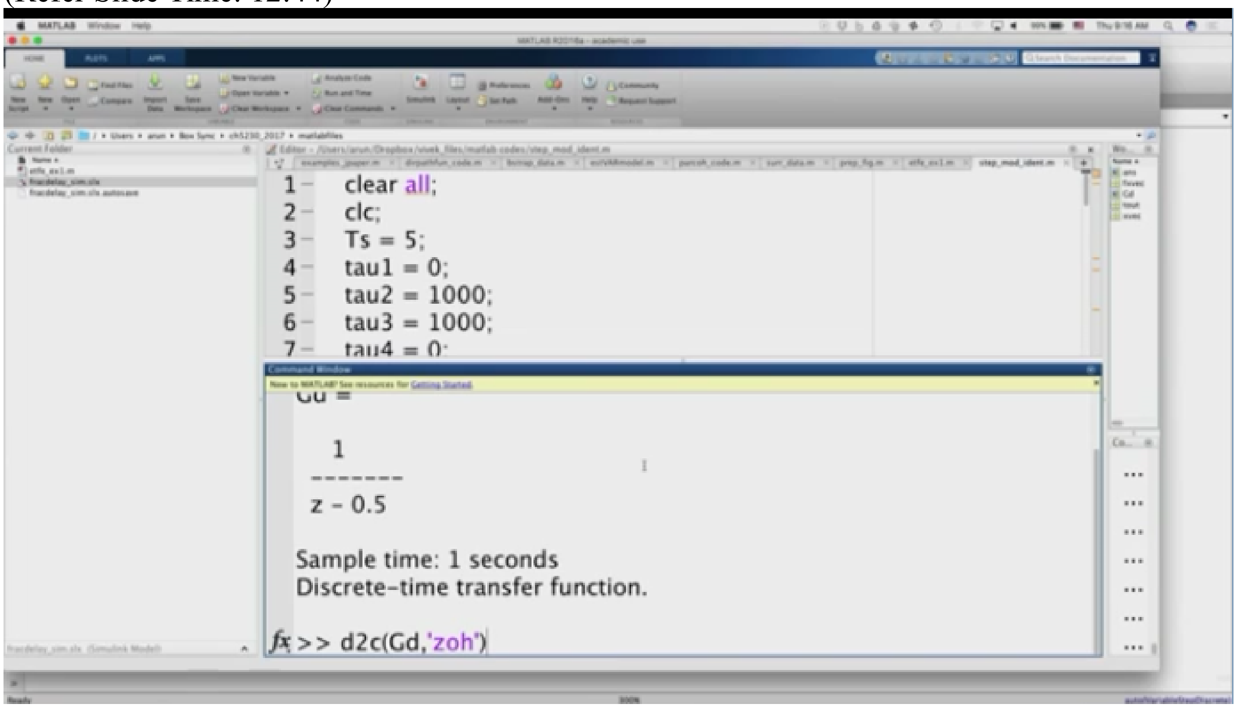

and you should verify if the pole calculation is correct, and the way to do that is take the pole of the continuous time system, we have set the sampling interval to one, so the pole is at 0.5, right.

What if I change the sign of the pole here? Which means now we’re looking at a discretize system with the pole in the left semi-circle, left half of the semi-circle, and as we discussed yesterday, we will run into an issue, because we can’t find an equivalent continuous time system that has the same number of poles and also has real valued poles, so when I ask MATLAB to do the inversion for me or the you know reverse mapping for me obviously you should be prepared to receive some scolding, some it will reprimand you for doing this, it will also warn you, but you ignore, so you can see there are bunch of warnings here, and it gives you this transfer function.

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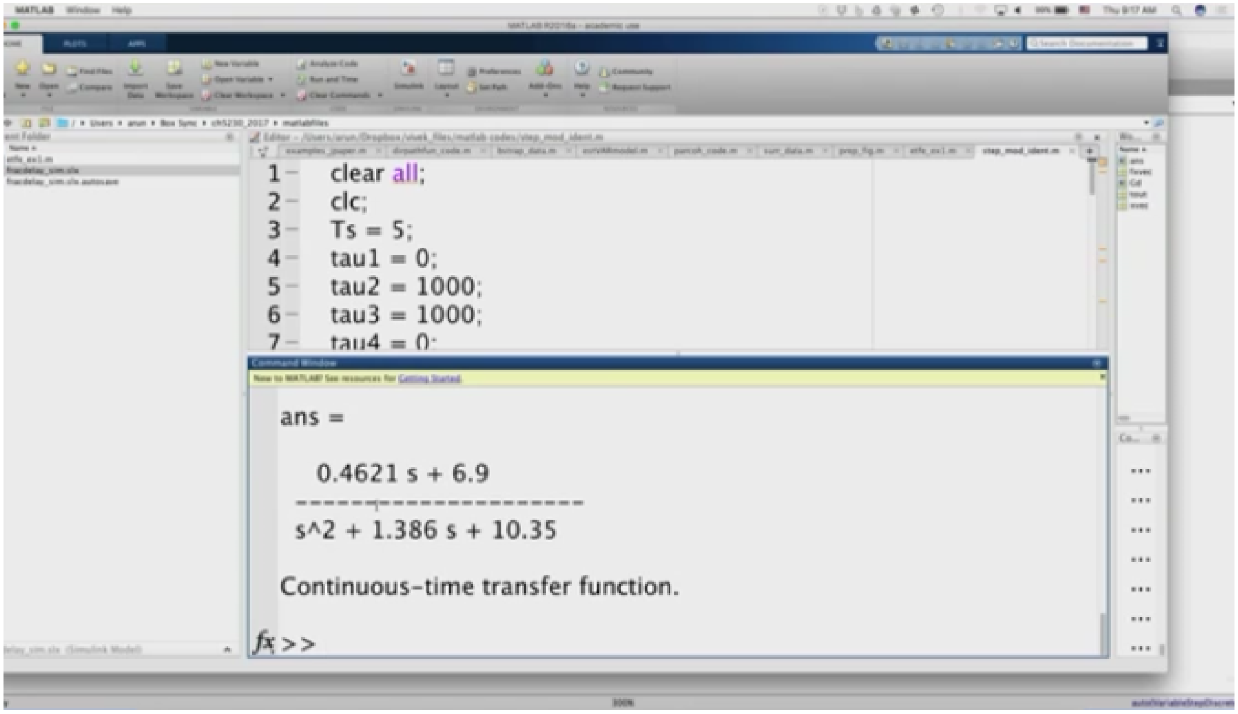

Why do you think it gave you a second order with a zero? If you look at the pole, so let's assign this to some GC, okay, got scolded no problem, now let's look at the pole,

