

CH5230: System Identification

Motivation and Overview 4

So these are some of the most important aspects of identification that will haunt us in every identification exercise, of course. And we are learning how methods of identification work. We may ignore the actuator dynamics and the sensor dynamics and so on. But in a real setting, if I were to ask you to conduct an experiment there is no escape from any of this. You will have an actuator, you will have a sensor and you may have to actuator separately, sensor separately, so as to get the model of the true process. But there are many applications and where you don't have to do that. Therefore this is perhaps optional. But the first fact is not optional, which is dealing with uncertainties that you have no escape. Even if you're building a discrete time model, you will have to do this. And one of the ways of doing this is, of course, as I said we now build a deterministic-plus-stochastic model.

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Motivation & Overview References

Overall Model

The overall model developed through identification is a **composite**, i.e., a **deterministic-plus-stochastic** model.

The deterministic model is driven by the physical / user input while the stochastic portion of the model is driven by a shock wave (fictitious and unpredictable)

Where the deterministic model it's a relative thing. What is deterministic and what is stochastic, is relative. The deterministic model explains, whatever you can explain as a contribution of the input. So what you have is... let's ignore for now the actuator and the sensor. This is your process. Usually we denote this with G and you are exciting this with u . And this generates some true response, y^* which I don't have. I don't have access to this. What I have unfortunately, I mean, is an access to a corrupted version which is what we call as y . Of course, diagram is incomplete, will complete this.

So what's happening here is, I would like to have y^* . I know the input that I have fed to the process. It has generated some response y^* , which I do not have access to, that's a true response. Instead I have access to a measured response. So I know y , but I do not know these two. Now I have made a big assumption here. What is that assumption? In drawing this schematic, I have made a very fundamental assumption. Right? So, I'm assuming linear super version of what?

Of the noise and the truth. Right? We call this as the additive noise model. And this is an important assumption on which a large body of literature rests upon. If you look at the literature on system identification or even any data analysis, it's not just the system information, in fact the good thing about taking courses such as system identification or time series analysis is a lot of the principles that you learn in these courses are universal, it doesn't matter. The moment you're dealing with data, whatever I have said until now is mostly true for any data analysis exercise.

You will never have the truth with you. There may be some minute amounts of v , the truth is only available in classrooms. Only when I, when somebody is teaching. Or when you are simulating, but reality you will always have a perceived truth. And that's true, right? If all of us are watching a movie, all of you are listening to what I'm saying, you are perceiving each of you has your own interpretation of what I'm saying, right? I may intend to convey y^* , but you have perhaps $y^* + v$. Right? So you have to now be able to figure it out over a period of time, what is a y^* that I am trying to convey. Okay, here I've made an important assumption, which is the y is $y^* + v$.

It need not be this way. Did anyone come and tell me, did some angle come and tell me, no. I just assumed it to be so. People have assumed it to be so. Why did they assume? Because life is simpler with this assumption. Is this only assumption that's possible? No. I can have another kind of situation. Remember there is another mathematical operation, see, basic multiplicative. Right? And that's also there is a body of literature that looks at this assumption. Though so this is additive. When I'm done with publishing some million papers on this assumption, I turn to the multiplicative assumptions. That's how life goes on.

Remember nobody knows the truth. That's the fun part of it, right. As long as your assumption is able to explain, do a good job of modeling the system, you are safe. If you look at even physics the classic example is the evolution of atomic models, right. If you look at a history as to how models for an atom have developed? People have hypothesized and then through means of an experiment they have checked whether this model makes sense. This assumption makes sense. Every model rests on certain set of assumptions. If it didn't work then that postulate was thrown out. That assumption was thrown out. Another model was proposed.

Do you think today, somebody has seen electrons going around the neutron around nucleus? Have you seen electrons orbiting? Have you ever heard of people be able to see, I found that electron is actually right now. It is just resting. It's just going to move in a few seconds. Nobody has seen. But then how did they accept certain models for the atom? Well there are other effects that are visible. Even though I cannot see the electron going around in the orbit there are other effects that are visible and I am going to ask my model, what is your prediction of that effect? I have experimentally observed something, if my model is able to correctly predict that effect then this model is acceptable.

