NPTEL

NPTEL ONLINE COURSE

CH5230: SYSTEM IDENTIFICATION

SAMPLED: DATA SYSTEMS 3

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Okay, very good morning. We'll continue with our discussion on discretization, so just to recap we learned yesterday what is concept of discretization, why we would be interested in discretization. And then we learned a method of arriving at the discretized model from the state phase description of the continuous time process and again I should say that discretization is fairly generic,

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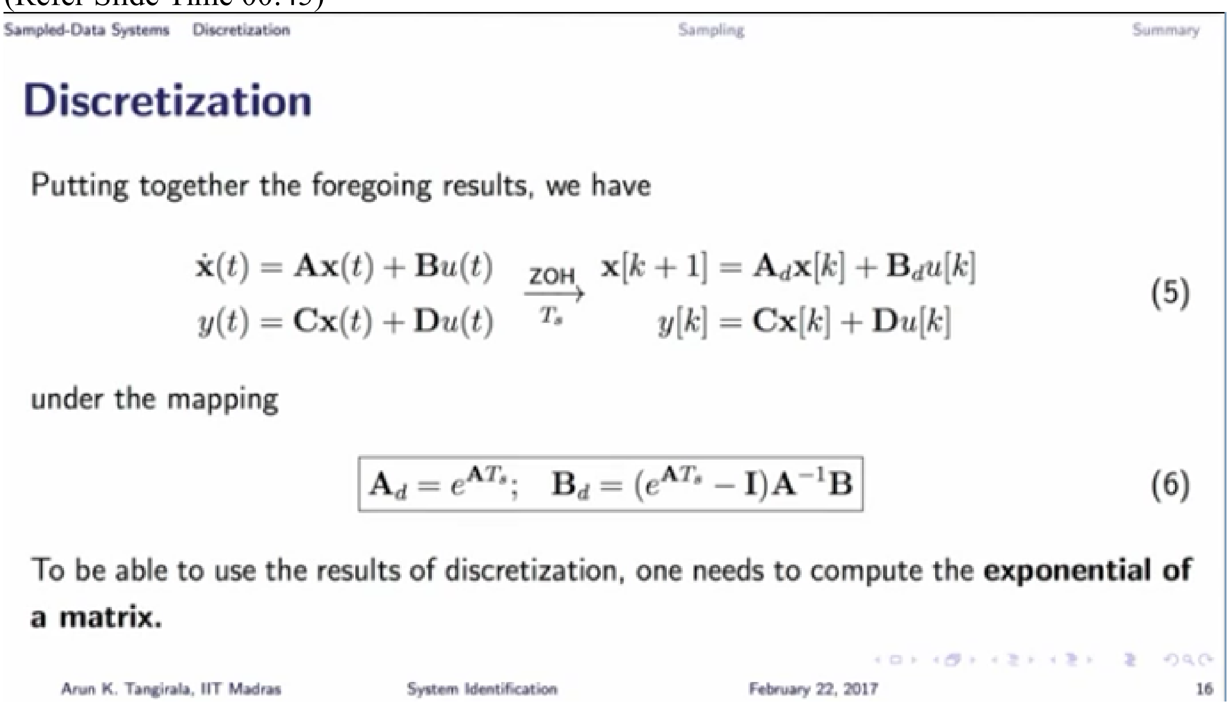

the particular discretization that we are looking at is under the ZOH approximation of the continuous time signal, at the input side, but otherwise as we mentioned yesterday there are many different ways of arriving at the discretized model, but whatever discretization method you come up with, there will be some error involved in it, but maybe for certain class of signals it will be exact, so this discretization that we are discussing is exact for step signals, and that is why this is called a step invariant discretization. And by default this is the discretization that you get to see in the literature.

And in MATLAB you arrive or you compute this discretized model using C2D, and if you were to type the help on C2D to bring up the help on it, okay, so in the mean time we can discuss this MATLAB is firing up. So if you bring up the help on C2D, you will see that there are many options ZOH, FOH and in my textbook you will also see mention of other kinds of discretization, many textbooks will present this as a matter of going from the Laplace domain to the Z domain, so in the continuous time the system is represented by G(s), that is the transfer function. And in the discrete time you have G(z), and many textbooks would say well how do I go from S plane to Z plane, which is different way of saying how do I discretize the underlying differential equation, it's just a different way of saying it.

And one of the things that you will find quite often is this Tustin approximation as well, which gives you another way of moving from the continuous time to the discrete time domain, this is not the ZOH discretization, in this kind of approximation you will arrive at G(z) by replacing S in the transfer function with some relation between S and Z, I'll perhaps talk about it a bit later, but this Tustin approximation is used quite often, not necessarily in arriving at discrete time models of processes but in deriving digital controllers given analog controllers.

See when control began, it began with the analog domain theory and somewhere during the world war and post-world war era when the digital revolution started to happen, people started thinking of digital control and that's where your computer base process control, or computer process control came into practice, at that time there were many questions that were asked and one of the questions that was asked was I have the set of analog controllers with me, how do I move from analog to the digital domain? How do I design a digital controller that achieves a same performance as the existing analog controller, and most of this analog controllers are PID controllers and for those of you who are familiar with PID's, you know PID's are also LTI objects, LTI systems and they have a transfer function description, so you will see in many digital control text books, a class of methods that derive the digital controller starting with analog PID control, when I say controller here I'm talking of PID controllers.

So they start with a analog PID and then move on to a digital PID using one of these approximations, they may not use ZOH discretization because the discretization there is different, right, but all of this qualifies to be called as discretization, any moment from continuous time to discrete time is discretization, here we are looking at ZOH discretization, but if every discretization as I said first thing involves an errors, second thing involves a mapping of the stability region, right, under ZOH discretization what we learnt yesterday is that the left half plane here which is the stability region, right,
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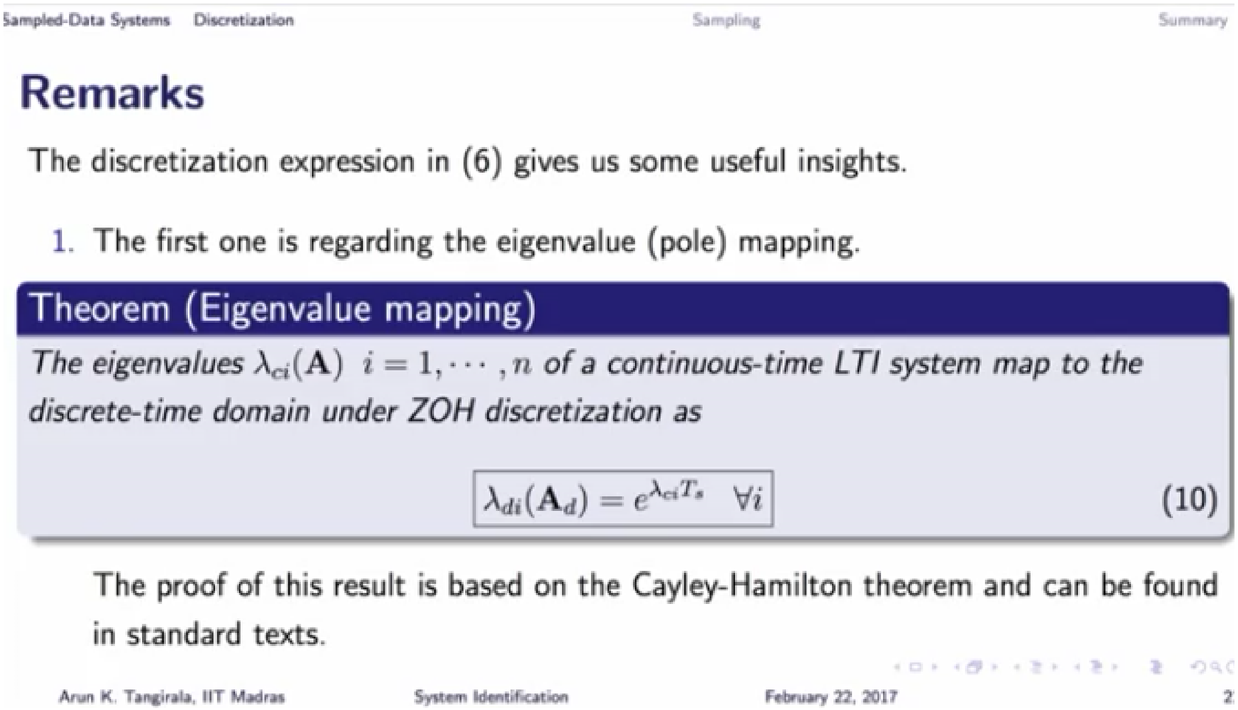