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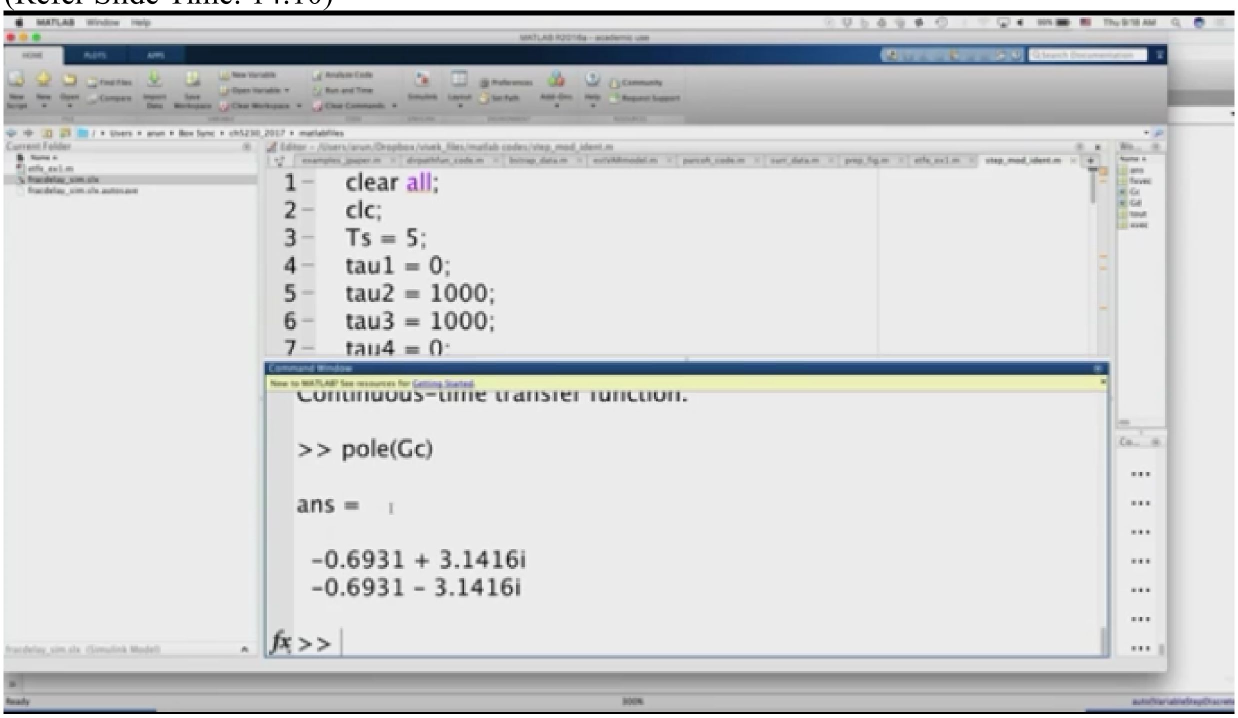

what has happened? Conjugate pole, it was not willing to give me a single pole with complex value pole, correct, so now why do you think I have a zero also here, because we know very well, does it cancel? It doesn't no? But it says that's what, you will get in life if you ask weird things, it says the relative degrees 1, I'll make sure relative, what is relative degree? Number of poles – number of zeros, it says relative order is 1 absolute order is 2 because I cannot give you a continuous time system with a single pole that is complex value.

What about gain? Is gain preserved? Is this correct?

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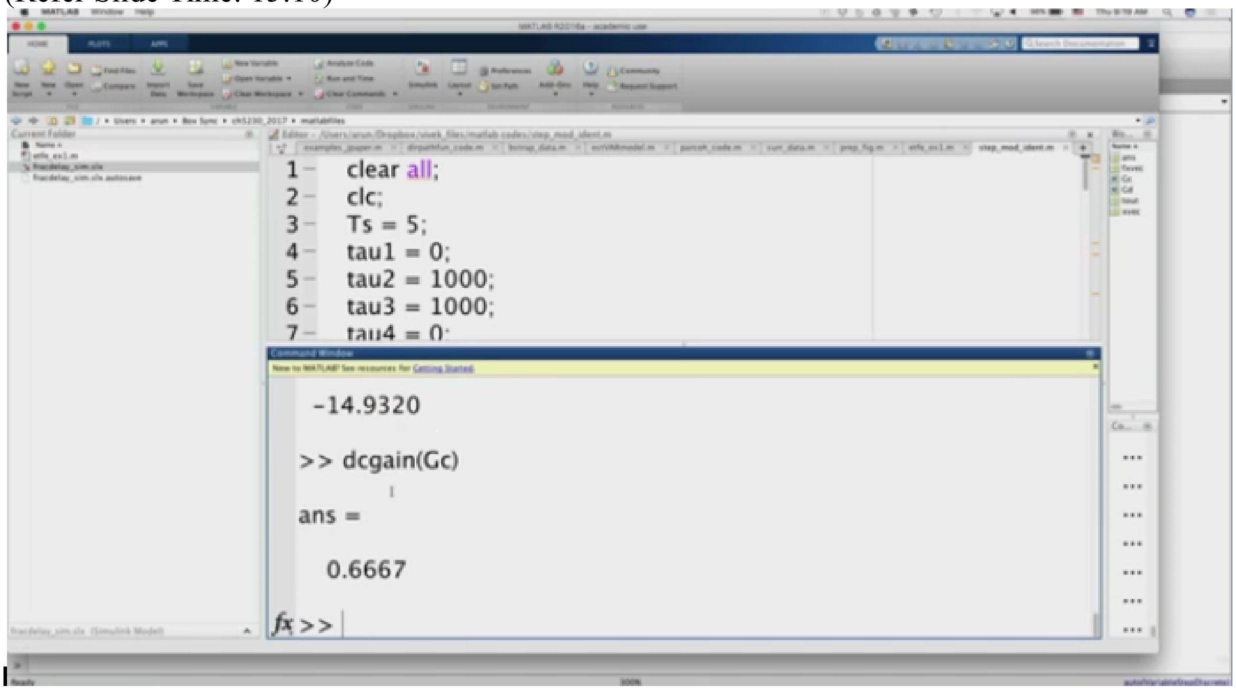

What is the gain of the discrete time system that we used? Should be 2, right? You said 2 because the pole is there,

