CH5230: System Identification

State-Space Representation 1

Very good morning. What we will do today is study state-spacedescriptions. I've already given you a brief introduction and some motivation as to why we would want to look at state-space models. Let us look at these two examples although they are not exhaustive in nature. They are at least representative of what will encounter. There are a few other reasons why we would like to look at state-space descriptions which we shall learn as we go along. So let's look at the first scenario, where we are compelled to look at state-space descriptions. And this scenario is perhaps one of the most commonly encountered situations where one would use state-space models as I had said yesterday, sates are these hidden variables. You can call them as hidden variables or unobserved variables.And you run in to this scenario many, many times and that's where the Kalman filter really comes into picture where it estimates this states or this hidden variables from available measurements. What we mean by measurements is, mainly the response of the system and inputs, if inputs are available. In system identification inputs are available. In a time series analysis scenario the inputs are not known are not measure,at least in the univariate time series case, where you only rely on measurements. So what are the situations in which we come across this kind of scenario where the states are hidden variables?

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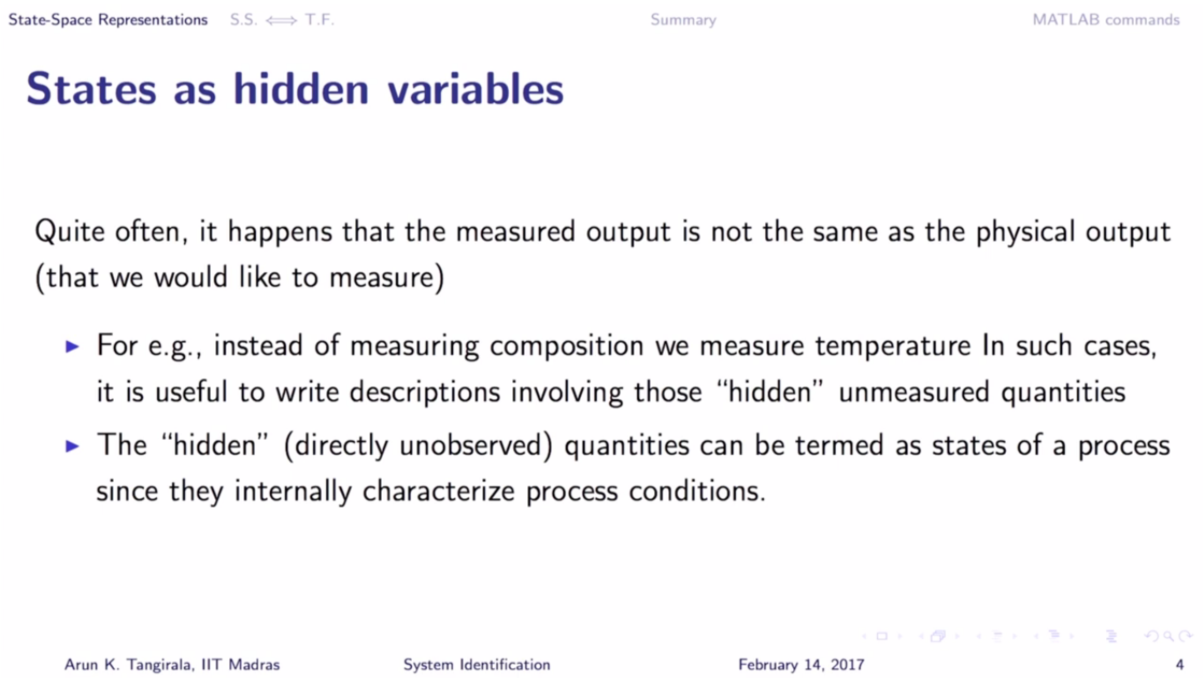

A standard situation is inferential sensing, what we call a soft sensing. Right. What do we mean by soft sensing? Well, there is a physical variable that I am unable to measure. And you encounter these kind of situations in many applications. In chemical engineering for instance, we still do not have an online sensor for measuring compositions that is one scenario or concentrations. There are gas chromatographs and so on, but their usage online is not widespread because of maintenance issues and so on. So how do you sense this concentration when you cannot have a physical sensor online? Another example is a molecular weight of a polymer. Or if you turn to combustion, let us say, you are looking at flame dynamics in jets and so on. The one of the critical variables of interest is this so-called heat release rate. Okay. And unfortunately there is no sensor that is able to measure the heat release rate. Again there we have to rely on other measurements to infer, whether it is concentration in a reactor or a molecular rate of a polymer or heat release rate or if you turn to cement industry, cement fineness, how fine is the cement. There is no online sensor, you'll have to collect samples of cement and take it to a laboratory, where you will measure how many particles are spread in a given unit area.