so this is the imaginary axis of in S plane and this is the real axis,
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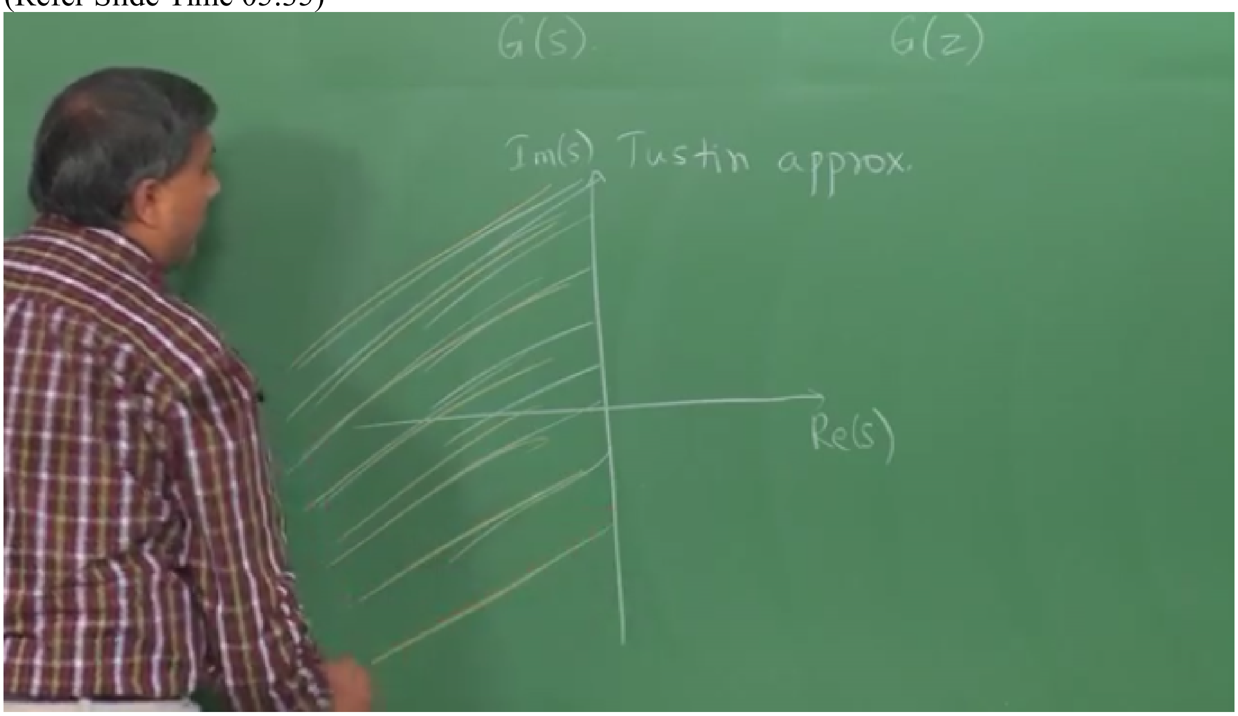

so all of this actually is the stability region, right, for the continuous time system and we know under ZOH mapping, we'll talk about the Tustin approximation bit later if time permits.

Under ZOH mapping the stability region for G(z) given by the eigenvalue mapping theorem, the stability region maps to the unit circle, okay, again this is not such a great circle don't feel unhappy about it,

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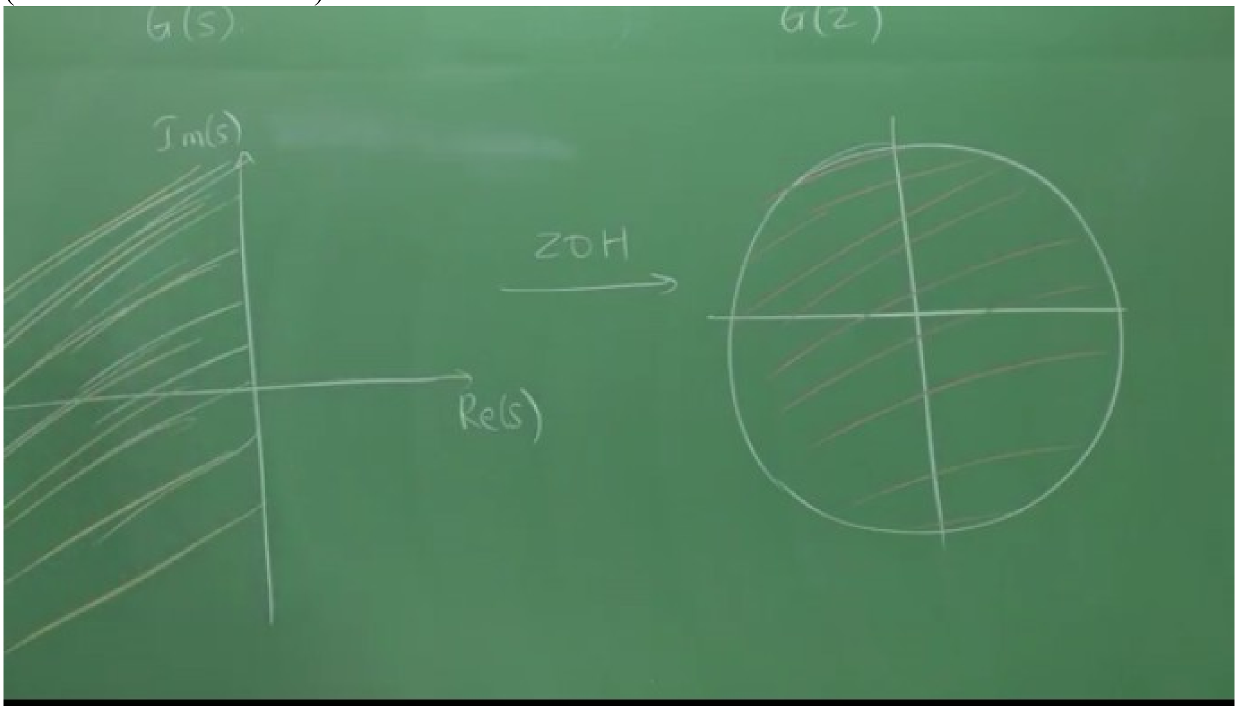

one of the course that I really did not love so much was engineering drawing, okay, so that's your stability region, we can do a post correction here to make it look like a circle and change the axis.

Now is that correct what I have drawn? It's correct or not? Right, sorry, I'm sorry, marginal once we can draw separately, so we can say the imaginary axis here, right, maybe we need a different color, the imaginary axis here maps to, and also remember that's sampling introduces periodicity, okay, that is another thing that you should not forget.

Now the other thing that should be kept in mind is that now you have the mapping, although we have derived it in the state space context we know eigenvalues and poles are the same, so if lambda I is the pole here, E to the lambda I TS is the pole in the discrete time, right.

Now all stable poles have a negative valued real part, therefore what can you say in general about the pole in the discrete time, okay, let me be more specific here, if you look at this semi-circle here, the poles can also take on negative values, sorry the unit circle here, the poles can take negative values as well, right, if you look at the left half of the circle, what do those, so what is this region here correspond to in this S plane?