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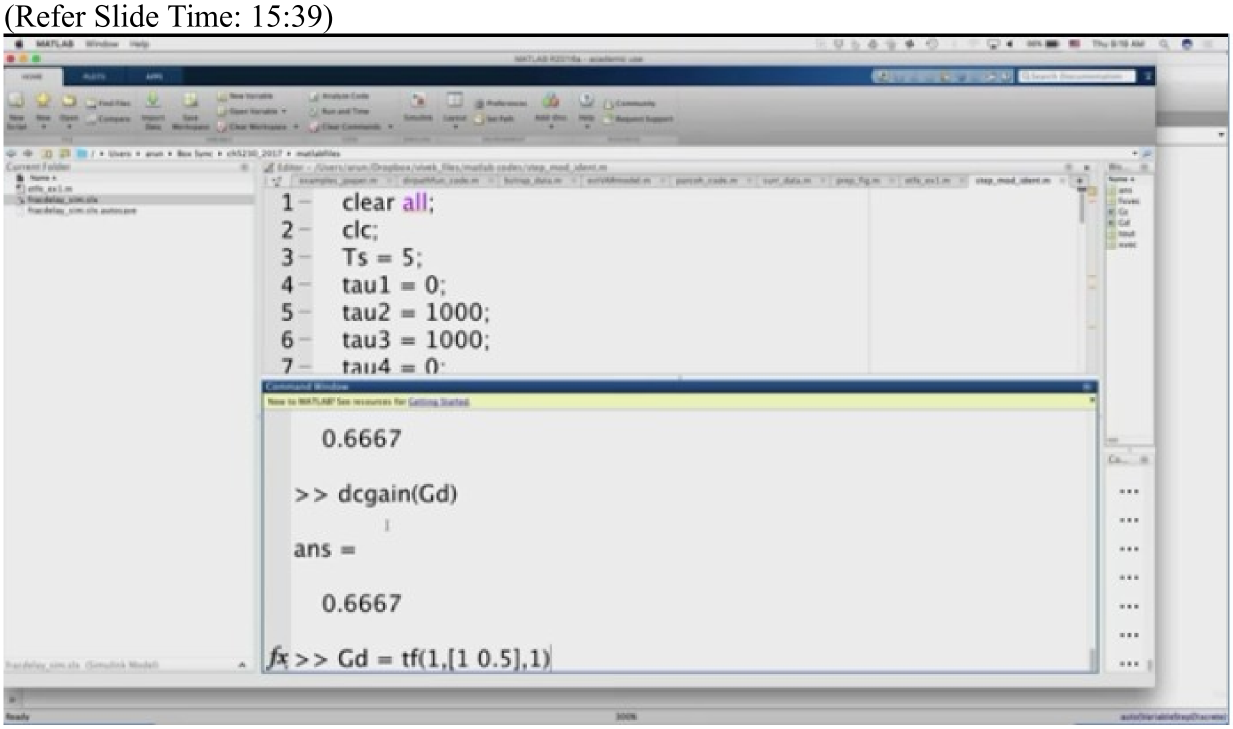

you thought the pole is at 0.5, the pole is at -0.5, what happens? You have 1/Z+0.5, right? Not Z, -0.5, so what's the gain? 1/1.5, right exactly, 2/3, so the gains are matched, it says as far as the gain is concerned I'll make sure it's matched, but don't ask me for more than that, this is the best I can get, and we can try even with 2 real valued poles, so here let's pick 0.7 and 0.4, but we'll change the sign, we'll say -0.7 and -0.4, right which means this coefficient should be 1, 1.1, 0.28 the denominator, so we call this as GD2,

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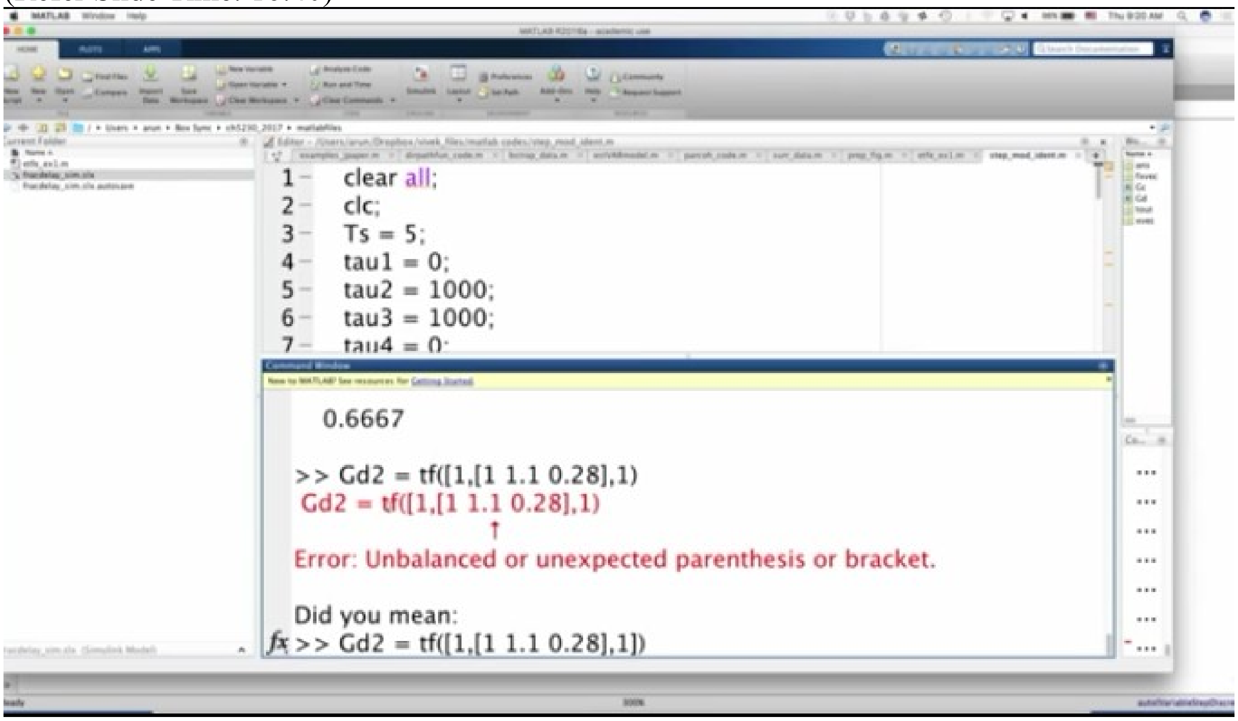