That's how things evolve. And so is the case here, if the additive model is able to do a good job of what I am out to do which is prediction, that's good. It's doing a good job. When I hit upon a class of processes where this additive model or additive noise assumption does not generate good enough predictions, then we have to question-- I may have to question this. Very often people don't question this assumption because there are other things to worry about in notification we'll talk about that shortly. There are other aspects that I have to question before coming, before questioning this fundamental assumption that I am making. You should not really go and shake this one, because if you shake that one then you have to turn

to a completely different theory, which may take a lot of time to understand and implement, okay? So, very rarely we question this.

Only if all the other things have been answered then we come back to this, okay. So now this G is called deterministic model. Here the word deterministic is used in a slightly different sense than what it is used in the traditional, you know, signal processing or a time series and in the signal processing world anything, any signal that you can predict accurately. Given the past is said to be a deterministic signal, is said to be coming out of a deterministic process. Here the term-- yes, the more or less that definition carries over, but in particular the deterministic part refers to that subsystem of your model or a process that is explaining the effects of input.

So I believe as an engineer, now, you should remember this particular framework is usually used by engineers and to a certain extent people working perhaps in systems biology and so on, but not in econometrics. In econometrics these kinds of distinctions are not made because they have enough [8:41 inaudible] to deal with. They just simply assume that the entire process is stochastic. Okay? Because that you know, they can't really think of a physical input that I can change accurately. There's nothing like that, all right. There is nothing to manipulate. Everything is observed. So the engineers are fond of this and for good reason obviously, right. I mean, I have a deterministic process and I'm observing it. And when I observe it the stochastic effects come into play.

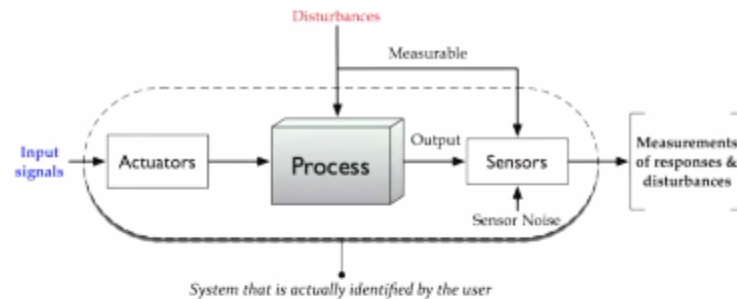
So this G here is a deterministic model and now I have to identification is all about splitting this measurement into two components. One as an effect of the deterministic model and typically G operates on u . I'm just writing it as G of u . But later on we'll change that and then there is a V , obviously there are infinitely different ways in which I can split y into two components, right. For give you a number 5, you can split it into 3 plus 2, 7 minus 2, there are so many ways in which you can split. But that unfortunately it is not that unconstrained problem. You have the input, so you know y star is not an arbitrary thing. It's coming out of the input and you have that input.

So that the problem is relatively better post than simply splitting y into two components. Given now y and u , I would like to split the measurement into two components or explain y as sum of two effects. One, the effects of the input and the rest. So the second assumption you're pointed out the first assumption, which is additive noise, which is correct. There is a second assumption that I have made, let me see if you can get that.

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System identified by the user

While the attempt is to identify the process, the **system that is identified** consists of **process, actuators, sensors** and the **disturbances**:



- ▶ Recovering the process model from the identified model is an advanced problem known as **continuous-time identification**.

In this framework, compared to the previous schematic that you had, right? So you see semantic here and you see the schematic on the board. You vaguely pointed out the additive part, but there is another assumption that I have made.

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Overall Model

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Okay. Right. So what is the difference if you look at it purely from a reader's digest kind of questions, spot the six differences kind of question. If you see, if you compare this schematic on the board with the schematic on the screen, you see that there's some difference. What is that, in terms of the schematic itself?