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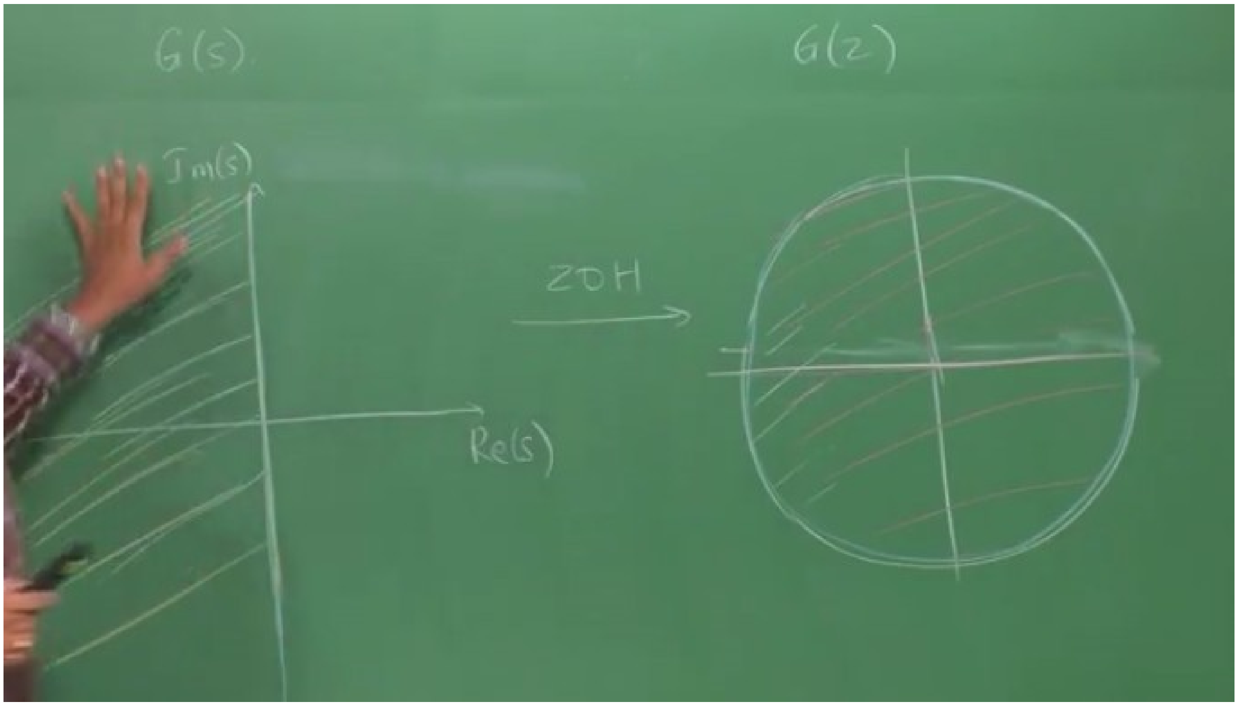

Or it's not possible, or is it possible? In other words let's pick a negative pole here so I have, let us say a discretize system I claim has a pole at -0.5, so now we are talking of the inverse mapping, right, what is the inverse mapping equation? Suppose we call this as lambda D, I mean this is lambda D let us say discrete time, this is lambda C continuous time, so what we are given here is that lambda, there is a single pole at -0.5 for a discretize system that's a claim,

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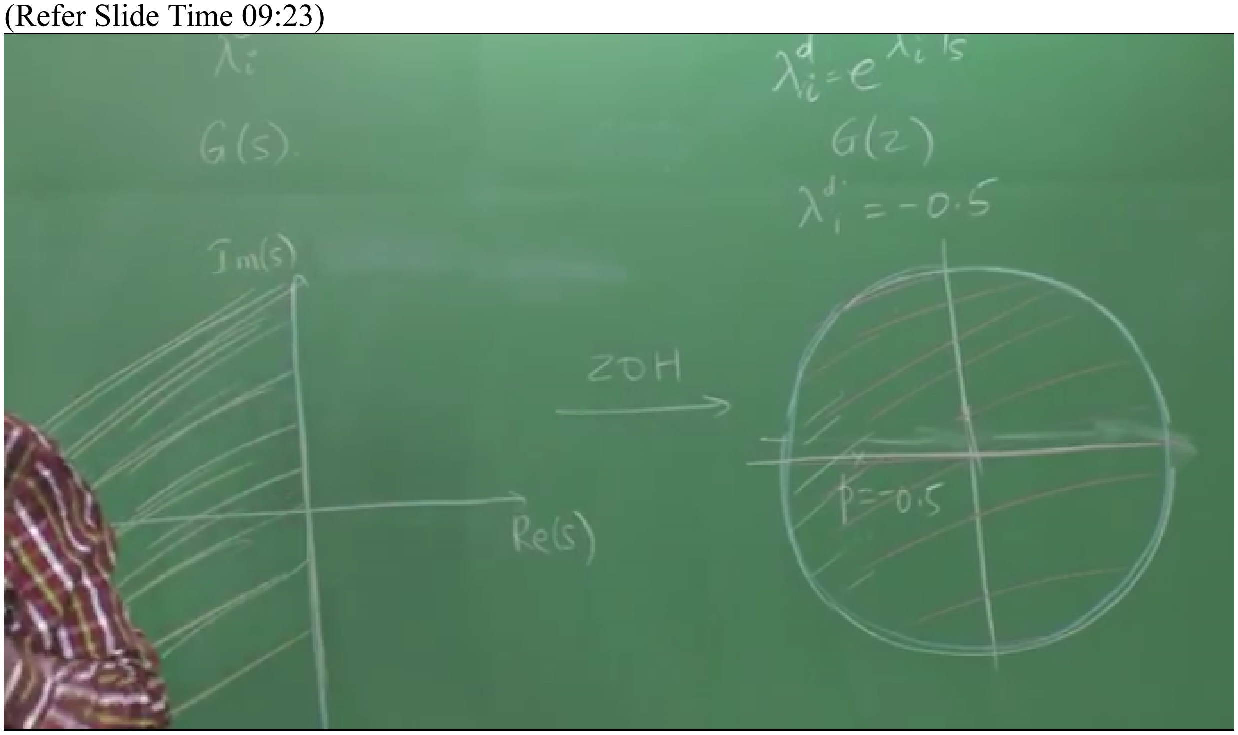

there exists a discretize system whose pole is at -0.5, it's a first order system.

What is the corresponding continuous time pole here? In general what is the inverse mapping? 1/TS, so this is the forward mapping is E to the lambda I/TS, and the reverse mapping or inverse mapping is 1/TS line of lambda DI, okay,

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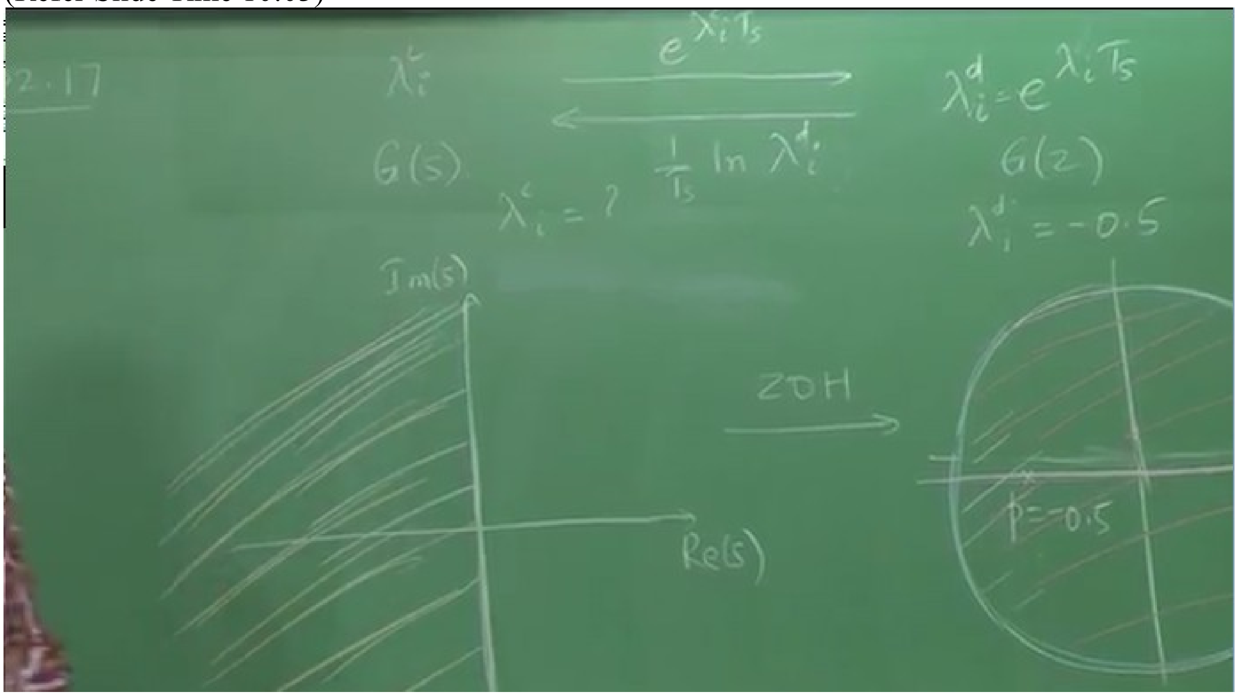

now I have given away most of the answer, why? Okay, so what? Logarithm of a negative number cannot be taken, seriously. Are you very serious about it? You can't take a log of negative number? Now way, you want to think about that answer again? How? So you have to give a complex representation to it, right, I would agree that logarithm of a negative number don't exist when you fear in high school when you say that, if you are not expose to complex numbers, I think these says complex numbers are introduced much earlier just because they thought maps is not complex enough so for high school kids.