okay, fine. No no no, something is wrong,

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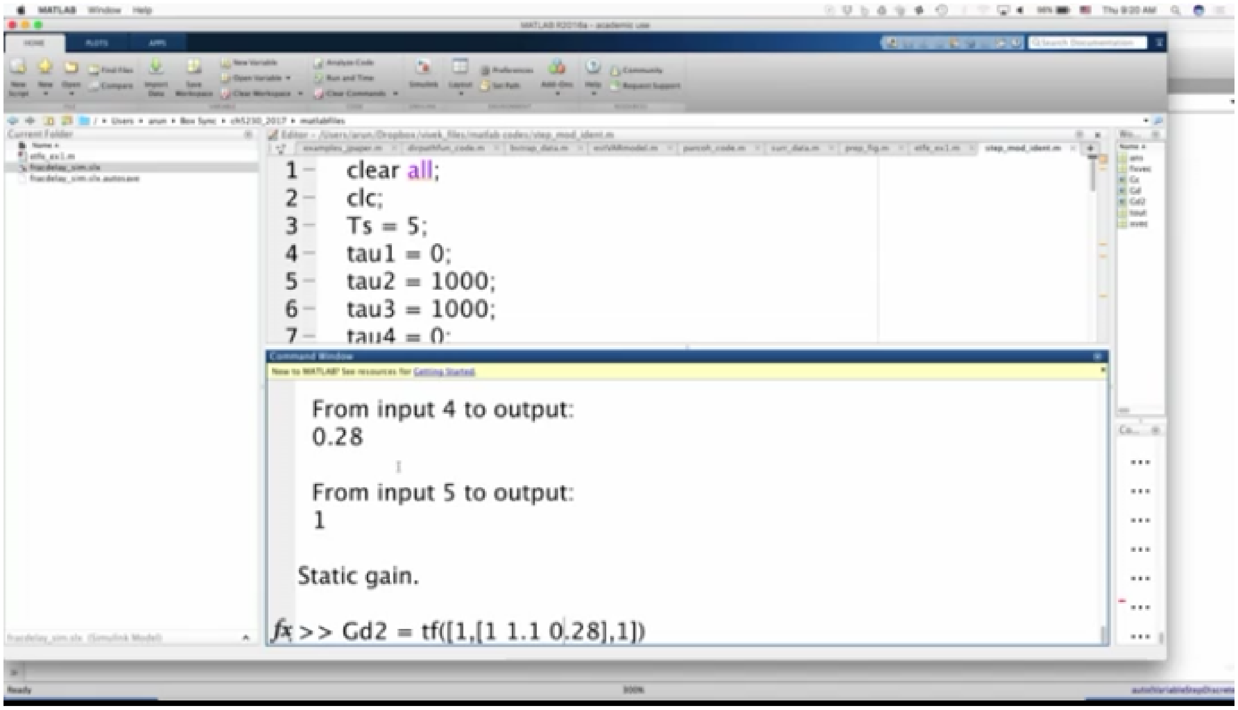

sorry. Oh oh sorry, okay, so this is the transfer function obviously we should be prepare to get some weird answers, yeah, so that was warnings and I'm happy about it.

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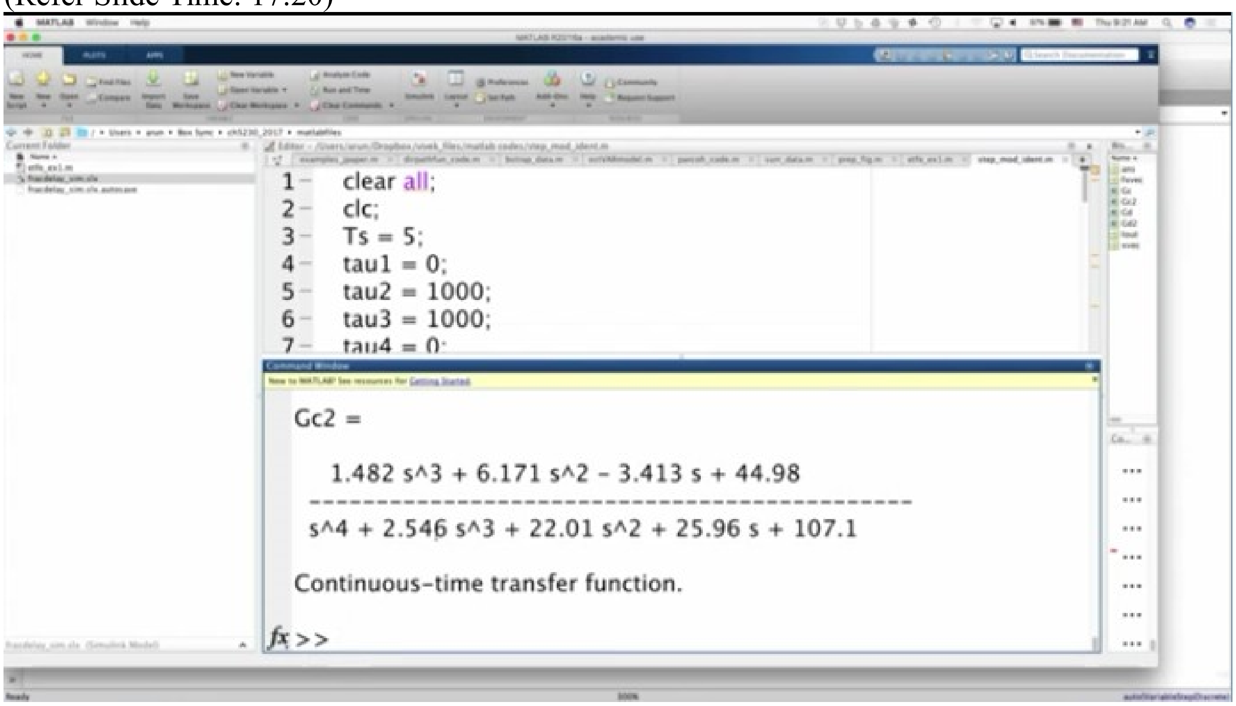

Now what happens? What is the order of the numerator polynomial? It's a cubic one, and the denominator is what, here the related degree is not maintained, right, so it's not really consistent, I do not know exactly what it does when it runs into this thing, I'll have to look at the documentation, I have not studied extensively D2C and we rarely use this, but I just want to show you that there is a routine and you have to use it with caution, so now we can ask once again what are the poles of this, right,

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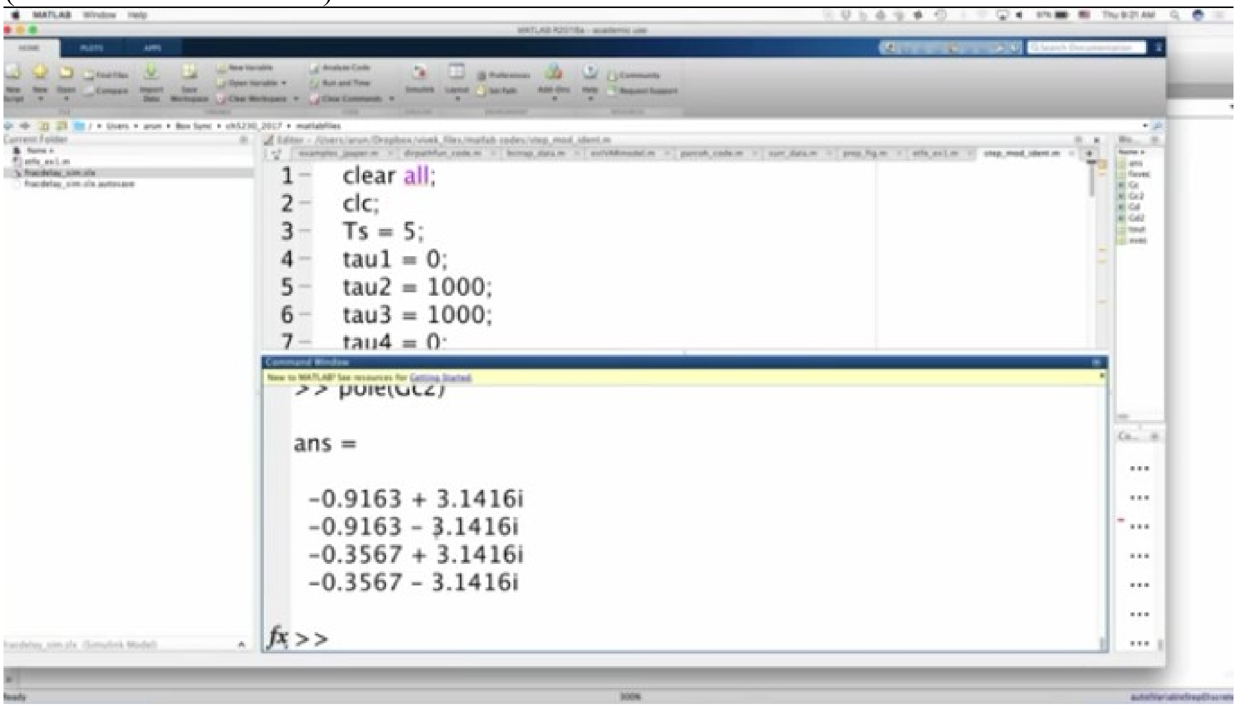