That's a measure of cement fineness. In all these situations, where you cannot measure the physical variable of interest online, you will have to rely on inferential sensing.In fact, a classic example is that of position estimation by a blind person. So if you look at people who are visually impaired. That's a technical word for today, for blind, generally we don't use the term blind.

So if you look at people who are visually impaired. They are unable to obviously, there main sensor which is their eyes are not functioning. Either poorly functioning or not functioning at all. In which case, we know very well that they rely on. What do they rely on? Sounds, right? So they rely on other sensors, the auditory system and then infer the position, right? In fact, many at times if I am, let us say, filling a bucket of water. I don't have to necessarily look at the bucket constantly to figure out if the bucket is full or not. By the sound of it I can actually sense. So this, all these situations correspond to inferential sensing. Now, in all these examples that we just discussed there is a variable that we want to sense but we are sensing something else. So the one that we want to sense is hidden to us, we are not observing it directly. We are observing something else. That variable that we are not sensing can be thought of as a state. Now, what is the basis on which a blind man or visually impaired person relies on sounds or I rely on temperature to infer concentration and so on? The premise is that there is a relation between what I observe and what I wish to observe. What I wish to observe is a state which is hidden from me. And what I am observing is measurement. So in these situations now we run into the, we can set up a nice state estimation problem, where we have a state-space model. There is yet another example which falls into this scenario, which is that of calibration. We are all familiar with calibration, right. What is calibration? I have an instrument either a thermocouple or a level sensor or any other sensor that I had. All these sensors, when I look at thermocouple or a level sensor and so on. What is the measurement that they get? Suppose I take a thermocouple. What is it supposed to sense? Temperature. Does it give me temperature directly? What does it give me? Some electrical signal, right, voltage some other instrument would give me current and so on, depends on the instrument typically its voltage.

But I'm interested in temperature. So the temperature is hidden from me. What I am measuring is voltage. That's a standard again state estimation problem, where the state is a variable that I'm supposed to observe and I'm not observing that I'm measuring something else. And I'm hoping that there is a relation, right? Otherwise, why would I use this instrument? I'm hoping that there is a relation between the temperature and the one that I want to observe. So let's look at such a situation.

Let us say, there is this process, which is, it's a first-order process and letsdenote the variable that I want to that I'm interested in by x. Incidentally this is a notation that we will use for states as well. You can think of this as a level measurement for example, we know that level systems follow first-order dynamics. Now, we also know that just now we discussed that we cannot measure level directly instead we measure voltage, the level sensor will give me some electrical signal which we call as y, right?