Now, correct, so you write a complex representation for -0.5, and then take the logarithm, the result is an imaginary complex valued, right, the imaginary number, so what this means is under ZOH discretization if I have a negative valued pole in the discrete time then the corresponding continuous time system will have imaginary pole, right, complex pole in fact, is that meaningful? Can I have a first order continuous time system with complex pole? Is there a physical system such as that? Does it make sense? What do you think? It's strange, it's weird you can't even imagine a physical system having that, I can imagine a physical system with two complex poles that are conjugate of each other, we know that poles always occur in conjugates, but here there is a single pole, its being maybe divorced from its conjugates somewhere or it's lost in some storm or whatever, but this is single complex valued pole, you cannot have a physical system like that, you can't think, if you get to think of it let me know, but we can't construct a physical system which has a complex valued pole in the continuous time, what does this tell us now?

Yeah, but what does this tell us about this mapping now? Theoretically we say the left of plane maps in the continuous time to the unit circle will be Z plane, but now we have realized any pole in the discrete time with negative real number doesn't have a counterpart here, so strictly speaking under ZOH discretization we should forget this region,

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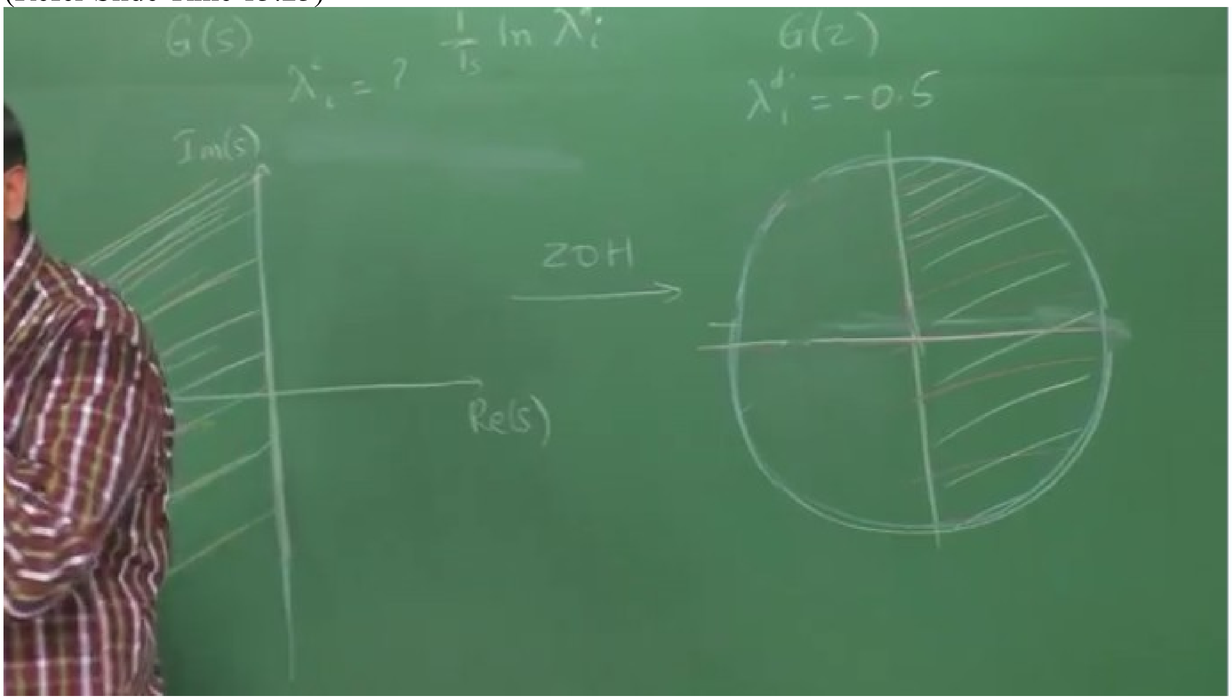

that means there is no way I can have a ZOH discretize system that results in an negative value discrete time pole, try doing that it's not possible, whatever you have here lambdas, you have E to the lambda I/TS right, that is a mapping, whatever negative value here E to the lambda is always going to be positive value, is that clear? So which means you have to be careful when someone claims that there is a discrete that, there is this discrete time system that has been obtained by a ZOH discretization of continuous time process, but this is only true for ZOH discretization, other discretization may involve other kinds of mappings, okay, now there exists naturally discrete time systems that can have negative valued poles, that's not a problem, but that may not correspond that will not correspond to a physically realizable continuous time process and if you claim it is come from a ZOH discretization.

And the stability region still happens to be the unit circle, in general if I give you a discrete time system, the stability region happens to be the unit circle itself, that doesn't change, okay, so this mapping here eigenvalue mapping theorem has a shed light on number of things, at least two important things, one how this stability region maps, this mapping business we have studied in mathematics also, usually we study this in either note 12^{th} standard math or maybe first year undergraduate map, conformal mappings and other kinds of mappings.

So very important to understand how things mapped when you are looking at either a discretization or a transform or whatever kind of thing that you go from one domain to another domain, okay.

So the second thing that has taught as is that there cannot exist as ZOH discretize system with negative value poles, clear or you are still in doubt? Nithya, okay? Yes, correct, no no, no no no, that's a big mistake to say that, so if you look at it, look at that is where you have to distinguish between a discretized discrete time system and the naturally occurring discrete time system, if you are examining at discretized ZOH discretized system or in fact any, in this derivation for AD the mapping that we derived, we have not assume any ZOH discretization, so any kind of discretization will result in this mapping, only the B part changes depending on whether you're using ZOH or not, so only when you're discretizing this is the story, so what is a question now?

Oh I see, okay, I understand what you are saying, correct, so yeah, you're perfectly right in that sense, correct, so we can extend this observation to even other kinds of discretization that's the point that he's trying to make, because and that's a very good point, because the expression that we have derived for AD did not make use of the ZOH at all, so we will embellish our statement further saying any kind of discretization that you are looking at can never result in a system with negative value poles, okay, thank you, good.

Alright, so now of course the discretization will have an impact on the zero locations and so on and that takes care I mean that is reflected in your B matrix and so on, we'll learn a few more things as we go along. The second part that we should remember is that now this is true under ZOH discretization that the gain is preserved, so the first result that we learnt is how the poles map, and the pole mapping sheds light on many things, one how the stability region maps, two what kind of discretize systems are possible, that means with what kind of poles, and thirdly as I said yesterday it will guide us on how to choose a sampling interval, which will talk about a bit later.

So the second result here is that the gain is preserved under a ZOH approximation which is very nice to know.
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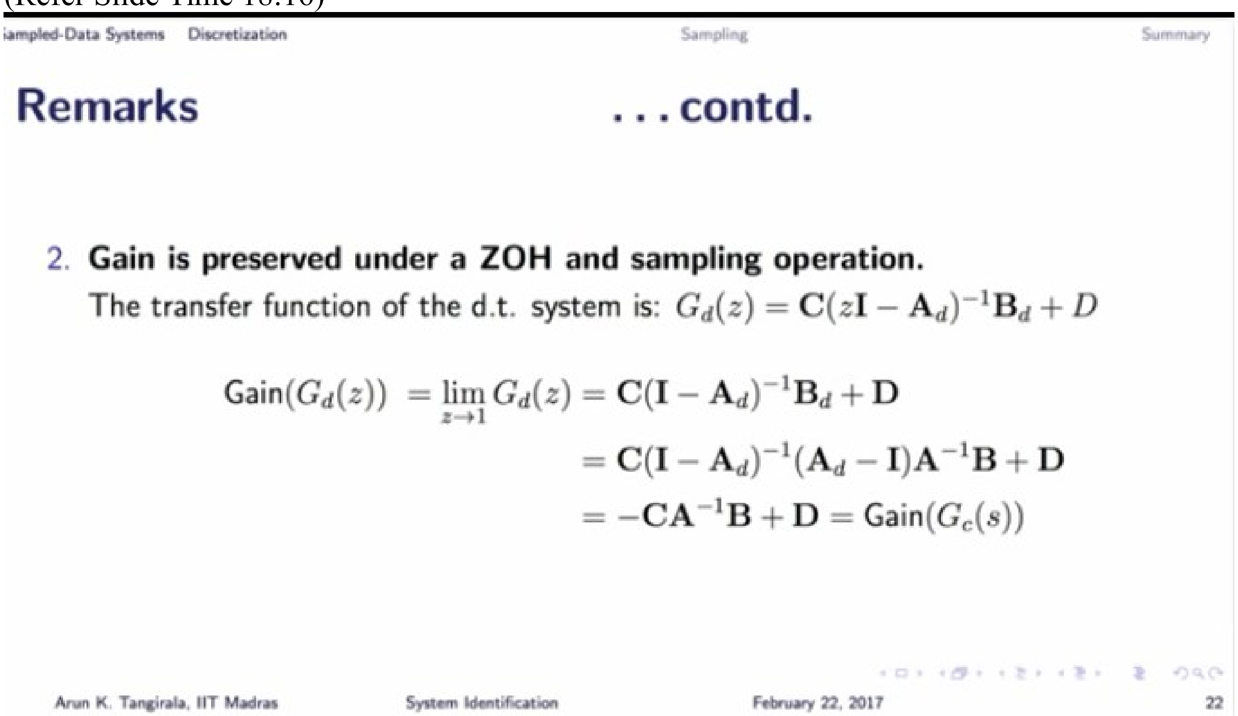

