CH5230: System Identification

z- Domain Descriptions 5

So we have now we move on to a formal discussion on stability. We have already talked about it previously but now we will state this formally in terms of the polls of the transfer function. Remember, we talked of two different forms of stability.

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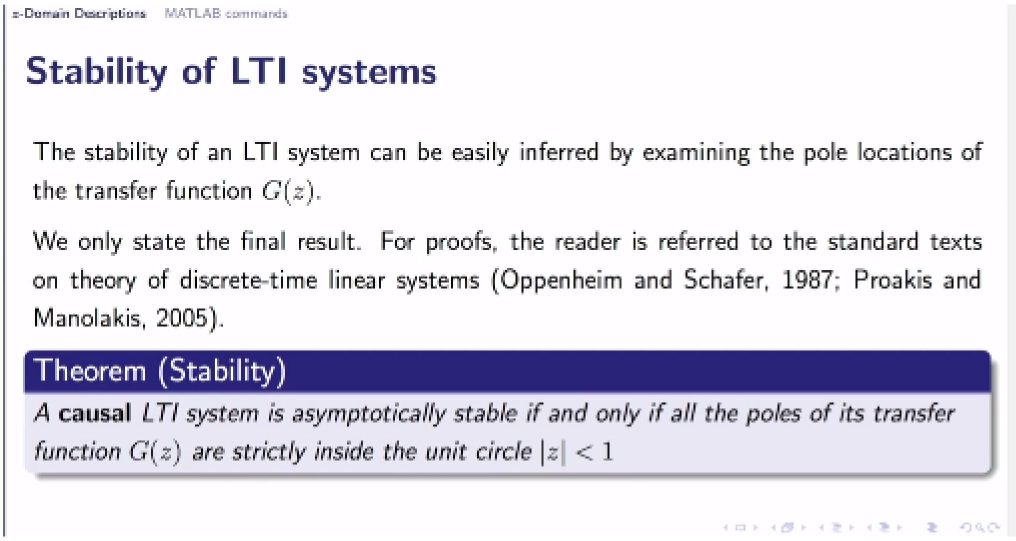

One is asymptotic, other is bebo stability. Asymptotic stability efforts refers to how the system responds to nonzero initial conditions. Bebo stability refers to the boundedness of the output for all bounded inputs. And we also said that both are identical except under some special situations. That means, if a system is asymptotically stable then it is bebo stable but not necessarily the other way around. And when is it that? It is not the other way round will quickly learn.

First let us look at this theorem which we are not going to prove but I've already given you some hint here, right? You can see that we are although we assume distinct routes. It's more or less clear. That the system, any LTI system is asymptotically stable. That means, we are looking at free response. If and only if all the polls are in unit circle. That means the magnitude of the polls are less than 1 because polls can be complex value. Now as I said, the other stability which is Bebo stability. And remember Bebo stability is not given in terms of the polls. How did we define bebo stability?

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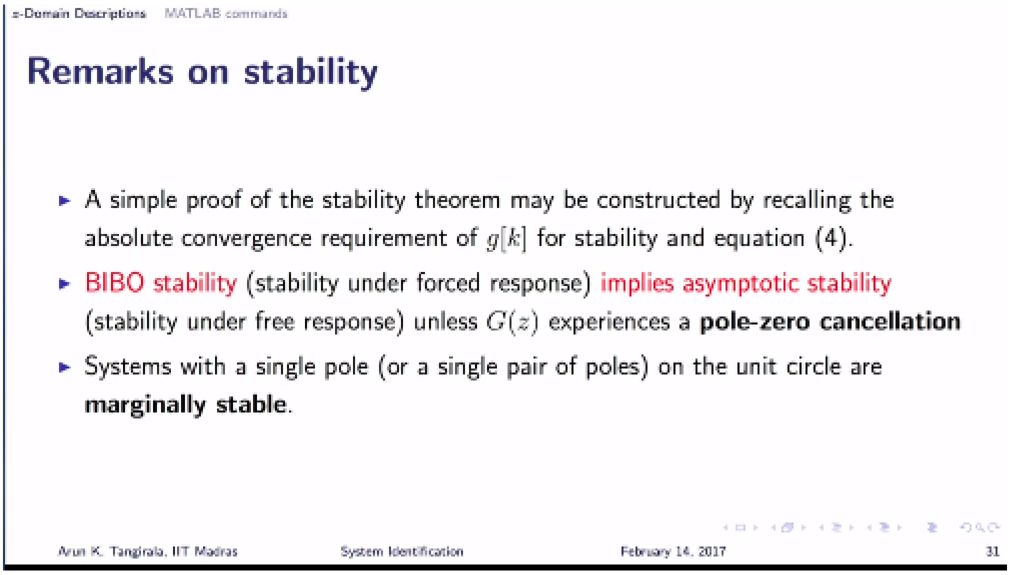