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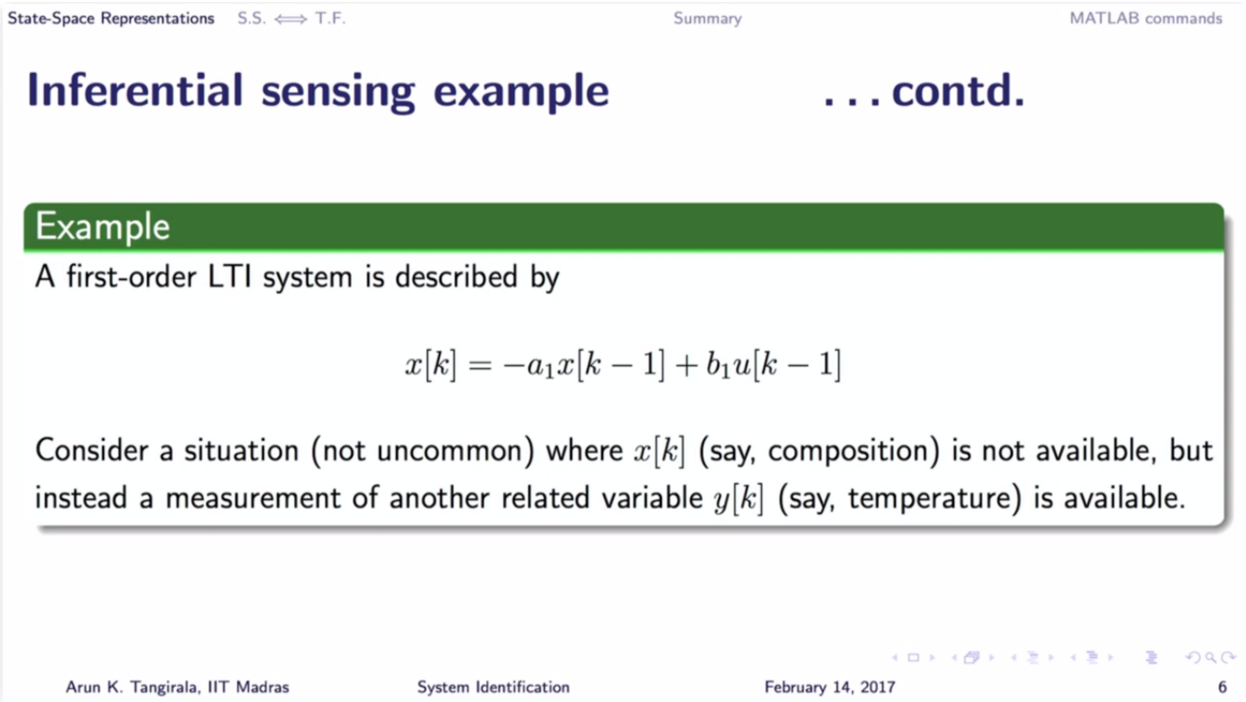

Let us for the sake of discussion assume a simple relation between the one that I'm measuring which is y and the one that I want to know which is x. And that is your equation number two here that you have y k equal c x k there's not numbered and there are saying the second equation here. So the top equation that you see here is the equation that's telling you how the variable of interest is evolving in time and the bottom equation is relating the measurement to the state. So we call this tope quation as a state equation because it tells me how the state is evolving with time. And the bottom equation as the measurement equation or the output equation there are different names to it. For simplicity's sake, we have ignored the measurement error.In a real scenario you would have measurement error also coming in. So when we talk of deterministic plus stochastic models we will bring in that component as well at a later stage. At the moment we are just, understanding how state-space models work. What is a mechanism and so on? It is conventional to rewrite a state equation as a forward difference equation, right. That is a convention that will follow it, because it tells me given the current state and the current input, what is the next state?

And the measurement equation tells me, how the present state is affecting the measurement. And this is going to be the typical scenario that you will see in state-space models.The state equations are always going to be dynamic models and in fact, they're going to be specifically first-order equations only. It is conventional to write state equations as first-order difference equations and that is why slowly we understand why the number of states equals the order of the system, right? If I have a second order, let's say, second order system, then in the next example as we'll see.

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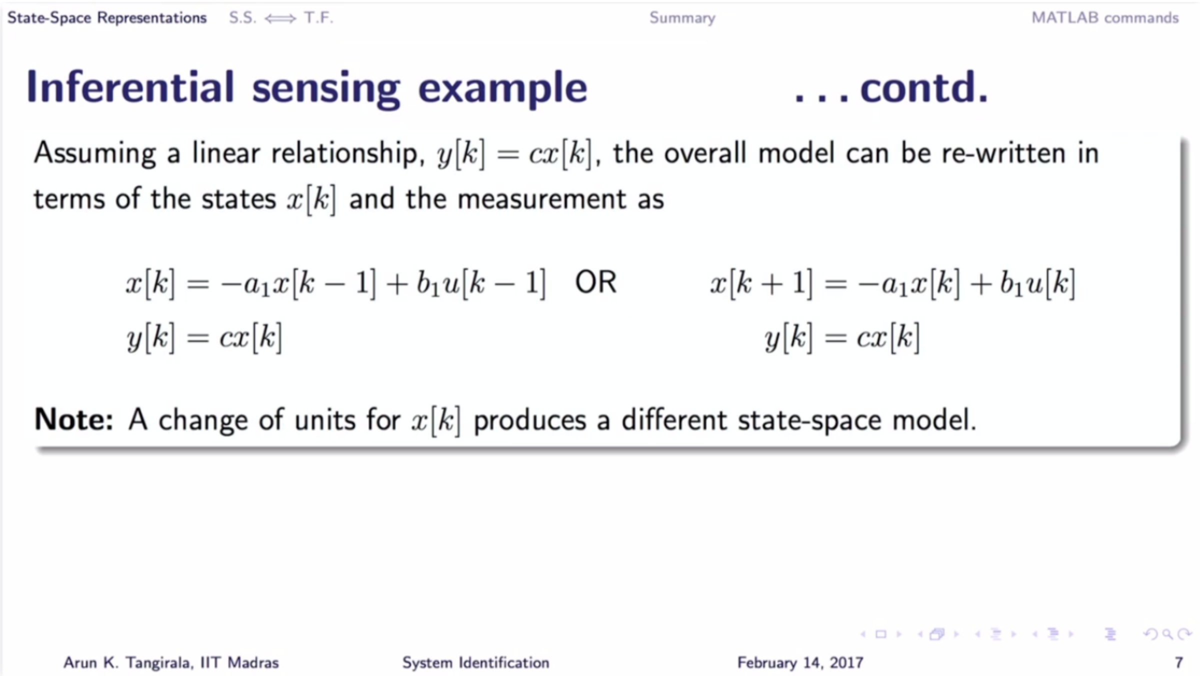