If this is not the case then I have to keep correcting for the gain all the time in digital systems, so whenever I use a ZOH and the sampler together only the gain is preserved, you can't really take out, you can't say only sampler and continuous time, you can't discuss that you have to talk about the three elements together, and the proof is fairly straightforward as you see on the screen, this is used in verifying for example if your discretization is correct one, two, to straight away figure out the gain of the continuous time system, so which is very nice to know.

Pole mapping I had to do a reverse mapping here, inverse mapping. Gain there is nothing to map, whatever gain I identify for the discrete time system is the same as the continuous time, so that's good news I don't have to put in any extra effort, so we have talked about poles, we have talked about gain, then there is one more vital statistic which is the zero.

Do the zeros map in the particular way? Unfortunately no, there is no expression that allows us to figure out how zeros map, that is zeros of the continuous time system to the zeros of the discretize system, there are some approximate expressions at very fast sampling rates, but otherwise none, in fact surprisingly a continuous time system may not have a zero, but the discretize system can have a zero, okay, and that phenomenon you get to see as you start dealing with second and higher order systems, first order system it's very nice, for a first order continuous time process you only have a gain and a pole, the first order corresponding discretize system also has a pole, gain is always there anyway let me not say that there is a gain and a pole, there is a single pole for first order in continuous time, and a single pole in for the discretized system as well.

But when it comes to second order continuous time systems the number of poles is always preserved, but the zeros are not necessarily preserved, what does this tell us? It tells us that the way the continuous time system interacts with the input, how? It is wired with the environment is different from the way the discretize system is wired, in fact zeros have another meaning also, if you turn to filtering they will tell you that the filtering theory will tell you that zeros tell you which inputs are being blocked, right, so if I have a continuous time system let's say which has a zero, let us say S-1 and here let's say I have S-2, here S+2 stable system.
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The zero for this continuous same system is located at one.

Earlier I said, just now I said that zeros tell you which inputs are being canceled out or blocked and so on, here to this system suppose I apply an input of E to the T, continuous time input, what is the Laplace transform of E to the T? 1/S-1 so, if I were to determine the response to such a signal, when I say it is blocked it is not really zero tau, but the effect of the input is cancelled out, E to the T is an exponentially growing signal, Karan, so Y(s) is we know G(s) times U(s), I have S-1/S+2 time over 1, S-1,

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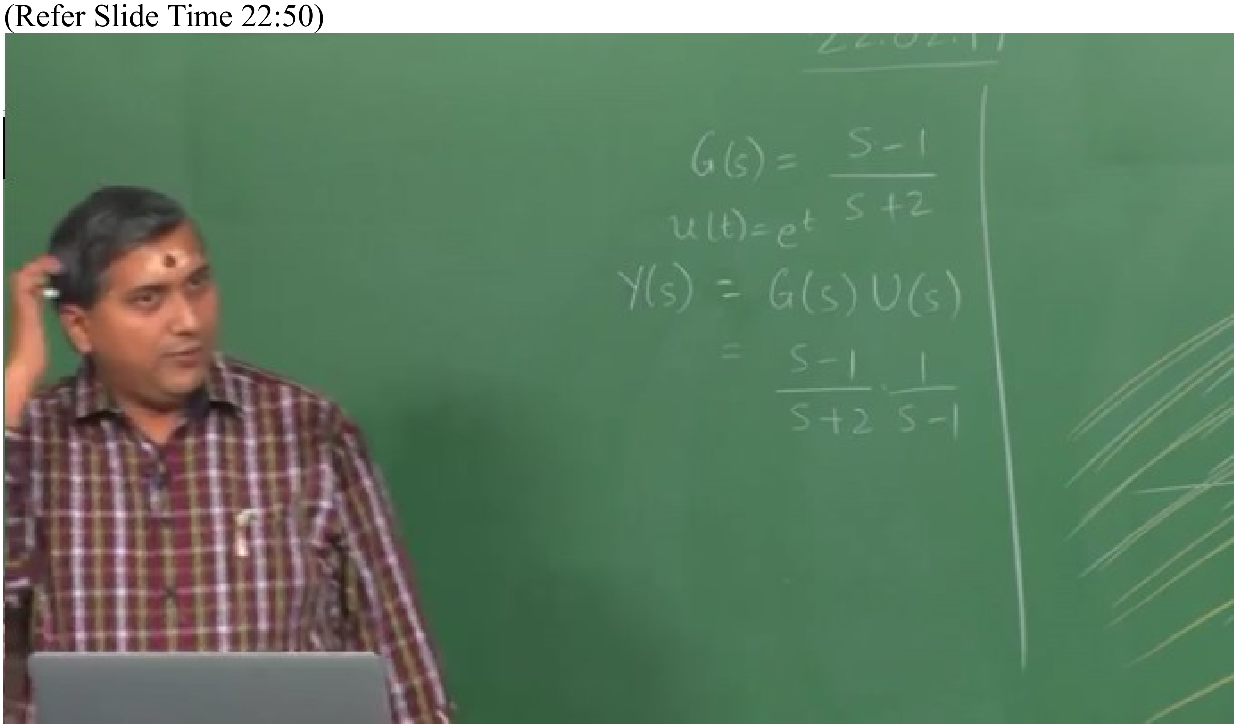

alright, and what the system does is it's able to handle that E to the T very well, and produce in fact you can simulate the system to the E to the T in your, in MATLAB for example, and see what response you get, that's all.

It's a very strange thing, right, it has completely killed the growing characteristics of or this explosive characteristic of E to the T, any other explosive signal it cannot handle, I mean it will produce an unbounded signal, only this input it's able to block, so the zeros have another interpretation in the sense that it kinds of squeezes the juice out of that input completely, and the system will produce its own transient behavior, the same applies to, I mean here I've used S-1, you could also use S+1, same applies to discretized, I mean discrete time systems as well, why are we discussing this? What we are saying here is if the system didn't have a zero for example, it's just 1/S+2,
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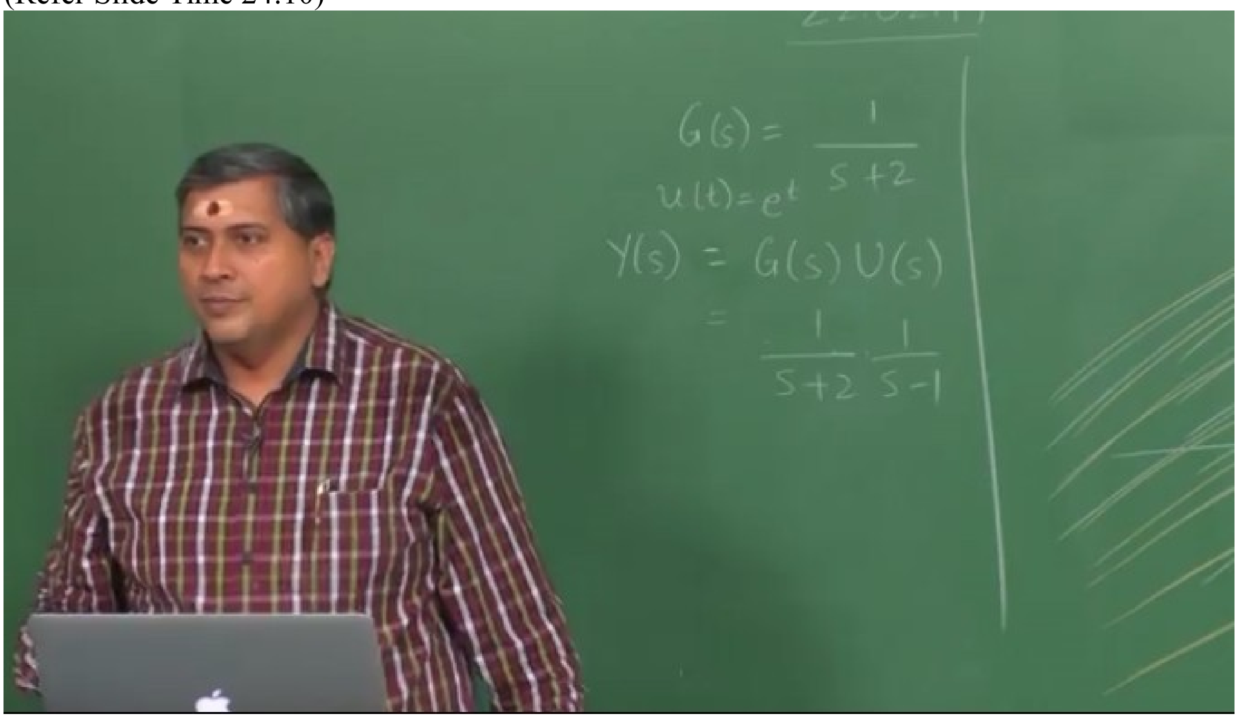