We said, a system is bebo stable if and only if the impulse response is absolutely convergent. Now you can show that there is a strong connection between the result that we just saw for asymptotic stability and the impulse response to be absolutely convergent. I'm avoiding that proof. But the connection is broken. That is in the sense it's not necessarily unique under some situations. What this means is that there is a connection between the impulse response and the poles of the system. Yes, because after all if you think of it, the poor solar system are telling you, how the system would respond under free conditions, nonzero initial conditions. Impulse is always like that. But is different. It depends on the numerator of the transfer function. Right. So if you look at the previous example, what would govern the impulse response? Yeah. Largely these but also in the right hand side, correct? Big part of impulse response would be shared by this. But there is this contribution also which means that there is a very close connection between impulse response and the poles but not a full one and therefore, bebo stability does not always imply asymptotic stability. Whereas other way around is true. Right. If the system is asymptotically stable then the impulse response is also going to be absolutely convergent. That's not an issue but not necessarily the other way around when? When there is it cancellation of unstable pole with a corresponding 0. What do we mean man stable pole? A pole that is sitting outside the unit circle, right. A pole that sits outside the unit circle will result in instability. If some how the system has been wired in such way that the Zero location exactly cancels out the effect of the pole. It doesn't cancel physically that pole. For every unstable mode, the Zero comes in and says, I'm going to really make sure that you don't go unstable. Right. That is what sometimes they say, when a man is mad. I mean, there is this saying which says that if this fellow is crazy, mad fellow get him married. You say, marriage will solve all kinds of problems. But seldom do they realize that marriage is also some kind of a feedback system. And if you're controller has been designed poorly then you can actually end up with instability. That is what we technically, legally call as divorce. No, I'm serious. So that is what is important to keep in mind. The control system is not just about systems it's also about human beings.

Anyway so that is something to keep in mind. We look at an example. If this zero is cancelling or the stable pole that's not an issue. Only when the zero is cancelling out an unstable pole then the system is bebo stable. That means in presence wife this guy is fine by himself he's unstable. So you girls should be proud of it. Then you had this other situation which is like a cat on the wall. That is we have talked about impulsive response is going to zero, decaying to zero then at the same time we have thought of impulsive responses blowing up. But then there is other case where the impulsive response neither goes to zero, nor blows up. It just decays and settles down to a constant value. Those systems are quite marginally stable systems. They are neither here. They're in the Trishanku swarga. They're neither here nor there. They're just waiting like this, many politicians waiting for the right opportunity to switch parties. Okay. So what are such systems. A classic example of a marginally stable system is an integrated with a single pole on the units circle. What is this integrator? Why is it called integrator? Any idea? I mean, I've given you the transfer function.