We will rewrite it as two first-orders like we used to do in solving differential equations see if you recall. If I am given a second order differential equation and I have to solve one of the ways of solving it is to break it up into two first-ordersand then solve them, right,where we will introduce artificial variables. That's once again where we encounter the notion of states and that is what the second example is going to be about. So let's complete this discussion on the first example here. So this is a situation which corresponds to inferential sensing and or soft sensing, which includes many, many applications including calibration. So you can think of c as being the calibration constant. Now, let me point out quickly the distinction between state estimation problem and a state-space identification problem. Okay. State estimation problem is very simple, I'm given these parameters of the state-space model, a1 b1 and c or you can say, a b c for now, right? I'm given this and I'm also given the measurement y, at over a period of time. And if there is an input I'm also given the input. So I'm given input and output sequence and I'm given a b c which we call as a model state-space model. And the goal is to estimate x k. That is what is the state estimation problem, as the problem, I mean as the name itself says I'm estimating States.

Another way of stating the state estimation, even in stating you have a state. So stating the state estimation problem is given the same information that is the measurements and the model figure out what the initial condition is. Because once they know the initial condition, initial state, I can let it propagate through the model and figured out what the state trajectories, right?

Either estimate the state at these instant or initial conditions. That's exactly what people are trying to figure out. Given the present situation, present scenario of the universe, can I figure out how the universe began? Well, you have big bang, zing bang all this big theories that trying to, their all state estimation problems in effect. But they're much more because I don't even know the model here at least I'm saying, I'm given the model. So people presuppose a model and then take the present situation of the universe into account and then try to backtrack and figure out what the initial condition could have been. As usual nobody knows the truth so people are enjoying. If you're happy with big bang theory you can keep reading on that. Then if there is something some other theory, what is the other theory? Doyou know any other theory?

[12:00 inaudible]

What is it? What does it say?

[12:04 inaudible]

Okay. That's another theory. You can come up with your own theories as long as it makes sense with your model and so on. So you can, people keep enjoying nobody knows you and I at least I don't know how the universe began, if you know it'll be great, but life goes on. But in state estimation it's a lot more, simpler and better. We do know in many situations at least qualitatively what the state is? I mean, whether the state estimates make sense. So that is state estimation problem for you and that's exactly what kalman solved. You may think what's the big deal about it? By the way, how will you estimate in this simple situation, suppose I give you a b c y and u, how will you estimate the state? Sorry. Yeah, this is here very simple, right, because there's no noise. Suppose there was noise, measurement noise, then you can't say, it is simply y by c, right? So,then you have to set up some kind of an optimization problem that is what is essentially state estimation problem for you. Now even under noise free conditions. In this example it is fairly straightforward to say that the state estimate is y by c. When you have more than one state, right? In many situations you may have three, four, or five states and your measurement is only one. For example in DC motor position control a motor position control, you can actually have three states. You can have theta which is angle and then you have angular velocity and then you have current. If you were to write the state-space model for the dynamics of that system, where you are applying a voltage, you would have three states and you may be only measuring one of them. Can you infer the other one, right? Then you have to ask, you have to raise more questions, here it is pretty obvious in the single state system.

