CH5230: System Identification

Response-based Description 11

So whether it is signal analysis or system analysis the two questions that we have raised, finite data and ability to compute only on a grid apply as well. So for this reason long ago in signal processing a new concept called DFT was introduced which still rests on the idea Fourier transform.

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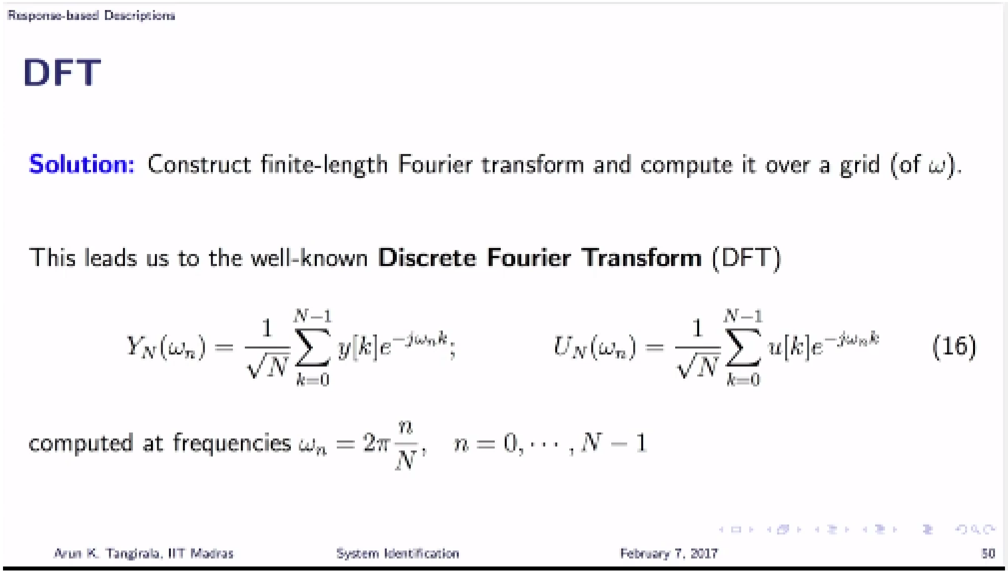

But now we have thrown away the time T. Until now we have said, these are DTFT. So this is DTFT. This is also a DTFT, Discrete Time Fourier Transform. This is also the DTFT. Why did we call the DTF, Whatever we written on the board because only the signal is discreet only in time but in frequency it's a continuous function but now having identified this issue, we would now like to compute g of omega or y of omega, whatever it is, whichever Fourier Transform you're looking at. We can only compute over a grid which means, I cannot compute y of mega but I can compute only y of omega at specific points and specific frequency values. Likewise, u of omega at some nth frequency and therefore, g as well.

Have you introduce now a subscript on omega? n. What is that n, it's a nth point. What is this nth point? What we are doing is, we are actually now saying, omega runs, let us say, this is 0 2 pi or you can say this is minus pi. Here is zero. And here is pi. This is omega access. I'm going to compute at specific points. Okay. So the nth point here, I pick this is nth point. Whatever. Any point that I pick is omega. So what are we doing now? We are sampling. We are sampling in not in time. In frequency. It's the same problem that people encountered with observing signals in time. If I want to store continuous time signals, there is no way to store contiguous time signals.

Can I store continuous time signals? What do you think? Is it possible? It's not possible. With all the digital thing that we have, we may store a very high sampling rates but the matter of fact is it's still a discrete time segment. Correct. So at that time people ask the same question. Look, I cannot store a continuous time signal. How do I now deal with this situation? Well, I'm going to sample it and I', going to observe it at a certain rate and the same question now. And there was a burning question that haunted many people for a long time. And then of course, it was resolved eventually with the sampling theorem. How fast should I sample? What should be the spacing in time? Can a simple once a day? Should I sample every millisecond? What is it? Then came the results in the form of a sampling theorem which is attributed to many people. [4:03 inaudible] which tells you, how you should choose your sampling rate?

How should we choose sampling rate for continuous time signals? Sampling frequency should be that is the maximum frequency. Right? Again, you see, the result was stated in frequency domain. You realize the power of frequency domain analysis. It directly says, if the maximum frequency in the contiguous time signal is f max then the sampling frequency should be at least, two times f Max. At least, we usually set very high. How much higher than that minimum. The same question now arises in this context as well. How fine should be the grid on omega? So the sampling theorem was resolved or it was arrived at by requiring that whatever maybe the discrete time signal whatever maybe the sampling frequency you choose, if needed you should be able to recover the continuous time signal by suitable interpolation, right.

