CH5230: System Identification

Response-based Description 10

Okay, so let's continue with our lecture. If you recall we were talking about frequency response functions and today we'll kind of close the curtains temporarily on that at least, for deterministic processes. One definition of FRF is that we have seen is that it is a discrete time Fourier transform of the impulse response sequence. Which straightaway tells us that the notion of frequency response function is meaningful and exists only for stable LTA systems. The alternative definition that I talked about in the last few minutes of the previous lecture is that the FRF can also be defined, is also the ratio of the DTFT of the output and the input and the way you arrive at this is by starting with the convolution equation.

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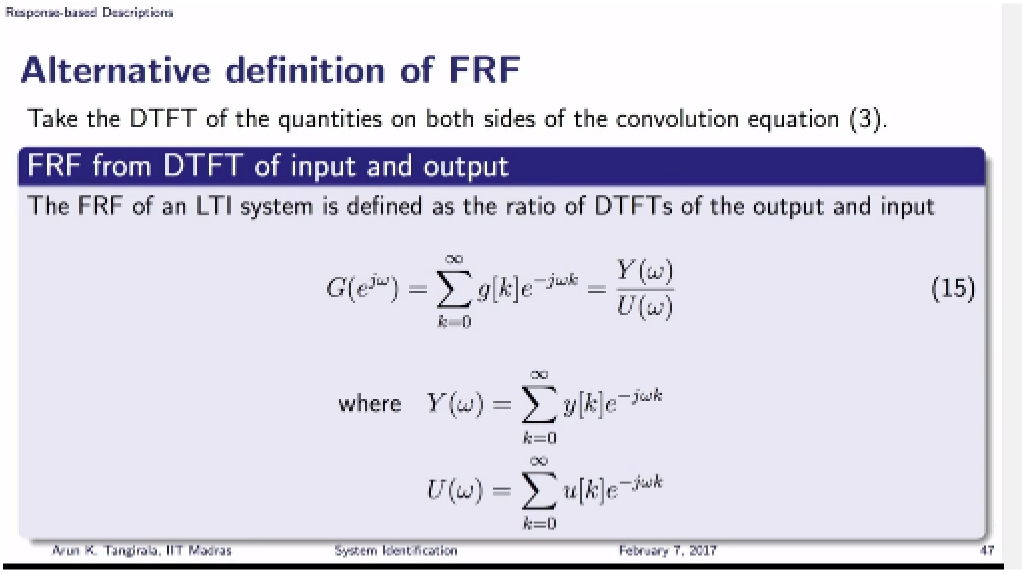

So you have the convolution equation and you take the discrete time Fourier transform on both sides of this equation f denotes the discrete time Fourier transform that is here on the left hand side, you would have sigma yk, e to the minus j omega k. K running for now, we can say from minus infinity to infinity because we are doing a theoretical analysis that gives us y of omega as per our notation. On the right hand side, you just make use of the properties of the discrete time for time Fourier transform. One of the standard properties of the DTFT is that the DTFT of convolution of two sequences. So you have two sequences there, an impulse response sequence and an input sequence. DTFT of the convolution is a product of the respective Fourier transforms. I don't keep saying discrete time, you have to understand we are referring to discrete time Fourier transforms. So we end up with g of omega. But it's good to write g of minus e to the j omega. There is a reason why I prefer to write g of e to the minus j Omega. Later on we may condense it to g of omega and that reason will become apparent when we talk of the Z transforms representation.

Okay, so this gives us the definition that you see on the screen, and once again here, we have assumed that the DTFT is exist of the respective sequences, right? And implicitly it is understood therefore that it is defined only for stable systems or systems which in which the transients died on. Okay. So all of this is theoretical definition. This is an alternative way of arriving at the frequency response function. Every definition such as this or any model representation that we have seen, like convolution equation form or the step response form or this form. All of these allow us to identify the system in different ways. That is one viewpoint that one can take of these different models that we have, right? Because although theoretically they have-- all they are all equivalent. Practically you may have different kinds of data with you. In many situations for example, this definition is amenable in many applications because directly you may have the Fourier transform of the output and input available to you from the instrument that you have.