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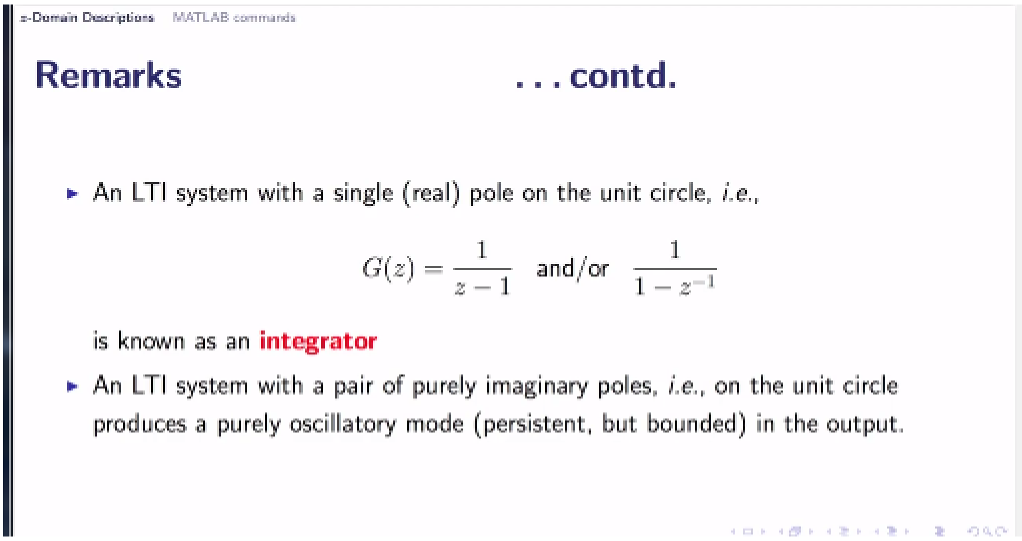

We'll go through a few examples shortly. Let's understand this. Why is this a system that has a transfer function of 1 over z minus 1 or 1 over 1 minus z inverse. They only differed by delay. Why are they called integrators? How do you arrive to that conclusion? No. Once it's previous inputs. Seriously? So let's write this here. One over z minus 1. Let's look at this. What would be the difference equation? Y k plus 1 minus y k equals uk. I'm I right? So now start writing at time zero. Let us say, at some point the initial conditions at zero and now you start writing. You back substitute, what would you get? So if you were to-- you can even write this as yk minus yk minus 1 is uk minus 1, doesn't matter. So if you back substitute, you'd get yk plus 1. What would be yk? See, yk plus 1 is yk plus uk. So yk would be yk minus 1 plus uk minus 1,Right? In fact, let's rewrite this, this way.

You see that. I'm sorry. Ignores. Okay. I mean you're saying, long division. Yeah. In fact, you can do a long division if you wish then you can show it's-- Correct. You can even do that way. There are many ways of doing it but this is a very nice, I mean, simpler way without involving any Taylor series and so on. Now if you recursively keep substituting, you will find that on the right hand side there would be a sum of inputs right from minus infinity up to k plus some starting condition. Ignore the starting condition for now. You just look at all the inputs that have begun from the starting of the universe to k the instant. They have simply added up so we few were to write this in the continuous time, you will see they'll be an integral. So that is why we say, it's actually an integrator. What are physical systems that have this kind of behavior? Any physical system that you can give? Basically an integrators and accumulator. Whatever input you give into this keep accumulating there is no output. It's like this person receiving salary and doesn't spend anything. Just keeps accumulating. Account keeps growing. What physical system can you think of? Any infinitely large storage system which has no discharge. Yes. Correct. Capacitor or an approximately integrating behavior at this large tanks which see seen industries. Which there is no discharge, huge cross-sectional areas. The way these integral I mean, near integrating systems will have heavy time constants, large time constants. A small-- Change in input it will take enormous time to reach a steady state. That practically you'll say, it never reach steady state. Theoretically this integrators will never reach it's steady state because they are, there poles are not strictly inside the unit circle. But as engineers we also have to think of approximate behaviors, not just exact behaviors. So there are these systems which have approximately integrating characteristics and those are these tanks with heavy cross-sectional areas or systems that take enormously long times to come to a steady state. Any system that has that you can think of it as an integrator. Now, this is not an unstable system per se. That means the response doesn't blow up. Now on the other hand if the system had two poles, repeated poles on the units circle. In fact at unity, not just on the unit circle, at unity. In other words, if you had a transfer function of this farm then this system is unstable. What is the impulse response of this? If g of z is 1 over z minus 1, what is the impulse response of the first one , for first system? No, no. You have-- we have solved, right? We have solved. I mean if you're confused you can also have if you want a Z here so that you can match no with the Z transforms or some standard signals that we have worked out. What is impulsive response? How do you calculate impulsive response given a transfer function? Take the inverse, no? How is transfer function defined? That definition has got nothing to do with the system. It is defined-- I mean with the stability of the system. It is simply the Z transform of the impulse response. Step so what does this tell us? For an integrator, the impulse response settles down at a non-zero value. It neither goes to zero nor blows up. Suppose, I have let us say, there is a Z here also. It doesn't matter. Numerator is not the issue.