How will you recover the continuous time signal? You have to interpolate. So they showed that there exists an interpolation formula that will ideally covered by continuous time signal for you uniquely if the sampling theorem conditions is satisfy. That is the main core. That's the core of that proof. Here also we should ask the same question, I have y of omega n, I have u of omega and so on. Whatever. Let's speak y of omega n. I have to decide what should be delta omega. Again the consideration is if necessary we may not do it. What would be the condition? If necessary by analogy with what we just discussed, I should be able to recover y of omega.

So from y of omega, right? So from y of omega n, from y of omega on grid points I should be able to recover y of omega if necessary. We may not do it at all. But you should prove that if asked for, you should be able to recover it. The key is that you should not lose information. There should be sufficient information in y of omega n for you to be able to recover y of omega. Whether you will use that information to recover y of omega is secondary. But there should be and then it was shown that the minimum spacing if you have n observations, the minimum grid spacing should be 2 pi over n in angular frequency or in cyclic frequency 1 over n. This is minimum. What is that big n there? The number of observations that you have.

So it's already now tied with the finite data thing as well. Both. Look at the DFT. And the DFT is now define addressing both these points. You look at the definition of DFT, it says, y subscript N. What is that N? It's a number of observations that go into computing your DFT of omega subscript n. What is that n? It's a index in frequency. So don't get confused between these two. And as you can see the summation doesn't run from minus infinity to infinity unlike, the DTFT. Forget 1 over root n for now that appears in front of the summation. So you have sigma yk e to the minus j omega n times k. And k runs from 0 to n minus 1.

Exactly, I have n data points with me. This is how I'll compute DFT. What have I done, you can look at this DFT in many different ways. One way is to think of saying, Oh, what I have done is, I have essentially restricted this summation to N minus 1. And then I got my DFT, is that correct? No. I have also sampled. Of course, 1 over route N is a choice. You don't have to have 1 over route N. Then I'll tell you the 1 over route n part shortly. But the main alterations I made are twofold. One, I have truncated. Two, I have actually sampled in frequency. Right, agree? These are the two alterations I have made to get my DFT. At this point, you may think that effectively what we have assumed is that the signal, whether it is output or input or whatever that sequence is zero outside this interval, right? That's what truncation would mean. When would you restrict the truncation like, we truncated the infinitely long convolution model to an FIR model. What was the implication? Impulse response is 0 upon that straightway. That interpretation was correct in that context but here you have to be careful. Just because I've truncated this summation, I limited it from zero to N minus 1 does not necessarily imply that I have assumed yk to be 0 outside that interval. You have to be careful. Is that clear?

Why? Because I have also sampled omega. I have also sampled omega. Therefore, you should be cautious before you bring in that interpretation. In other words, by restricting the summation in DTFT to just N observations in time does not necessarily mean that I have assumed yk to be 0 outside the interval. We call this as in signal analysis, we call the signal extension. What is it that you are assuming about the signal outside the interval of your observation. Did you assume it to be not existent? Did you assume it to be periodic.? Did you assume it to be constant? There are so many extensions possible because nobody knows. You only know from 0 to n minus 1. You have not seen it before. You're not going to see it after. So what kind of extension have we assumed here? We have assumed the signal to be periodic.

That means, it come as strange thing to you. But recall what I said in the previous class. Sampling in one domain in time introduces periodicity in frequency. Right. We said, the moment I am dealing with a discrete time signal the Fourier transform became periodic. Now we can show that sampling in frequency is equivalent to periodicity in time. In other words, by working with DFT instead of DTFT, I have assumed that the signal is periodic outside. That is with what period? Of the length of observation. In other words, I've assume the signal to repeat itself after and it was repeating itself before. This is an implicit assumption when you compute DFT. It is not necessarily that everyone who uses, who computes DFT is aware of this but it is extremely important to be aware of this. Those who are well versed know this fact and that can introduce a lot of artifacts into your signal analysis.

So by working with the DFT, what you are implicitly assuming? How do you work with DFT? Well, there is an algorithm. There's no new transport. Algorithm called the Fast Fourier Transform., FFT that comes as a part of every computational package, numerical package. FFT is just an algorithm to implement DFT. So FFT for all practical purposes DFT only. By working with DFT, you are implicitly assuming whether it related or not or state it or not loudly enough that the infinitely long signal that you have not observed, is a pediatric extension of what you've observed Okay. And that can introduce a lot artifacts.

The artifacts introduced by this start vanishing as the number of observations becomes larger and larger. Obviously. as N goes to infinity then DFT in when it comes to summation and even when it comes to spacing tends to DTFT. How? Because when N goes to infinity, this summation will go up to infinity, right. Or you can say, k equals minus N by 2 to N by w. Doesn't matter. This summation now will resemble, will be that of the DTFT. Not only that what about the spacing? Becomes a continuum. Right, spacing goes to 0, omega becomes a continuum. So as N goes to infinity DFT tense to DTFT. All right? So these are some very, very important aspects of DFT, which is what we work within practice. Now you may ask the question, can I use a finer spacing than what is being suggested? The minimum spacing is one over N or 2 pi by N. Can I use a finer spacing? Can I get more information? No.