Okay, and the other way of looking at it is that in many vibration applications, on mechanical engineering applications, and if perhaps, a few aerospace applications. You may excite the system only at some specific frequencies in which case again you can use this kind of definition to arrive at g of e to the g omega. The other definition that we had is as I've mentioned is that g of e to the j omega is the Fourier transform of the impulsive response, right. So now you have to ask, which definition is more amenable to identification, although theoretically they are one in the same. What do you think? Which is more identification friendly? So let us say that this is second definition, this is first definition, one or two? Two, because typically we have input, output data with us. I can directly compute the DTFT. We have not yet come to that. We'll talk about that now, the next. And then arrive at an estimate of the FRF.

The first definition is amenable when you have the impulse response with you. You could also argue this way from the input, output data. First I'll estimate the impulse response coefficiency, right. I'm I right? We have seen already-- although we have not learnt how to do it but there are algorithms available to do that for us. I could estimate g the coefficients and then indirectly estimate FRF. The question that you should ask in your minds is, whether both will be of same quality? We may not answer that completely today. But you should raise this question in your minds, whether applying this definition here will give me a better estimate than this one which is being estimated in a indirect way. First I estimate f and then I estimate the FRF. What do you think? At least, preliminary thoughts. It can either way. We don't have-- at least, you may not have learnt enough material to concretely argue but let us hear out some thoughts on that. The errors may carry forward in a different way. Okay, Good. Good thinking. Any other thoughts? You can think of. Good. That's one way of looking at it. Any other thoughts on this? What do you think? Ultimately when you estimate. Remember, with every estimate that you report it has to be accompanied by an error assessment, an average error assessment. So you have to also look at how easily you can compute the errors, average errors in your estimates. If you look at-- it if you go by this definition what is a source of error in this definition?

When you use it in practice? Can you think of what are the sources of errors? Because that will also now pay way for the next part of our discussion today. Suppose, I want to use this definition and practice because I'm going to be given input, output data. Do you think first of all, I can estimate FRF accurately using that definition? Let's go step by step. Let's take some baby steps. Do you think that you can estimate FRF accurately? Our goal is now the estimate, right? In identification is all about estimating g and the noise model from input, output data. It's all about that and it will always be about that. It's only g in different forms we're estimating. One is impulse response, other is step response, other is FRF and so on. With respect to FRF can we estimate it accurately? Why there is a silence?

You refuse to answer on February 7th or something like that, I mean what is it? Do you understand the question? Very simple. Can I estimate FRF accurately, given input, output data and practice using this definition? It can be yes or no. That's all. Why? Good. Good. Okay. So you can have measurements errors. What else?

Sorry. Finite sample length, good. Where is finite sample length coming to picture? I'm not sampling in frequency anywhere. Where? Did I talk about sampling in frequency here?

By taking Fourier transform.

What did I do?

We have to work with limited samples.

Okay. What can that do? Good. So to complete your answer this y of omega that you see on the board, what is a definition? It is based on infinitely long output sequence which I do not have. How I'm I going to use this definition in practice? I have to evaluate this given output data. I have to evaluate this given input data. But can I use this formula as is? What do you think? [11:09 inaudible] Can I use it as is? No. Unless you think it's a very obvious question that I shouldn't be asking. You can answer it quickly. What is the difficulty, I have a finite amount of data only. The output that I have is only about 100, 200 whatever but finite number of observations.

Whereas the theoretical definition assumes you have infinite data with you. So the point is, when I want to use this definition, the first thing, the first problem that I encounter is that I have finite data. Right? If I were to write the issues, finite data is an issue. If I give you infinite data will be able to estimate accurately? Your FRF at all omega. At any arbitrary omega? Yes. but at all Omega? There is difference, right? At a specific omega and at all omega. You understand the finite data problem that I cannot use this expression as is but I would still like to use this because this is the definition. We'll have to figure a way out of using finite data and still working with this definition. We'll come to that. So we have identified it as a source of problem or error, potential error.