Now what is the impulse response of this system? RAM,that means now the impulse the sponsors start to blow up. So one pole at unit circle, you can tolerate. And that why in the final value theorem also some interpretations are usually given in many textbooks. When you see, you have limit Z minus 1 times y of z. Right. This entire thing is y of z. This is to say that one pole on the unit circle is tolerable. Z minus 1 times y of Z should not have any more poles on the unit--at unity. Which is to say that whether it is a signal or a system, the poles that you can accommodate before the system becomes unstable. The poles that you can accommodate at unity is only one. Repeated poles will result in instability. So remember that. And then there is this other situation where there is an LTI system with a pair of purely imaginary poles. You can have such a system, right. In that case, what can you say about the system? What you can say is that it will have purely oscillate remote. This is also true in the continuous time case. The stability conditions for the continuous time case is slightly different there. I mean, quite different. In fact, there you would say, the poles should be left half plane and so on. So don't get confused.

But as far as this condition is concerned both in the continuous time case and the discrete time case when the poles are on the imaginary axis, alright? And obviously then there have to be a pair of imaginary axis and on the unit circle that is important. Both, you can have it on the imaginary axis anywhere. That is a restriction now that is a difference between continuous time case and discrete time case. In the continuous time case, the poles can be anywhere on the imaginary axis and it will be considered marginally stable. The system will produce pure oscillations. It's like your spring mass system without dumping. Ideal situation, no friction, nothing. Here the imaginary poles on the unit circle produce a purely oscillatory mode.

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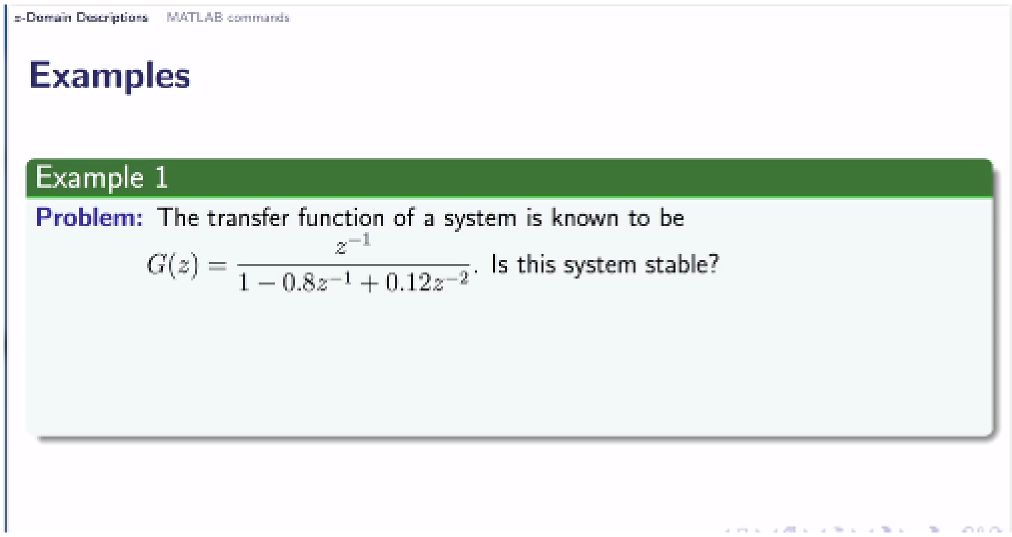