then the system will not be able to do much do anything, whatever the input is given it will reproduce at the output, discrete time system will also behave the same way, because we just now said first order maps to first order, no zeros introduced.

On the other hand, if I have second order continuous time system and I discretize it using this, under this ZOH discretization, I won't keep saying ZOH it's understood we are looking at ZOH discretization,
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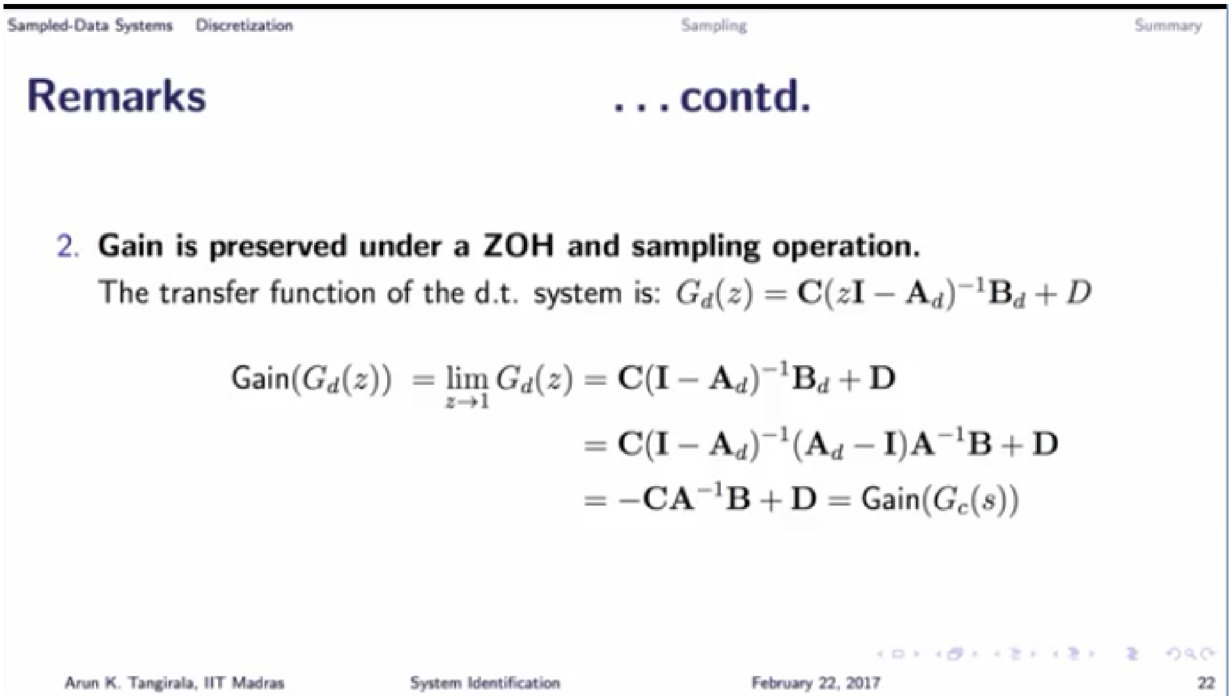

you can find that there is a 0 coming up, I'm not going to go through the theoretical calculation its' very easy to show that you will get it, in fact let's do that, although I'm not going to derive the final answer for you, but it's easy to show, suppose we pick some second order continuous time system, right, and I want to figure out what is the corresponding discretized system under ZOH, how do I do that?

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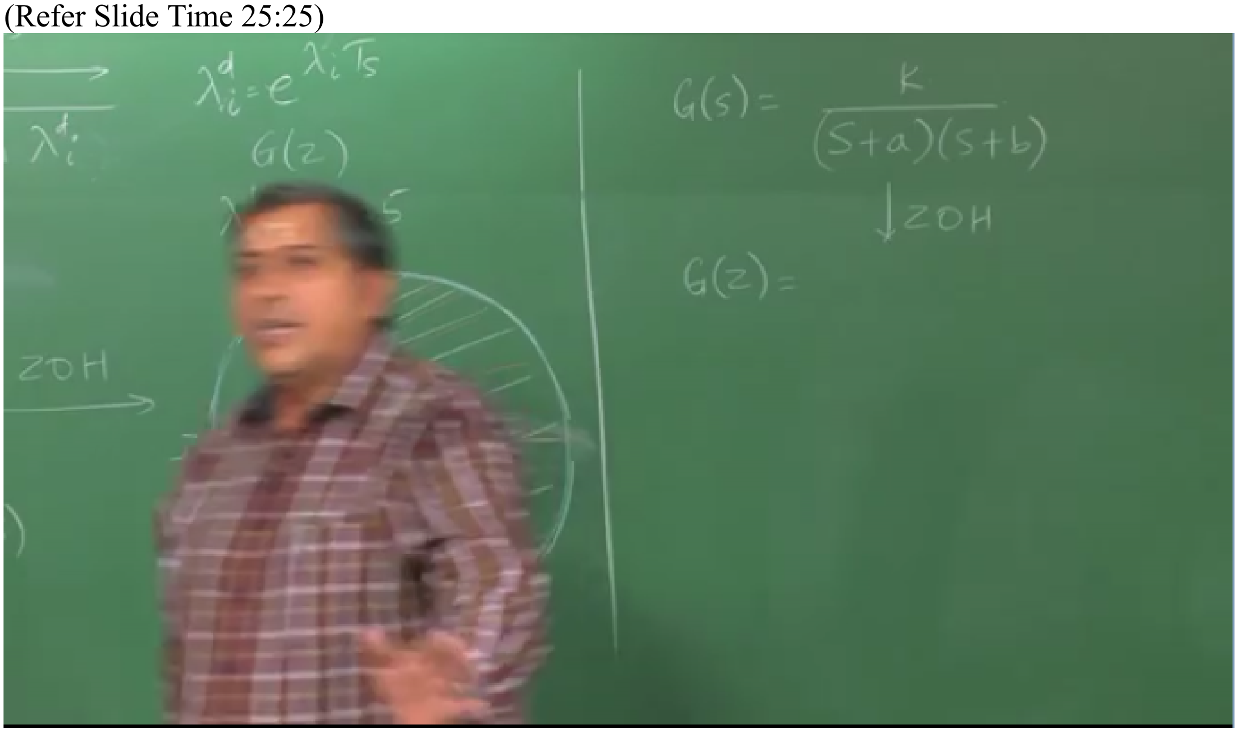

With the method that I have learned what will I do? I'll write a state space description for this, and then use the state space formula, and then from the state space I arrive at G(z),