so these are the poles and you can see that the imaginary part is the pi, and that has come about because of this negative valued poles that you are looking at.

And the pi comes about because there is a negative sign, how do you represent a negative number as a complex value, complex number, so that's where you are running to this pi, did we have the same story for the earlier one? Right, so you'll always have this pi, and that pi as the imaginary part of the pole, that's because you have to represent a negative number as a complex number and you will have to E to the J pi, okay, so that is the story that you will run into weird systems when you have a real valued poles, we could actually pick poles that are complex but with still negative real value, and we'll do that as a final example and then move on, okay.

So let's pick for example here 1/(Z-P1)(Z-P1 star) and we can pick values for P1 as let's say -0.5 + or – J, imaginary right 0.5J so P12,

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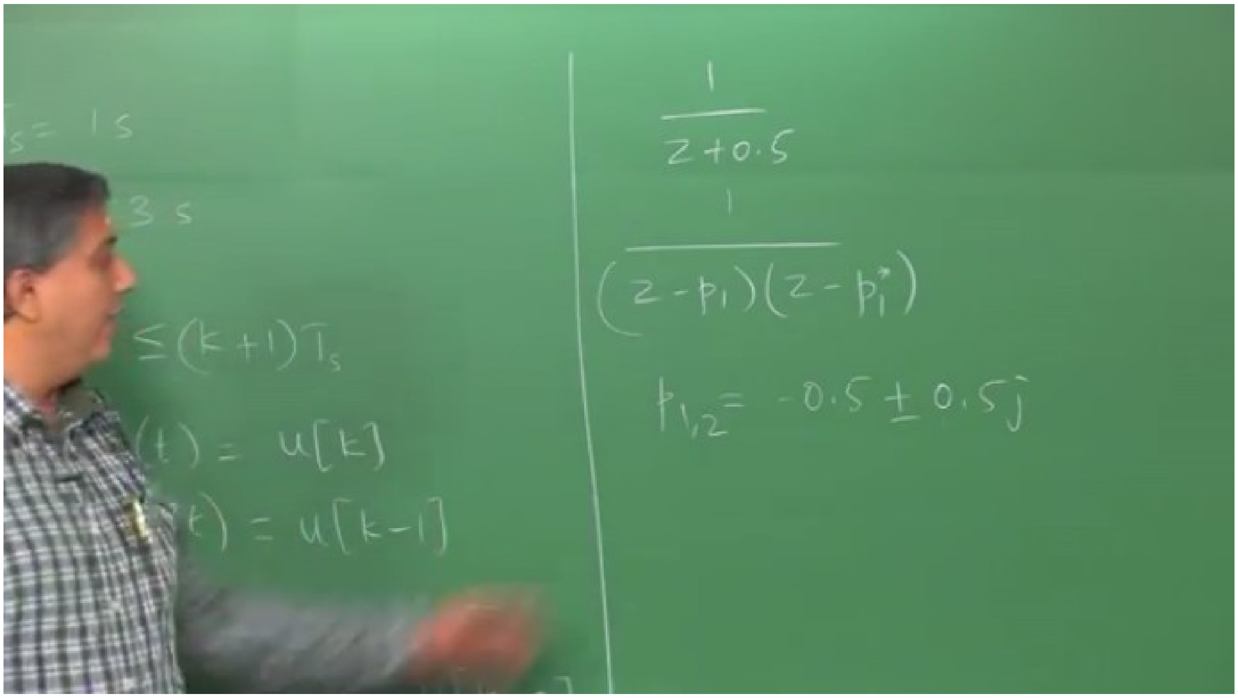

so let's say these are the poles of the discrete time system, so what would the coefficient? 1, and then it would be 1, why? -P1 + P2, P1 + P2 is -1, right, what about the product right? P1 times P2, 0 point, sure? Okay.

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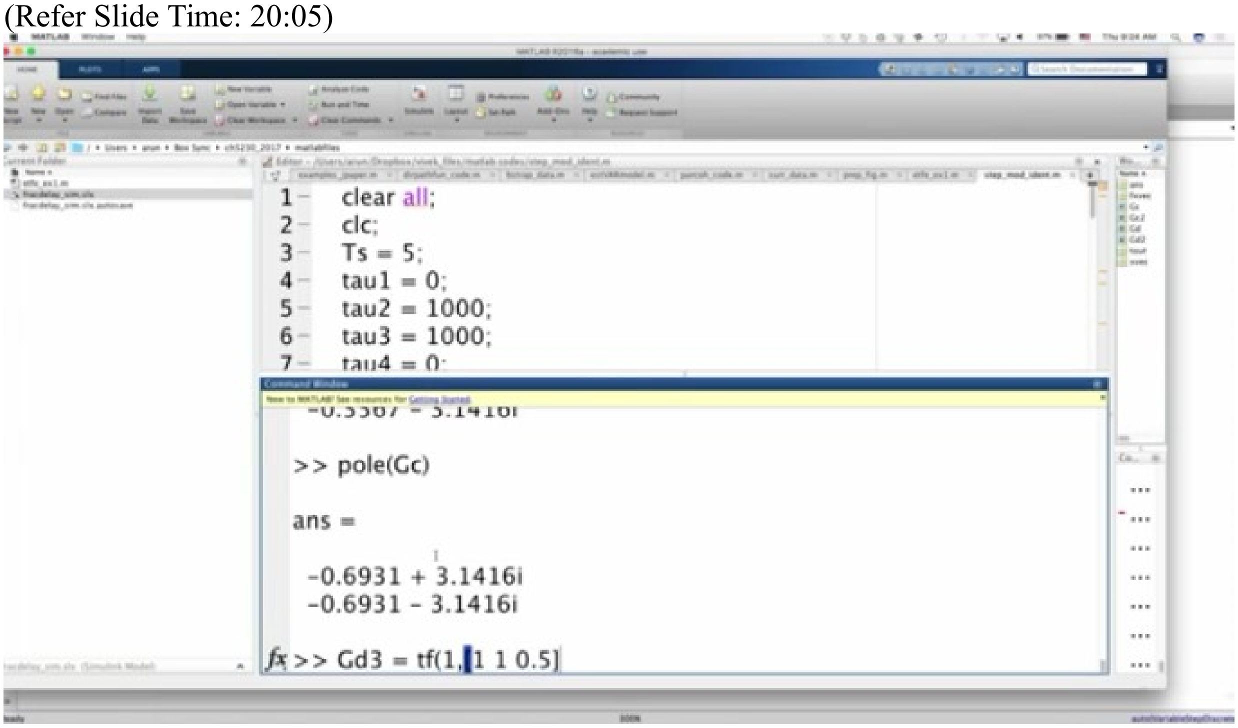