So persistent but bounded. The response it will just keep oscillating, it will neither blow up nor settle down. That is such systems are also called marginally stable systems. Okay, so let's look at an example here. Here is a transfer function, question is whether the system is stable? What is your answer? Z inverse or 1 minus 0.8 z inverse plus 0.12 z inverse square. Where are the poles located? Okay. Good. Here look at 0.6 and 0.2 and the system is both asymptotically and bebo stable. Why do we say that? Because there is no zero pole cancellation of-- first of all, there are no unstable poles. That itself means that they are stable. What about this situation now?

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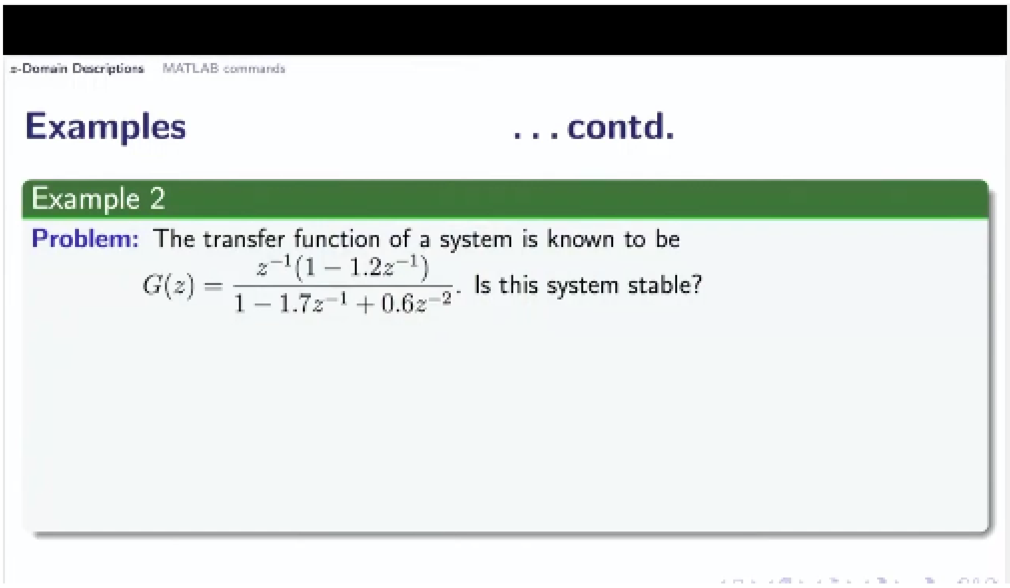

How many poles and where are they located? 1.2 and correct. So obviously it is asymptotically unstable. But there is a 1.2 sitting at the 0,right. So that lady is actually going to suppress this unstable mode. That 1.2, 0 location at 1.2 is going to suppress the unstable mode at 1.2 and in fact, you should take as a simple thing. By the way I forgot to show in MATLAB, I will do that. As a simple exercise go back and create a transfer function object in MATLAB and see, how the impulsive response for the system looks like. By the way, you can create transfer functions in MATLAB using TF command. I've shown you that but I'll show you once again. You can create an either in terms of Z or Z inverse. So let's go back to a previous example that we worked out. for example here and let me pull up MATLAB here. Increase the font size. Visible? Or you want me increase it further? Okay. Fine. So how do I create this. Well in fact, maybe some of you may still have some difficulty. Okay, now it should be safe.