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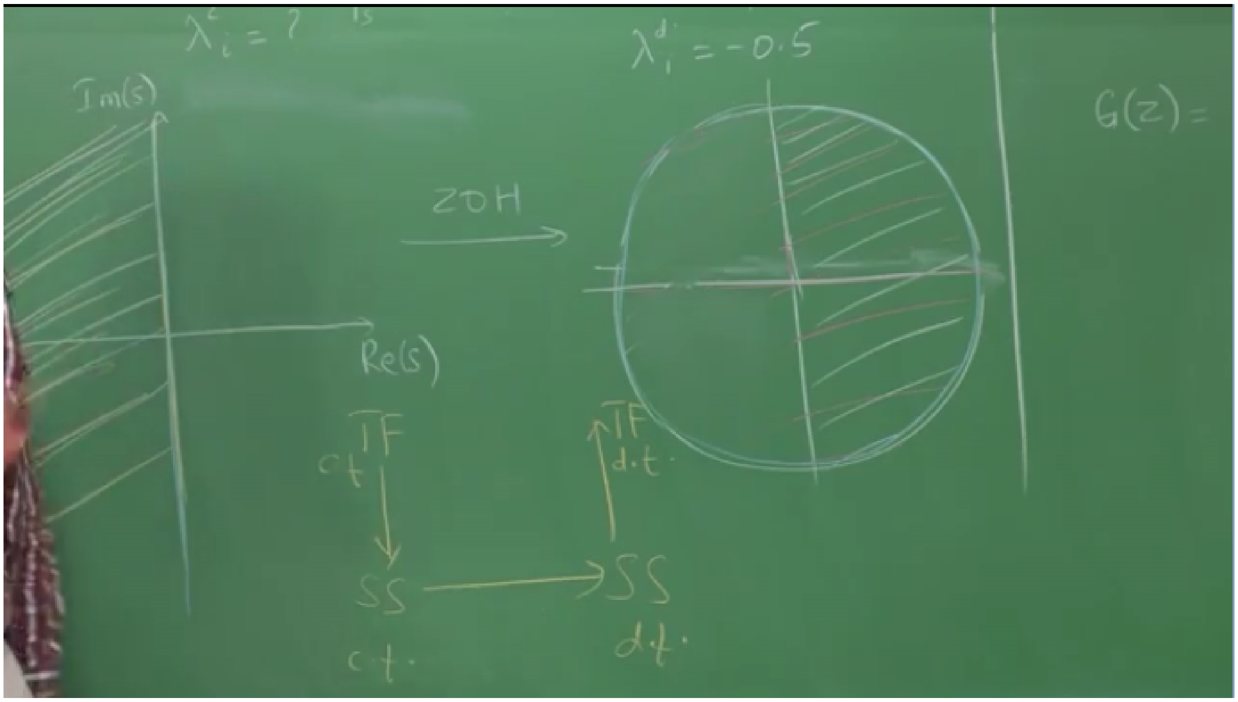

so it’s like a very circuitous route that I can follow, so I’m given a transfer function description in the continuous time, I write a state space representation in a continuous time, go to state space representation in discrete time and then arrive at transfer function in discrete time, very circuitous route, fortunately there exist another route which directly takes it from transfer function to transfer function which we will learn very soon.

But even without learning that we can understand that a continuous time system, second order continuous time system without a zero can result and in fact in general results in a discrete time system with a zero, how do we see that? In a very quick manner how can you see that? Without actually deriving the expression for this, any ideas? That delay is okay, delay won’t introduce the zero, normally in the past years that I have taught and I have asked the question on discretizing a second order continuous time system, students with religiously use the method that will very soon discuss, and then arrive at G(z), there is nothing wrong with it, but it is going, and it’s a bit laborious to do that.

It’s easier to break up the second order system into two first orders, right, so imagine this two first orders being made up of, sorry, the second order being made up of two first orders in parallel, which means I’m going to write this as C1/S+A + C2/S+B, what is the advantage? Do you see, do you forcing the advantage? I’m sorry, I can operate them, correct and then arrive at individual discretize systems, discretization is a linear operation, okay, correct, so that is the idea here by breaking this up into two first orders in parallel, I can derive now the individual G1(z) and G2(z) and then say well G is G1+G2,
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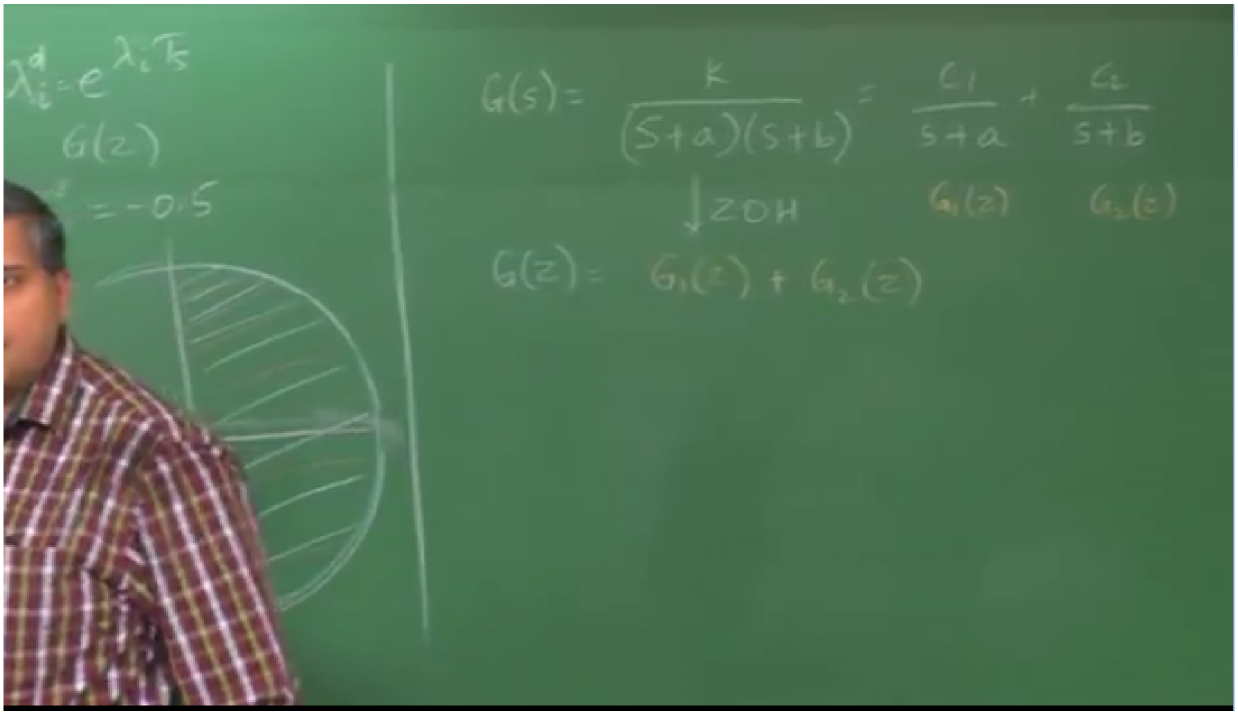

what is the advantage? Because first orders are very easy to remember, all you have to do is gain and pole mapping, I don't even have to implement any formula, straight away I write the pole for G1(z) and adjust the numerators as the gain is preserved.