First-ordersystem it's not a big issue at all or even you look at the other example that we discussed yesterday, where we said that there is this liquid level-- the heater example. So this is a stirrer and here you have a heat input and there is a flow out. And I'm, I have height and then there is this temperature, because of uniform stirring, we can assume there is a same temperature outside. Now here if I look at temperature dynamics, we have already discussed it's the second-order system you will need bore temperature and level. State, two states to describe the temperature dynamics. Now the question to you is, suppose, I measure temperature only. Okay. So, our states in this example are? I'm going to write a continuous time function here. These are our states.And now the question is, suppose, I only measure temperature at any time t. Which means our major output equation simply becomes 0 1 x, right.

In place of c you have 0 1. Now the question to you is can they estimate the other state which is a liquid level? Physically, think about it. Leave the model aside, physically if I give you the temperature measurement alone and the model of course, I'll give you the model, but don't worry about rigorously analysing the problem. I'll give you the model for the temperature dynamics, the second-order state-space model. I give you the temperature measurements would you be able to infer the level? Yes or no? Really?Why? I give you the temperature and the flow also, the inlet flow also that actually doesn't make so much of a difference.

[16:26 inaudible]

Sorry.

From different instance of time [16:30 inaudible]

Yeah. That's what? When we say temperature not in a single instant? Please don't think that I'm going to give you a single instant, I'm going to give you a temperature profile over a period of time. Do you think you can solve it? Physically, is there information for liquid level in the temperature. The answer is yes, there is information about the liquid level in the temperature, because there is a connection, right? This is only qualitative. Sometimes, I mean, final answer will be given by your model as to how these states and measurements are related, but if you just look at it physically, then it's possible. What about the other way around? Suppose, I give you a model for the temperature, right? For the temperature dynamics and I will give you liquid level. Will you be able to estimate the temperature? Is it also possible? So more than the answer I'm interested in triggering your thoughts on a concept called observability. The question that we are asking is whether the states that we are not observing directly, can we observe them from the given measurements. What do you think governs this ability to observe the states, whether it is this example or some other example, some other application. In general what governs our ability to estimate states from measurements? What do you think?

[18:23 inaudible]

Sorry. The measurement model and the state equation model, correct. We will come back and answer this question briefly although I am not going to dwell in detail on the concept of observer ability as of now, but we will just qualitatively discuss and draw some hints on how I can figure out whether a system is, whether the states are or we say the system is observable or not. In other words given measurements whether I can infer only states. When we complete, after we complete the second example, when we give the general state-space description at that point we will talk more on observer ability, but it is a central concept in state estimation.It is a central concept in state estimation because unless you are guaranteed that you can that the system is observable, you can't even actually set up the state estimation problem you can, but you will never get any solution to it. Now this is state estimation problem. What is state-space identification problem?What do you think it is?

Given the states[19:37 inaudible]

Given the states how lucky we would be.

[19:40 inaudible]

If you're given the states, that's it.You know, the problem is simplified and life is a lot easier. The biggest [19:54 inaudible] states are hidden from you.

[19:56 inaudible]

That's all. And the, you have to necessarily identify the states whether you are interested in the states or not. The state-space identification problem is a superset of the state estimation problem.In the state estimation problem he said we have given the model and the input output data and the objective is only to estimate the states.In state-spaceidentification same story in identification what I mean given only input output data. Nothing additionally is going to be given to you in state-space identification. I am given input output data but now the task ahead is to estimate the model also. So I have to get my a b c also.And in the process I may have to end up estimating the states.So a state-space identification problem is a joint estimation, joint state estimation and identificationproblem, model estimation problem. So it's both and you will run into these situations many, many times.Which model are we talking of the model between the states and the inputs, the model between the measurement and the states, everything put together the state-space model.

So now you understand the difference between state estimation and state-space identification problem.State-spaceidentificationproblem is much more challenging. You are just given input output data you have to estimate so many things. You have to estimate a b c in this case a b cscalars, in a general case a b c maybe matrices.And in fact in the full case that is when you have the deterministic plus stochastic case you will have to in addition to this estimate the noise covariance also. So there is a whole lot of unknowns that you have to estimate in state-spaceidentification.And that is why it's a lot more challenging. Okay. Now, just to point out that the state-space model is not unique coming back to the example. If I change the unit for x k, I'll get a different values for a b and c, at least, yeah, for all. In fact if I change the units of x all the parameters a b c are going to be affected. But the overall input output relation is going to be the same. What does this mean? It means that there is no unique state-space model.