So let's make sure that it is indeed what we wanted to create, fine, now once again, oops,

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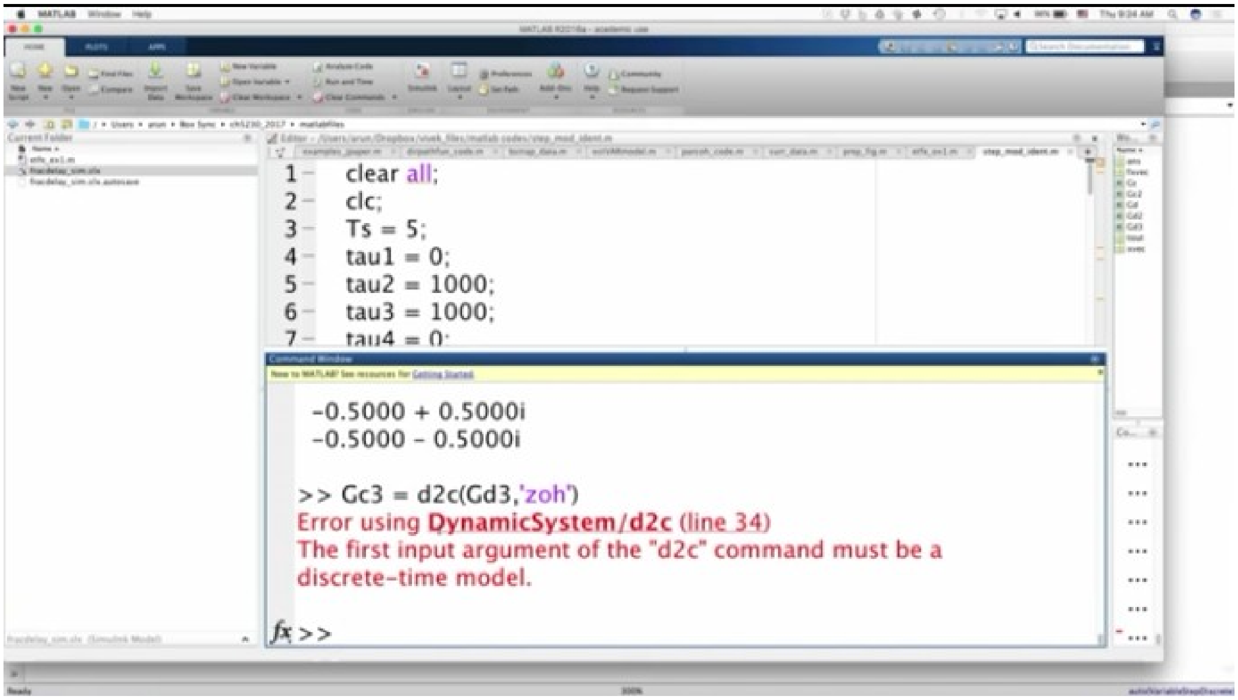

oh I forgot, sorry, correct, now it works, did it gave me warnings? No, now why did it work?
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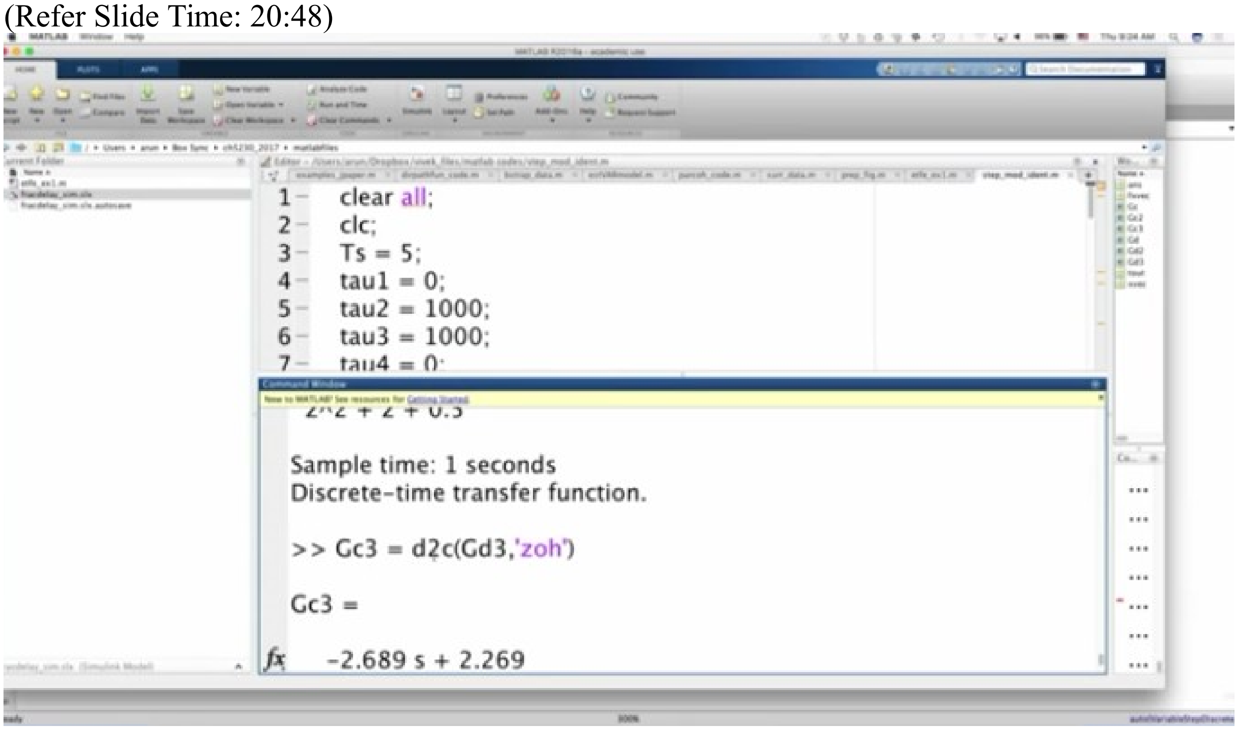