That's all? What are the contributions as per the schematic on the screen? What are the contributions to the measurement input? Of course, that you have taken care off on the board. Then you have the effects of unmeasured disturbances and then you are disturbances, let us put them together. And then you have sensor noise. But on the board I don't show so many channels of contributions. I just show input and the rest. So, what have I done? I have lumped, correct. So I have lumped, the effects of all those uncertain things that cannot explain. They're going sit in v . And that we call as stochastic because we believe that there is no mathematical model that can explain describe the evolution of in time. Because v consists of effects of noise, which we have already talked about, it has a random nature to it. And then effects off unmeasured disturbances that I'm treating as random.

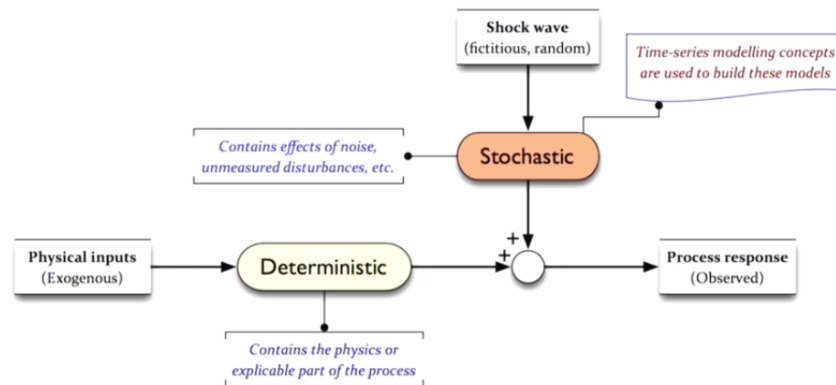
Everywhere I am treating things as random. If there are measured disturbances, of course, I can add one more term, we'll ignore measured disturbances from them. So what we are looking at here, it's a lumped additive model and that's going to be the framework for the course. And in fact for a large part of the literature that you see on system identification, you see this kind of framework. I'm lumping all of them into V . Now, just now we said that identification is all about-- we said its building a model, but now look at this perspective. It's all about splitting y into two components, one as an effect of the input and do the rest.

We, just now we said consists of effects of sensor noise, unmeasured disturbances or you know, whatever, let's ignore measured disturbances, so unmeasured disturbances and sensor noise. But now in the statement that I just made I have added another component to v . I said whatever cannot be explained by $G u$ goes and sits in v . Which means, if I've made a modeling error in G , if I have assumed a wrong mathematical form for G , the effect is felt in v . Right? So, v now contains three components. One is noise. Other effects of...

Good! Now, I have is modeling error. So that's the, so now what you see on the screen is the same thing that as you see on the board.

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Overall Model



A good identification exercise separates the deterministic and stochastic parts with reasonable accuracy. One should not be contained in the other.

We have labeled something as deterministic. We have labeled something as stochastic. There are they are relative. Now in a good identification exercise, you would ideally do a so-called correct job of separating, segregating y into effects of input and effects all the remaining. That is effects of unmeasured noise, unmeasured disturbances and sensor noise. We would try to minimize this portion or take this to zero, if possible. We don't want this to be sitting in v . But if I have done a poor job of modeling, G , then that would be reflected in v . Right? What do you mean by poor job? As a simple example, suppose the true process is a second order system.

There are many physical examples of a second order system. Right? Can you give an example of a second order dynamic system? [15:20 inaudible], spring mass, right? So each of you has seen in your own domain second order systems, the second order dynamics. Suppose I fit a first order model. What happens now, obviously I have missed out on one time constant, on one more. That is reflected in v . Is there a way to figure out that I've made a mistake? The good news is, yes. I can, using statistics I can act statistical measures I can figure out. By looking at so called correlation between what has been left out as we call residuals in system identification and the inputs I can figure it out, if I have made a mistake in G .