What is the gain of the continuous time system? Really, what is the gain of the continuous time process that we have written on the board? Why there is so much silence? K/AB correct, I thought I heard only K, sorry, okay, so K/AB, so all you have to do is for G1(z) you just have to adjust, so this would be S, it will be Z here minus, where is the pole now? E to the A? ATS? – ATS,
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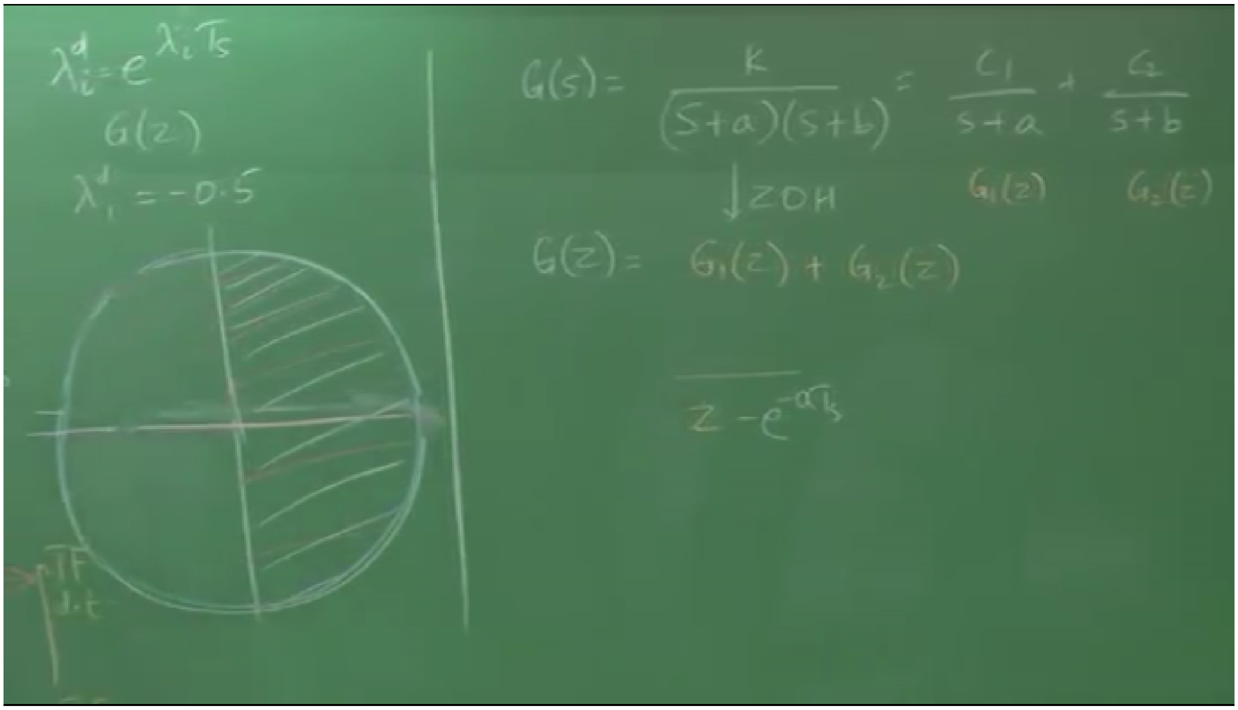

pole is at –A, right? So the denominator part has been taken care of, and I just have to adjust this numerator such that the gain of this is the same as the gain here, so here the gain is C1/A, I can find out what is C1? So I just call this as some K1 prime,

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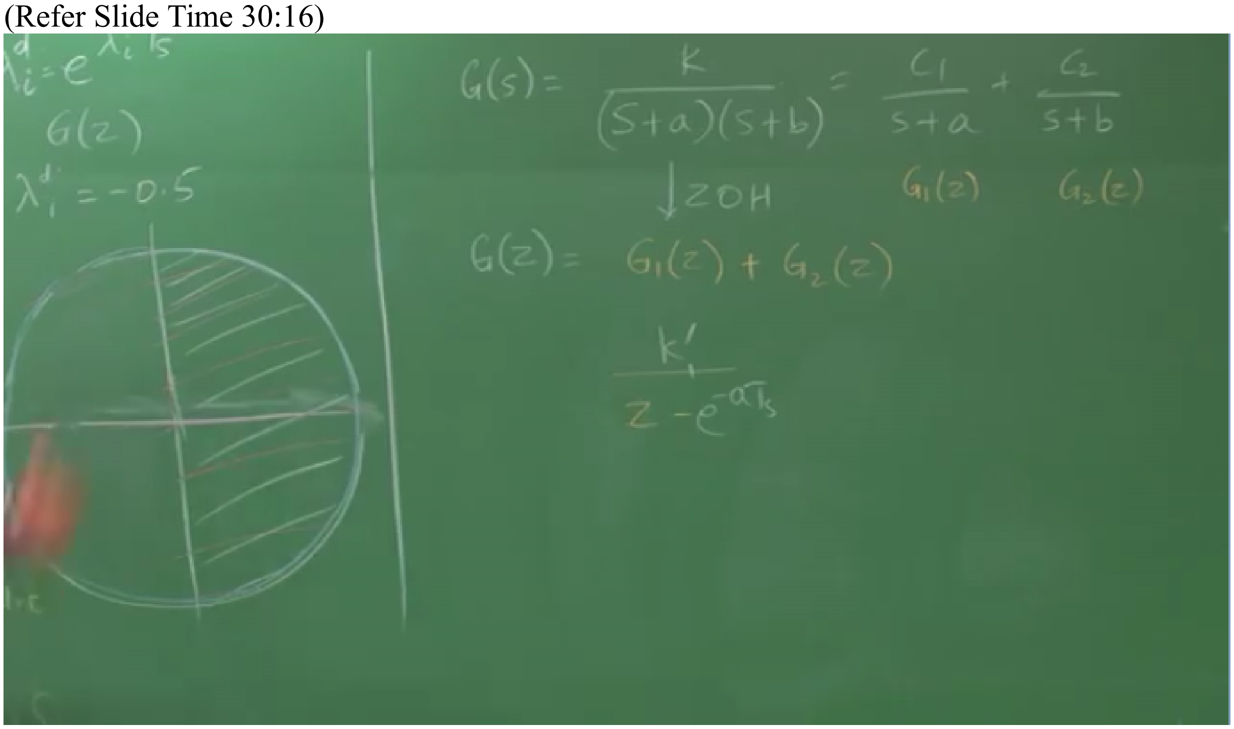

likewise for G2 then add them up, right, so K1 prime/Z-E to the –ATS + K2 prime/Z – E to the –BTS, clear?

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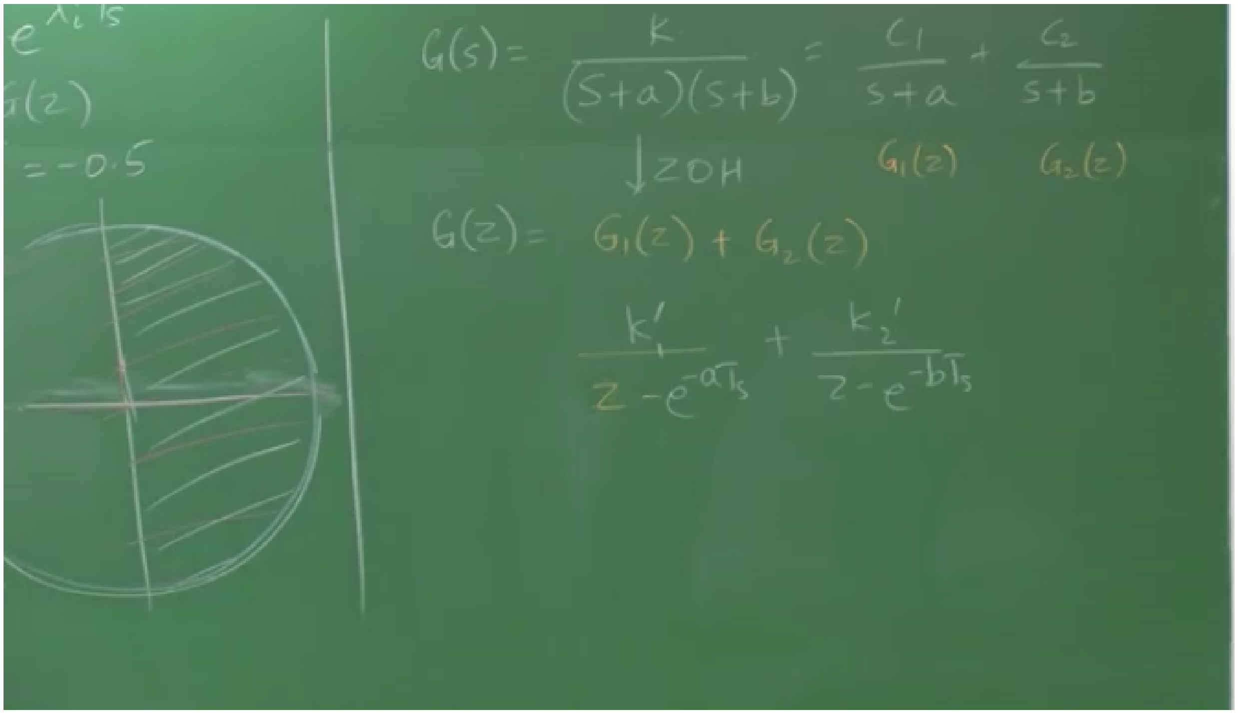

Now can you see that the zero can emerge? Without evaluating K prime and so on it's fairly obvious now that the discrete time system has a zero, the continuous time case also you can say that maybe zero no, but C1 and C2 are chosen in such a way that there is no zero, there is no such requirement for the discretize system, so this is a very simple way of realizing that even though the continuous time system may not have a zero, the discretize system can have a zero. Any questions?

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