You can clearly see, the number of observations has an impact on your ability to resolve two frequencies. We say, frequency resolution is now 1 over N. Right. The frequency spacing is 1 over N. So the number of observations clearly limits your ability to resolve frequency. The more the number of observations, the better is your ability to resolve two different frequencies in a given signal. And when you extend it to the system identification, we'll come to that shortly but we are now just understanding the signal part. Once I understand how things are apply to the signal part, it becomes very clear. What happens when I take it to g? We are only talking of y now u and so on. You should also now understand why this is named the DFT? We have thrown away the T, correct? Because now it is discrete in both time and frequency. So it's understood. When I say DFT, it's understood that I'm dealing with a discrete time signal and that I'm dealing with the function that is discrete in frequency.

Now let me explain 1 over route n part. The standard definitions do not involve 1 over routine N. If you look at the classical definition of DFT. But over a period of time there's 1 over route N was introduced because when you want to recover the time domain signal. Always you should remember this. When I'm transforming a signal, you should always ask, whether I am able to recover the signal from the transform? To give you a very simple analogy, non mathematical analogy, transforming is like, taking-- working with completely different domain. So if you have let us say, you've take a cloth, your shirt or your dress. If you soak it in water, it's like transforming. But before soaking it in what it, don't do on to ask whether I'll recover my cloth? Yes or no?

Otherwise, I can soak it in anything. Suppose I soak it in some substance, some liquid which leaves permanent marks and you will never be able to erase that. You will not soak in that. There are reasons for soaking it in water but still I will ask always this question, if I soak this cloth in water, will I recover it? There are many cloths, which a material which shrink, right? We ask this question when we buy cloths in the cloth store. Will the material shrink, if I wash it? Yes, madam it will shrink. We have already-- therefore accommodated for that. Okay. It's two kilometers longer than what it is supposed to be. Okay. Well just joking. But the length has been adjusted accordingly. Some material don't shrink. If you are able to recover the cloth as is through an inverse transform. What is a inverse transform? Drying. Then if drying is able to get you back the cloth you soak, then you say, you have recovered the cloth. All right?

Likewise here, I'm taking the signal from time to frequency domain with a reason in mind. If necessary I should be able to come back in time. So from y of omega N or u of omega N or YN of omega N. Suppose I want to recover yk or uk and so on any signal I should be able to do it. Can I come back? Yes. So I can come back using what is known as IDFT, inverse discrete Fourier Transform. How does IDFT formula look like? Very simple. Yk can be recovered from v of omega n using a similar looking expression. If you have used 1 over route n here to obtain your y of omega n then you can-- you should also use a 1 over route N here. And remember this n also runs from 0 to N minus. Because you're computing at that many points, right. N equals 0 corresponds to the DC component N, n being equal to N minus 1 is the maximum frequency you can detect. Beyond that if you start evaluating, you would just be repeating your calculations. So we restrict our delta this omega n. Omega actually runs from n is 0, 1 over N. In general it's n over N. If I say, omega n here, it's a set of frequencies here. Where N runs itself from 0 to N minus 1. So do you see the similarity of the expressions here just e to the minus j omega n, you have an e to the j. It's a conjugate. That's all. Otherwise they are looking very similar.

Suppose, I don't have 1 over route n here then I need to have 1 over n. And that makes it extremely difficult for students and their writing the exams because that is the time where they are under maximum tension. The general rule is, when you are under maximum tension whether you're a student, faculty it doesn't matter. When you are under maximum tension, you will be able to give least attention. Okay. To the correct details. Tension takes away attention. Remember that.

So you want to know, whether 1 over n, that is a time actually you start being very fundamental. This 1 over n should it appear here or in the forward transform? So you spent 10 minutes thinking and if you make a mistake the marks are also divided by. So that's a problem. Okay. So to avoid all kinds of confusions, if your use a 1 over route n you just have to remember only one thing that I have to use only a conjugate. Well, that's not the only reason. I'm not saying this was introduced for pedagogical reasons. What you can show now is that, with this 1 over N route here and here, you can show that the two norms are preserved. Okay. Of course, in both cases summation run from 0 to n minus 1. So this is called an energy preserving transformer or a unitary transformer, where the square two norms are identical in both domain. If that's not the case, that is if I don't have 1 over route N here and I have 1 over N here, this relation holds but with a slight modification. Clear? So much about this DFT. Actually requires much more but these details are enough. Any questions on DFT? So now that we have understood DFT of individual signals, it's time to introduce the ETF, Empirical transfer function.

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