Because the poles are already complex valued so I can represent the complex valued pole with the complex number representation, so it's fine, it's only when I have pure real valued poles that are in the left semi-circle I cannot find an equivalent continuous time system, so here it has given me the right answer, in fact I can check for the poles of this GC3 and ask for let's say,
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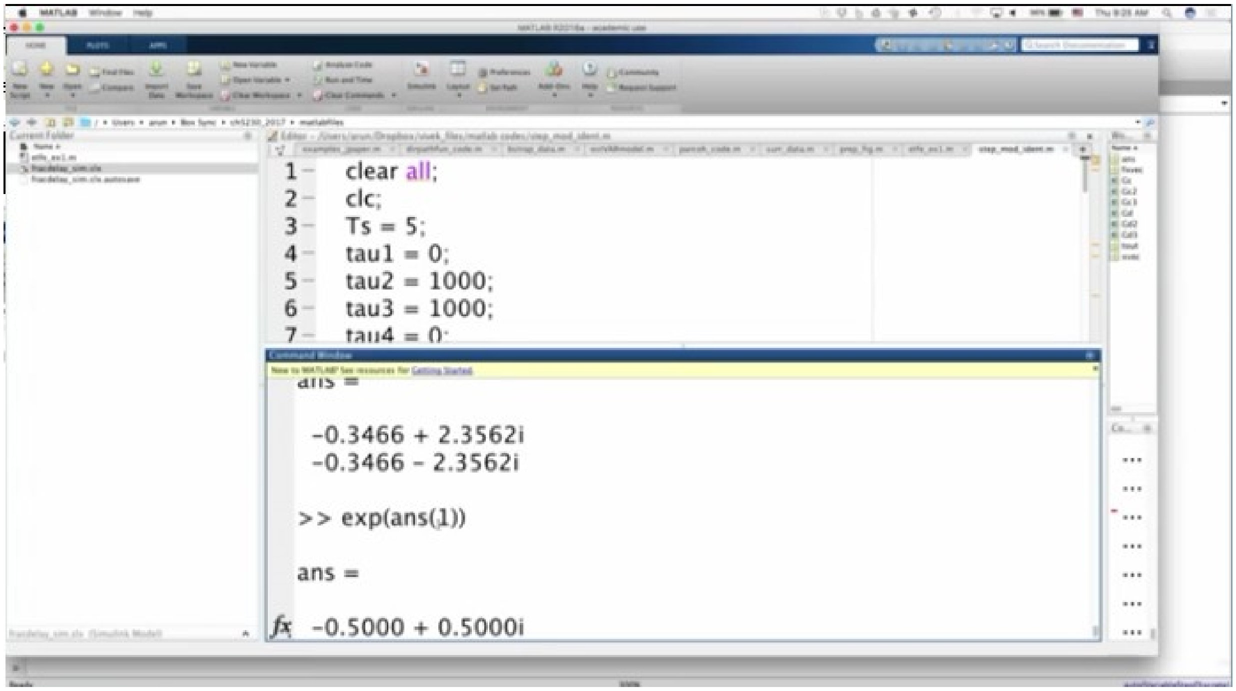

that’s it, right, so I just picked one of the poles and it gave me perfect mapping, so you get the message here when you go back from discrete to continuous, if the discretize system has pure real valued poles and they are negative you cannot find an equivalent continuous time systems, so that’s a story, okay.

And there is a one more thing that I wanted to just give a correction to, I had written an expression for Tustin’s approximation, that’s actually not correct, Tustin’s approximation is arrived at, you must have read in many texts the mapping S to Z,

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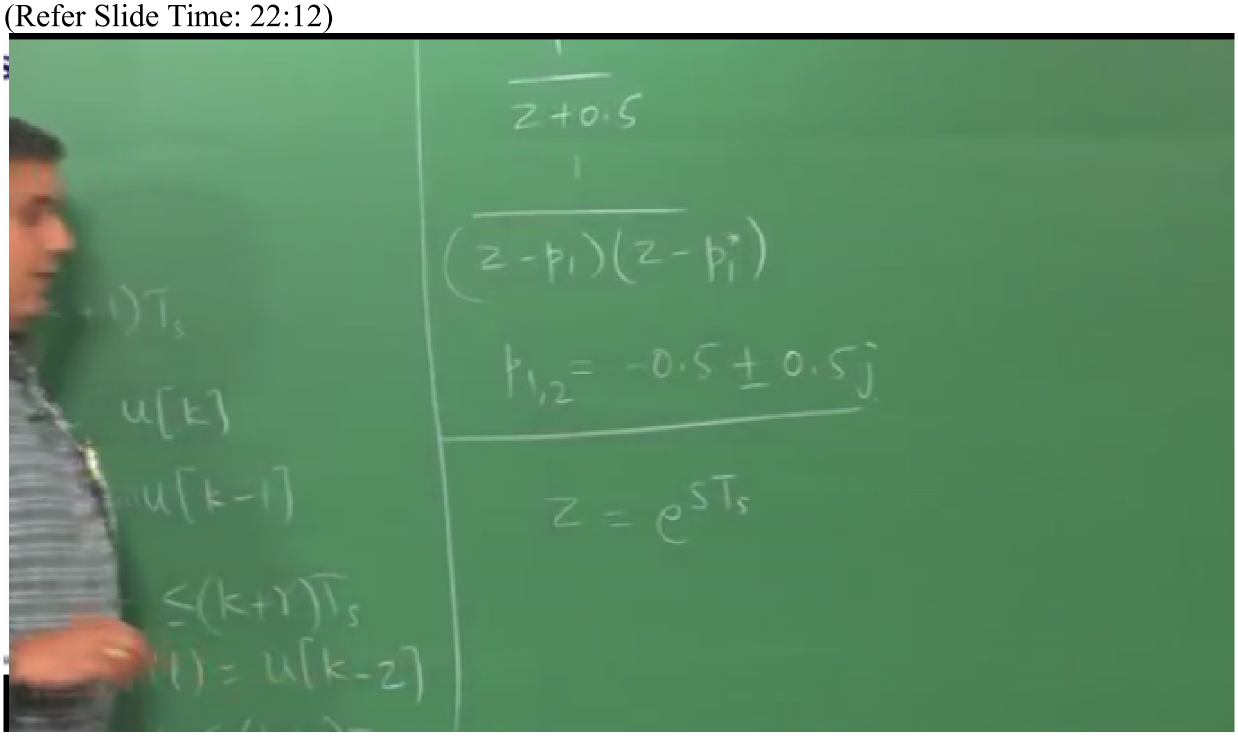

you know in some digital control textbooks or the way you go from S to Z plane and you must have read some kind of approximation sediment, this also fits in nicely with your eigenvalue mapping, if the pole of the continuous time system is lambda C, then lambda D is simply E to the lambda C times TS, Tustin's approximation first write this as E to the S TS/2 over E to the – S TS/2,