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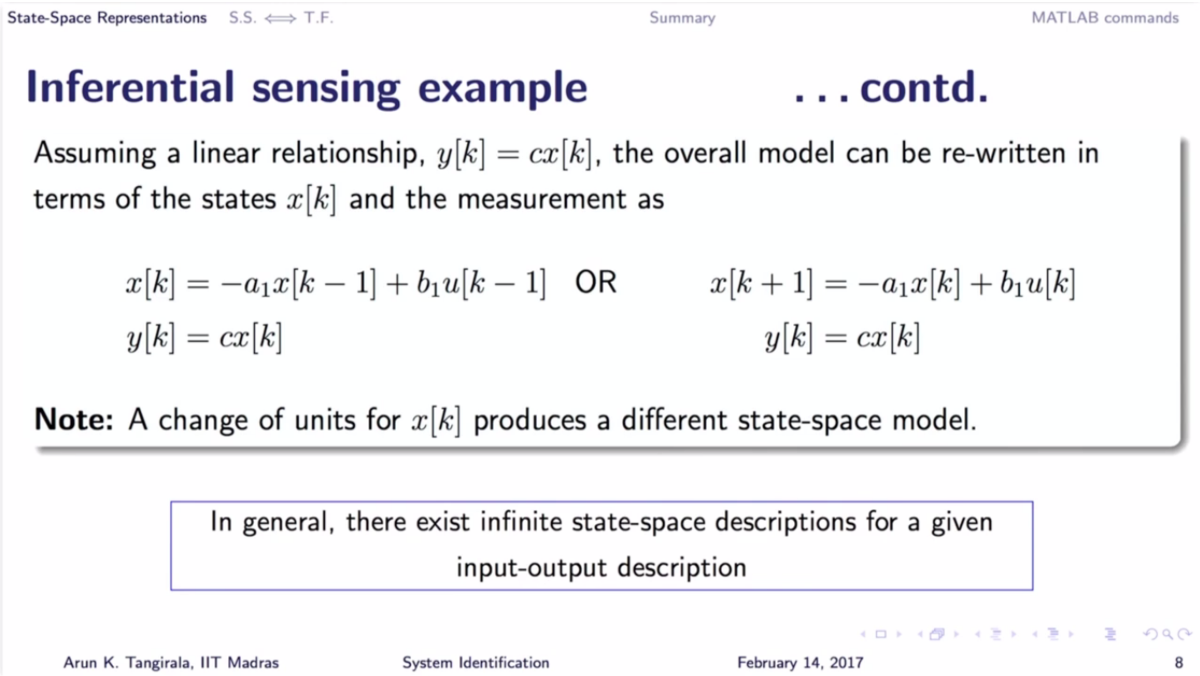

Unless what? Unless I fix the unit for x, I say that I want to estimate the temperature in Celsius only. Given the thermocouple voltage, which is why? Then it's unique. What we say is unless I fix the basis for the states, the uniqueness is not guaranteed. And that is what distinguishes between a freely parameterised state-space model or a non-primitive state-space model and so-called structure state-space model. We will learn as we go along. So let us look at the second example which is once again going to help us understand the notion of state, but from a different perspective. In the previous example the states where hidden variables.They are physical variables that I want to sense.They are not necessarily spurious.So there is a solid reason why I want to set up a state-space model there. Here in this example, I have a second-order system, in fact. And now I can imagine the second-order system as being made up of two first-orders. Why do I want to do this for various reasons? Analysis becomes easier, solution becomes easier and so on. So here it is more for mathematical reasons I want to consider the second-order system as either two first-orders in series or two first-orders in parallel, remember I can decompose it in many different ways. So here let us assume that the second-order is made up of the two first-ordersin series, right? And of course, straightaway you know there is no unique way of doing this, because I can always take a constant from g2 and account for that in g1.That itself brings about non-uniqueness. Any factorization will have that non-uniqueness.