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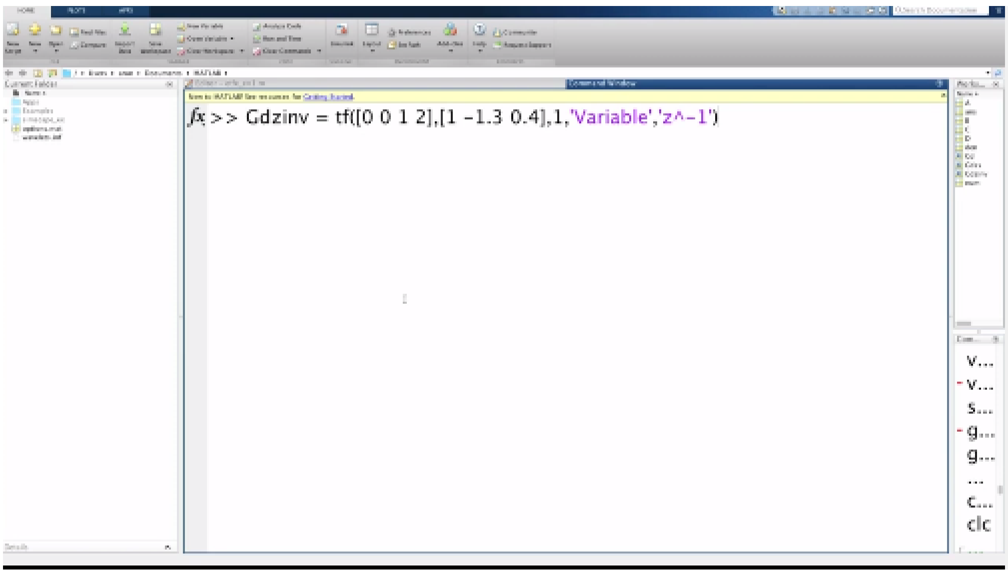

So I have here gd zinv. Normally, I would have if I don't specify the variable it is created in terms of Z. Which means that this transfer function here has to be rewritten in terms of Z and you have to do that. Given as is you want to write the transfer function, what you want to do is give the coefficients as they are given in increasing orders of z inverse. Same but the way we have written, numerator, the first argument is numerator. Second argument is the denominator. And what is the third argument? Sampling interval because it's not specified. It doesn't matter. You have to specify some sampling interval. Otherwise you wouldn't know that you are dealing with a discrete time system. And then you specify the variable. All you have to do is say, variable z inverse, once you do this, you'll get this Gdz. It matches with what has been given in the problem.

Now you can ask for pole agrees with our calculations, correct? And 0 again agrees with our calculation. dc gain, right. We can even ask for an impulse of Gdzinv. So that is how the impulse looks like, okay. Stable. As a simple homework problem now, your take, go back to this other example that we just discussed. This one, create a transfer function object. Look at how the impulse response looks like. Okay, come back and tell me if it is bebo stable.Yes. Just one pole . One pole at unity on the unit circle, yes. It will shoot up. Correct. That is why-- you're correct. You're right. So that is why it's marginally stable means, what happens is as with respect to the impulse response viewpoint, it's a system which doesn't actually blow up. It doesn't mean that it is actually stable with respect to all inputs. Stability, when we talk of stability for linear systems regardless of the input it should be stable. Therefore this integrator does not qualify that stability condition. If you look at the other systems like the Z over z minus 1 to the whole square, whether you give impulse, whether you give step, it is going to blow up but this one is a special class where to the impulse it doesn't blow up but two other kind of input step and so on it can blow up. So that is why it is marginally stable. Okay. But that's a very good point. Yes. If you give a step it'll blow up. That means if you talked about storage systems as being examples. Step would mean that I would increase the flow and hold it there. Beyond a certain point it'll blow up, it'll all over flow. Physically it'll over flow, mathematically it'll blow up. Oaky.