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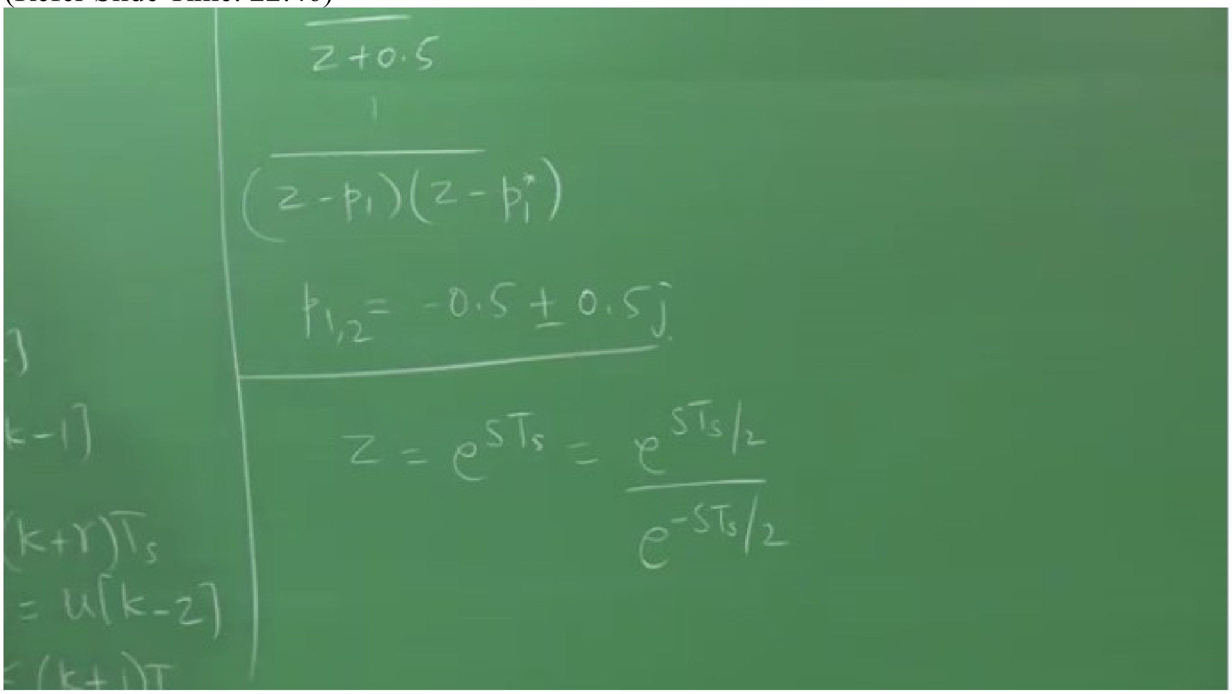

at this stage there is no approximation, and then you introduce a first order approximation here, so you write here 1+S/2TS or if you don't like you can write STS/2 anyway, so 1-S/2TS,

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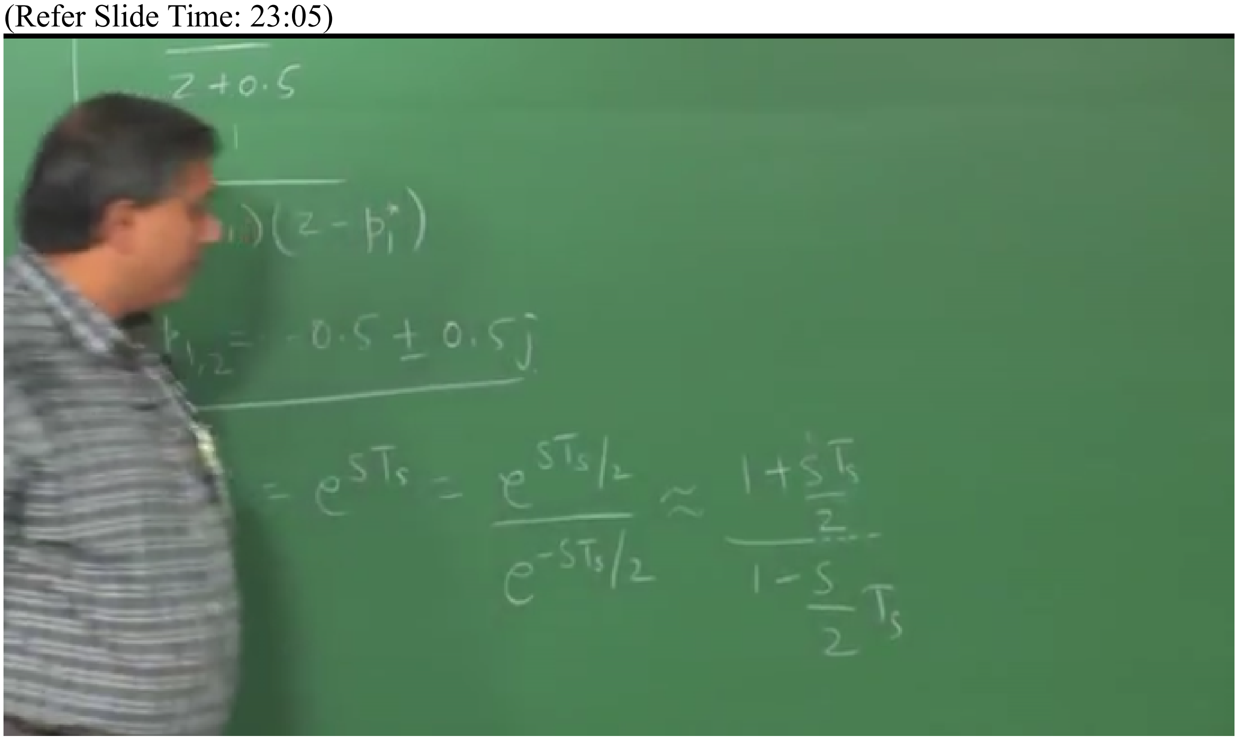

from where you can derive the mapping, I mean you can rewrite this in terms of S to Z, this is S to Z, Z to S also you can rewrite, so if you are given G(s) you can use this approximation to arrive at D approximation, so the expression that I gave you in the previous lecture for Tustin's approximation is wrong, okay.

My textbook does give you the correct expressions, and there couple of other approximation is also the textbook discusses, but we will be only concerned with ZOH in this course. Now let's quickly wind up this entire topic by discussing sampling, so you have learnt how to discretize at a given sampling interval and when they hold devices zero order hold.

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