The second question is can I compute this FRF? Can I estimate this FRF? Even if I were to give you very long data which you can consider is practically infinite. Let's say, I give you a million observations, will you be able to compute or estimate, FRF at all omega? All Omega. What is a range or which I would compute FRF, range of frequencies? Sorry. Yeah, minus pi to pi or 0 to w pi. We have spoken about this last class, right? So we will only worry in this interval. Notice that there's a square bracket here and there's a parenthesis here.

Which means, you would compute at one of the endpoints. You don't have to compute at both endpoints. Okay? So can you compute FRF at all Omega? Yes or no? Why? Because it's a continuum. Correct? That cannot be a source of error. But that is also a practical limitation. This we definitely expect to get results and some error in the estimate. Even if there is no noise. Suppose, that I've given you the cleanest measurement that you can assume practically that it's free of measurement error. Still this can introduce an error in the estimate of FRF because I may have to truncate this summation or I have to do something, right. I have to construct. I have to at least compromise on some kind of approximation.

So the second issue is, can only compute and at discrete domain or on a grid. Grid of frequencies. As I said, the second aspect doesn't lead to necessarily any error, it's only our inability to compute on a continuum that doesn't result any error necessarily. It is the first one finite data that can result in error. And of course, as one of you have already mentioned, you have measurement error, right. So these are the three issues I have to worry about when using this definition in practice. Did we worry on similar lines with respect to impulsive response as well?

Did we modify the convolution equation accordingly? We did, right? We said, the convolution equation assumes infinitely-- infinite number of IR coefficients I will not be able to estimate. So there are two parts available, either truncate the convolution model. How do we estimate, we are not discussing but we potentially identified a source of a problem of using that in identification. See, you have to understand every class in the big picture of system identification. At the end of every class it's a good habit for you to ask yourself, where does today's class stand in the big picture of system identification? And at this moment we are in the small subfield of models and trying to understand, what models are available? What mathematical descriptions can be given, so that when I am presented with data I can choose a particular mathematical form to identify the system?

But we don't want to study. Okay, these are the models and then this is estimation theory everything we want to nick well. That is why any model that we study we will ask this question ultimately when I use this model, what are the potential difficulties I can face when I actually sit down to identify the process. And that's what we did with the convolution from and we said, infinite unknowns cannot be estimated. So we will either choose to truncate arbitrarily well, with a reason, good reason or we will parameters g to identify a different kind of model which we have not yet spoken off, we'll speak about that as well. Here when we are studying their frequency response functions again we are asking. Okay, I have thought to the definition of FRF. Yet, another way of describing a LTA system. But when you actually sit to identify, what are the problems you can face? And we have listed the three difficulties. Let us keep the measurement errors aside for now, because we are still talking of things in the deterministic world.

In the deterministic world the first to apply, finite data and inability to compute or a continuum of omega. For this reason, we introduce what is known as the-- this is not new. It is not just that somebody discovered in identification, it has been earlier thought of in signal processing long, long ago. And obviously because of the strong connections between signal processing and system identification, you run into similar problems as well. So suppose, I'm not looking at system identification. I'm only looking at Fourier analysis, frequency domain analysis of a signal. The same story will apply. Suppose, I was only given some signal y.

Forget about all of this and I want to compute the Fourier transform because once I compute the Fourier transform I can construct power spectrum, figure out what frequencies are present and so on. Even in such an analysis, you will be presented with these two issues. Right. Forget about input, output. I'm just given some outputs, I'm response of a system. Like, I showed you for the liquid level case study, powered spectral density. How did I compute a power spectral density? I was only given finite data, right? I did compute Fourier Transform but went-- did I go by this definition? Not necessarily as is but some modifications of equate.