So that is an important point and good that you brought that up. As far as engineering applications are concerned we think of any system that has extremely slow time constants that means, a pole even close to the unit circle. We will come back to this point later on when we deal with--When we get into real identification. Where the pole is not at 1 maybe at 0.995. technically it is stable but practicality it is so slow. The step inputs won't blow it up but you say, for all practical purposes, I will approximate it as an integrator if needed. Okay. That is why we think of this. We discuss this integrator as a special case but you're right. For all other inputs that that hold on. I mean, where the inputs that actually have a new steady state, the system is going to give you a response it'll blow up. Okay. Good thinking. Alright.

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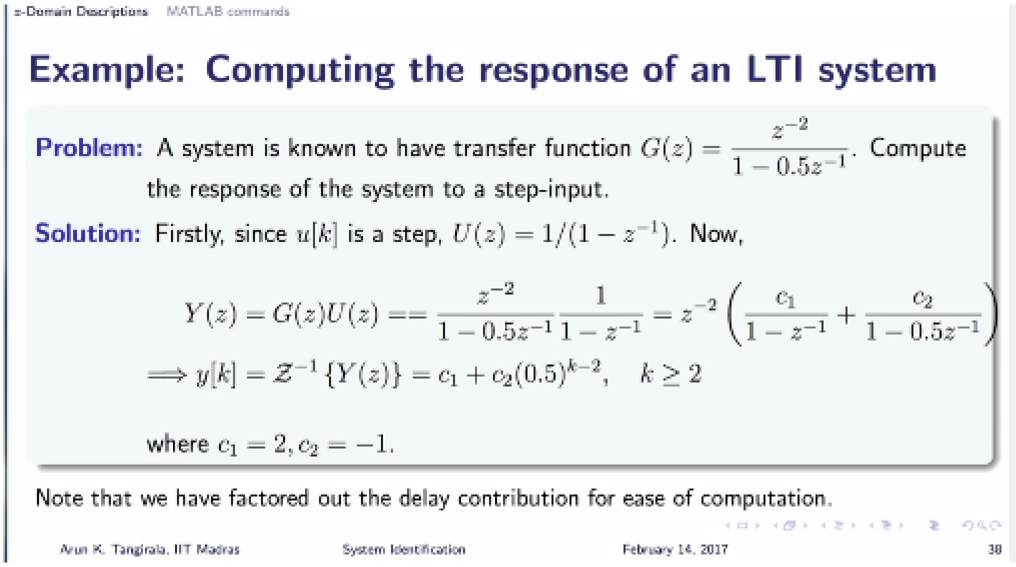

Let's quickly look at the response computation parts. Until now we're talking about stability computing the response is fairly easy. I'm just going you through this example, how you would use the transfer function to compute the response. This is the time cell function given to me z to the minus 2 over 1 minus 0.5 z inverse. It has a delay of two units. That is something that you should notice. Therefore sometimes when you compute the response of systems like this keep the delay aside and take the delay free part and compute the response. Include the delay after you have computed the response. So that it doesn't mess up your calculation. You understand what I'm saying. You can factor out Z to the minus 2 and keep a side. Very well remembering that that will only introduce a delay. It won't affect the dynamics at all. Correct. Now calculate the response of the system to the delay free part which is 1 over 1 minus 0.5 z inverse. You can do it that way. In fact that is exactly what I have done in this calculation effectively. So here I am interested in the step response. So I take u off. I mean, I compute u of Z, plug into the expression for Y of Z, do a partial faction expansion. The rest of it is inverse z transform now when I'm performing the inverse z transform notice that I have factored out z to the minus 2. So that I know it only causes a delay. So I compute the inverse z transform of the quantity within the brackets which is going to be C1 plus c2 times 0.5k and shift everything by 2 units. That's all. So that is going to be the response. Again, you can verify this solution with the help of MATLAB numerically, I'm not saying symbolical.