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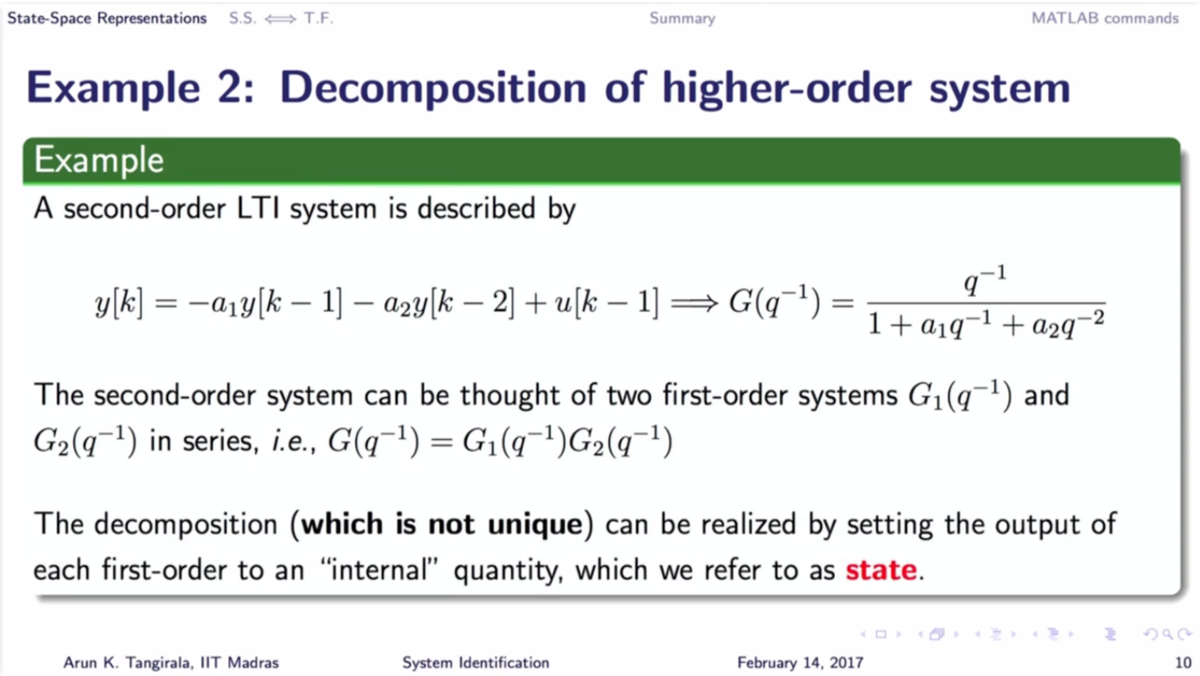

So let us look at one such decomposition.And in one such decomposition we assign the output to the state. So we're imagining g to be made up of two first-orders in series, right?

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Example 2: . . . contd.

Thus, we introduce

$$x_1[k] = y_1[k]; \quad x_2[k] = y_2[k] = y[k]$$

where $y_i[k]$ is the output of the i^{th} subsystem $G_i(q^{-1})$. Then, the input-output equation can be re-written in terms of the states as

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} p_1 & 0 \\ p_1 & p_2 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u[k]$$

$$yk] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

where p_1, p_2 are the poles of $G(q^{-1})$

So we're saying here is G and we say that this G is actually made up of G1 and G2. This is the input uk, G producers response y in this case G2 will be responsible for generating y. G1 receives the input u.So here this is the hidden output as far as this decomposition is concerned. Let's call this as y1and we call this as one state. You say that this is one hidden variable from me.

But again states need not always be hidden variables you have to understand.In some cases part of the states are hidden. So here one state is hidden from me the other state, remember how do I know how many states are required. One clue always is that order of the system. States are remember, those variables are dynamically change with time.If you're given a differential equation straight away you should be able to say what is the number of states that are required? If you're given a differential equation also you will be able to do it, but as we will see again shortly when you have delays in discrete time systems we have already discussed this yesterday, there can be an increase in the number of states.So it may not be so obvious, when you are given a different equation with delayed inputs as to how many states are required. Some more thinking is required and you will get to the answer. Always therefore, before you jump into writing a state-space description if there is an opportunity figured out how many states are required, you need to do that.

In an identification problem it's completely different.In identification I'm only given data.But if I'm given a differential equation or a difference equation and I have to arrive at the state-space description, I have the answer with me, I just have to think. But in state-space identification I'm only given data, I have to know speak to the data to tell me how many states are required. I have to make the data speak to me. That is where we run into some methods, which ultimately use the singular value decomposition to tell us how many states are required, but at the moment you don't have to worry about that.