That is something that we learn in the example that I am going to touch base on tomorrow. Okay? So that's a beauty. There are statistical tools available to me that can tell me, whether I have made a mistake in G . Now, the story does not end there. Fine, I have made use of the statistical tools to figure out, what is it? An appropriate model for G , mathematical models G . It doesn't end there. There is one more component that I have to model. Remember v . I have to, if I have to take into account that there is this V and if possible, ultimately what I want to do with this model,

I want to make a prediction. I've gotten the effects of input, but maybe there is also some predictability in V . Which cannot be explained by the input, remember, the source of these disturbances at least under open loop conditions? By the way we are only going to deal by enlarging 99 % of the times with under open loop conditions on. Close loop identification is a lot more challenging, because the v is correlated with

input all the time. In a closed loop setting the measurement is driving the input. Which means V is a part of you. It makes it more complicated than what we have right now. So we'll assume open loop conditions.

Once I have done a good job of G , then I would like to turn to V and ask well, is there some predictability there? And that's a pure time series modeling problem. Because the causes of here unknown. What is generating this noise, what is generating this effects of unmeasured disturbances? I do not know. The causes are can be many and even if I know I may not be able to measure them. Right, where as you fix up measure disturbances can be taken care off. Therefore I have to turn to time series modeling to model V . And then you run into this very important result in time series analysis. Which says that under some conditions, you can think of v as being driven by some fictitious input called white noise and that's what you see in the schematic. Where there is a box name that labeled the stochastic, yeah.

We imagine now, so there is a further imagination, it's all about imagination. That's what exactly I told you yesterday. It's all about abstraction. I imagine v to be now generated by some shockwave which I cannot predict. It could be an earthquake, it-- whatever it is. But there is some unpredictable signal that is driving V . It's just an imagination, purely fiction, but a useful fiction, unlike your pulp fiction and so on. So, it's a useful fiction and then the time series analysis theory or the random process modeling theory tells me, how to identify the stochastic model?

Given that-- given this architecture and we will, of course, I'll review certain aspects of that at a suitable time. In the first one third of the course or maybe one fourth of the course, we will focus on what are the models are available for the deterministic system? And then we turn to a quick review of the stochastic models. The nice thing about this architecture that you see is now I have an input output kind of input output framework for both y^* and V . Both are now can we modeled as an input driving the respective components.

But there is a big difference the input that is driving y^* is a physical input that I know. Whereas the input that is driving V is fiction, with some known statistical properties, mainly the uncorrelated structure. So remember the biggest challenge in identification is to be able to separate these two. Okay. At least today I should tell you what these courses about contains and quickly talk about the grading scheme. So, what is a formal study offer?

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What does a formal study offer?

Through a formal study of the subject of identification, we can answer many generic questions that arise in empirical modelling

- ▶ What type of models are possible? Which one(s) to choose?
- ▶ How do we “fit” a model that “explains” the data?
- ▶ How to “correctly” account for the deterministic and stochastic effects?
- ▶ Will the experiment influence the model that we fit? If yes, in what way?
- ▶ How do we set up and solve the problem of estimating the unknown model parameters?
- ▶ What kind of experiments should we design to obtain a good quality model? and several other related questions.



You see in all of this note can be done by simply feeding data, throwing it into a software, we'll use MATLAB. And you can actually feed the data to the MATLAB software. In fact specifically we'll use a system identification tool box, which has all the routines that you need. I can just supply the data to the routine and give some user inputs and get a model and keep going. Well, if that is all you want to do you shouldn't be in this course. And unfortunately if that is all you plan to do, I'm afraid most of the times you'll end up with the wrong model.

You will not even know, what you are doing, why you have chosen that model, whether the data has quality in it-- information in it to fit that good model, what is this routine doing, when it report say errors, whether the errors-- what assumptions have gone into calculating those errors. Whether the method that have been-- that has been used for estimating the model parameters is that appropriate for your situation. So many questions for which you will not have any answers and the formal study will offer you all of that right from designing your experiment to building your model.

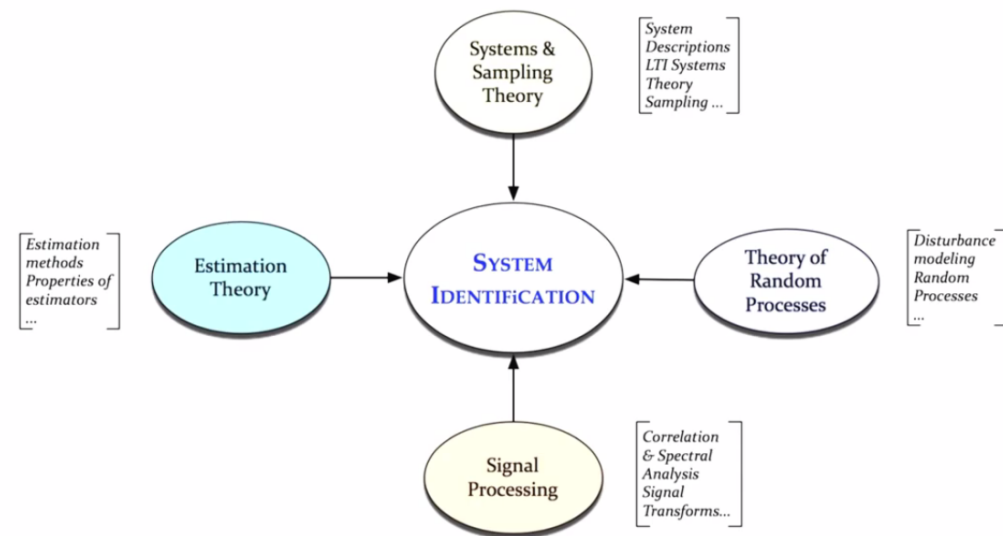
That's why you should do a learner job and informed job of identification rather than a blind job offered. If you're going to do once or twice in your life, then you don't need this course. But if you're truly interested in knowing how models are developed in a systematic manner, that's why I say, system identification can be also thought of a systematic identification. You're going to do things in a very systematic manner and the advantage of doing that. One main advantage of doing that is not only-- are you clear in your mind at each stage of system identification.

What you're doing? When I say each stage data acquisition, data preprocessing, choosing the model structure, estimating the one, there are so many stages. At each stage you would have clarity on what you're doing and more importantly when you'd end up with the wrong model, you will have an idea of what could have gone wrong. Unlike a very blind user who has no exposure to the theory of identification. You would be-- And that is what distinguishes between a learned analyst and just a blind analyst. This is not just true of system identification. It's true of machine learning everything right. Right?

so we'll answer many of these questions that you'll see, what type of models that are available whether I'm choosing the correct estimation algorithm. Have I made the right assumptions on the noise model, have I chosen the correct estimation algorithm. What are the consequences of choosing a certain noise and plant or this G is also known as a plant model together and so on. And so there are so many questions for which we can obtain answers in a formal study.

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Foundations of SYSID



And in order to understand system identification, remember system identification is not a subject by itself, it's a confluence of different fields. Especially four different fields. This slide is not to scare you, but to tell you, what is required and that we will touch base on all of this, right. So if you look at it there are four fields that give birth to this monster called system identification. One is your systems and sampling theory, which we will review in the first part of the course. And then you have the theory of random processes, you know why it is there, now. If I were to show you this slide in the beginning you wouldn't follow, why it is required?

And then you have signal processing, because you want to process data, you may work in a domain, a new not in a time domain but in a transformed domain. And there are so many other things that you have to learn in signal processing correlation and so on. And then of course at the heart of system identification is the estimation, parameter estimation. So you need foundations of estimation theory. This course is going to give you basics of all these four. Mostly whatever is required and that is how the textbook that we are going to follow and the textbook that we're going to follow is, the [@Principles of System Identification: Theory and Practice@](#). It's written by a man called Arun Tangirala.

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Course contents

Topics	Chapters
Background: overview, systematic procedure, motivational case studies.	Chapters 1-2
Models of deterministic LTI systems: discrete-time convolution models, response-based models, difference equation descriptions, transfer function and state-space models, discretization	Chapters 3-6
Stochastic processes: Review (auto- and cross-correlation functions, white-noise process and ARMA models) ¹	Chapters 7-12
Input-output models for Identification: non-parametric (step, impulse and frequency response) and parametric models (ARX, ARMAX, OE, B-J)	Chapter 17

And don't get scared by the size of the book. But now that you start through this class, you will understand why the book has been written that way. I assume that you don't know anything. You don't have any basics, in any of these fields. And which is mostly true, even if you have taken a course it had its expiry date, right? You may look at the grade and be proud of it. But the question is whether you remember the concepts and mostly in the curriculum that we have not only in this country but across the globe, you wouldn't find an engineering curriculum that has all the four fields catering these four fields. There are about 10 copies in principle available in the library.

Maybe 2 are not available someone would have borrowed it on a long term basis. But yes there are copies of it available in the library. And then there are other things that I don't intend to talk about. There are other copies also. Okay. But I will leave it to you to discover and I'm sure you've already discovered, some of you. Right, so the course contents as they're posted in the outline, I've just taken the portions of that and put it in the slide here. Quickly and I'm not going to read every line here, so we'll initially go through the models of deterministic LTI systems.

What models should I've consider for G? Remember you have to pay a lot of respect to this, it is says G, all the time it is G. Right, so you know why in Hindi Ji means, you have to actually pay a lot of respect. So you have to pay attention to the deterministic linear systems theory. Which is what we'll spend time on and I've given the chapters-- corresponding chapters of the book. We'll have a quick review of the stochastic processes. And then put together you can see a big jump in the chapters, but that is the way-- because of the way the book is arranged. We put together to see now, what are the input output models that we are going to consider for identification.

And once they have this together, I need to know how to make a prediction. And you may wonder why should I go to prediction without even estimating the model? Unfortunately it's a Catch-22 problem, in the sense unless I know how I'm making the prediction theoretically, I will not be able to estimate. Why, because estimation of all these models rests on a very important optimization formulation, which says

fine these models such that the prediction errors are minimized. Because there are so many choices, right? So I have to find the G and the other stochastic model. Alphabetically what comes after G? H and that's the kind of convention we are going to follow. The stochastic model is going to be denoted by H which is going to be driven by this white noise e .

So in order to estimate G and H, I turn to an optimization formulation essentially estimation, which minimizes the prediction errors. Unless I know how to theoretically compute the predictions, I will not be able to even formulate the optimization problem. So we'll assume that the model is given and ask how I would make predictions. Once I have done that then I turn to estimation theory. And that constitutes the bulk of this course. And that probably has a significant overlap with time series, but our focus is going to be different in this. And then subsequently followed up with application of this to identifying so-called non parametric and parametric models, we'll get to know what are those.

And then finally we learn some statistical and practical aspects of model building, which is very important. Right. And if time permits we would look at some exploratory topics. I would definitely spend some time on identification or state based models. It's considered an advanced subject by enlarging this course we look at input output models. Okay. That is, what is the focus of this course but towards the end we will turn to states based models. The algorithms associated with identifying states based models [have 28:49] quite involved. It involves linear algebra and so on.

Typically that's considered an advanced topic and for good reason, but nevertheless we'll spend time on it. That process will have a quick review of Kalman filter. Towards the end if time permits we'll look at some exploratory topics. Now, I've given all the chapters there are additional references that I've given.

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Course contents

Topics	Chapters
Statistical and Practical Aspects: time-delay estimation, diagnostics for model quality checks, residual analysis, model validation, handling drifts, outliers and missing data; input design	Chapter 22
Identification of State-Space Models: Kalman filter, subspace identification methods, Grey-box modelling	Chapter 23
Advanced (exploratory) topics: Recursive and closed-loop identification.	Chapter 25

I'm not saying that you should only subscribe to my book. There are other resources. Please feel free to refer to any resource that helps you. If you don't find this book really to your liking, it's not palatable and

so on. Then you can always come to me and ask for a better resource. There is no hard and fast rule, but very important that you attend the lectures. Do not please think that since the book is available you can skip lectures, all right. And go through the course outline, the course outline has links to other resources and as I said the primary software for this course is MATLAB.

So you should have some familiarity with MATLAB, if not please do develop some familiarity with it. Our institute has a license, so you don't have to really put an additional effort to procure it by other means. There is an institute by TAH license. Fortunately this batch, you are lucky. We also subscribe to the system identification tool box. So do that, have it installed.