CH5230: System Identification

Response-based Description 8

Today we will continue our discussion on the Frequency Response Function and then get introduced to difference equations which are the parameter questions of this response based descriptions. So if you recall yesterday, we said the Frequency Response Function tells us how a Linear Time Invariant system responds to sinusoidal inputs of a known frequency. And particularly we derived this expression in 14 [0:47 inaudible] which is also you can say a definition of the Frequency Response Function.

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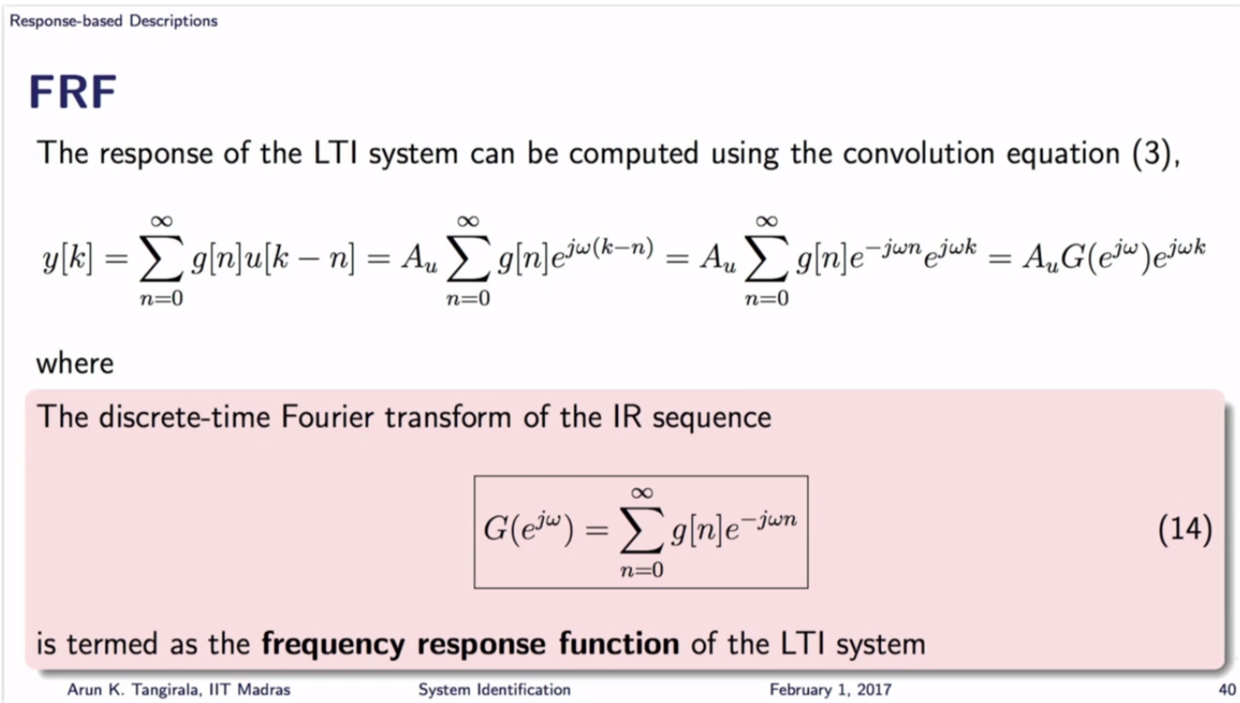

It is, as I said yesterday, the discrete-time Fourier transform of the Impulsive Response function. You must have encountered the notion of Fourier transform at some time in your career, either that you know you know the first year course or the second year course, most of the times you get to see continuous time version but in some courses you must have been exposed to the discrete-time motions as well.

So in general if I have a sequence x, let's say some x k. Then it's lead DTFT is usually denoted as X of omega, where omega is the angler frequency is defined as the infinite summation here of x k times e to the minus j omega. Of course this DFTF has enormous number of applications and depending on the context that you are looking at you can interpret this summation in many different ways. Right.

In signal processing for example you think of the DTFT as breaking up the signal into its constituent sinusoids. If you imagine that the signal is made up of sins and cosines. Again it's a useful imagination and then subsequently that is used for computing the contributions of frequencies to the overall power of the signal. If you recall in the liquid level case study I had shown you the power spectral density of the input and of the output that comes from the Fourier transform. We first compute the Fourier transform and then from the Fourier transform we construct the power spectral density. I'm not going to go into that, sorry at this moment in time but that is one of the prominent uses of the DTFT. As for as signal analysis's concern.

And as far as system analysis's concern the DTFT finds enormous use once again in frequency domain characterization of system. So you have a tool that is used for signal analysis and then one for system analysis, of course signal signals cannot exist without systems. But the reason for distinguishing between signal and system analysis is many times in data analysis I'm only looking at signals, and then when I want to say something about the system, then everything in the system. That is one aspect of it. The other aspect of it is pure theoretical analysis to begin with like the one that we are doing right now. We are theoretically characterizing a linear timing variant system. And that's what we'll focus on.

So there are certain conditions under which the x of omega exists because right hand side is a summation infinite terms and we know very well mathematically for this summation of any infinite times to exist some convergence criteria have to be satisfied. Do you recall now, by any chance, what is a convergence criterion for x of omega to exist. So, x k should satisfy some criterion.

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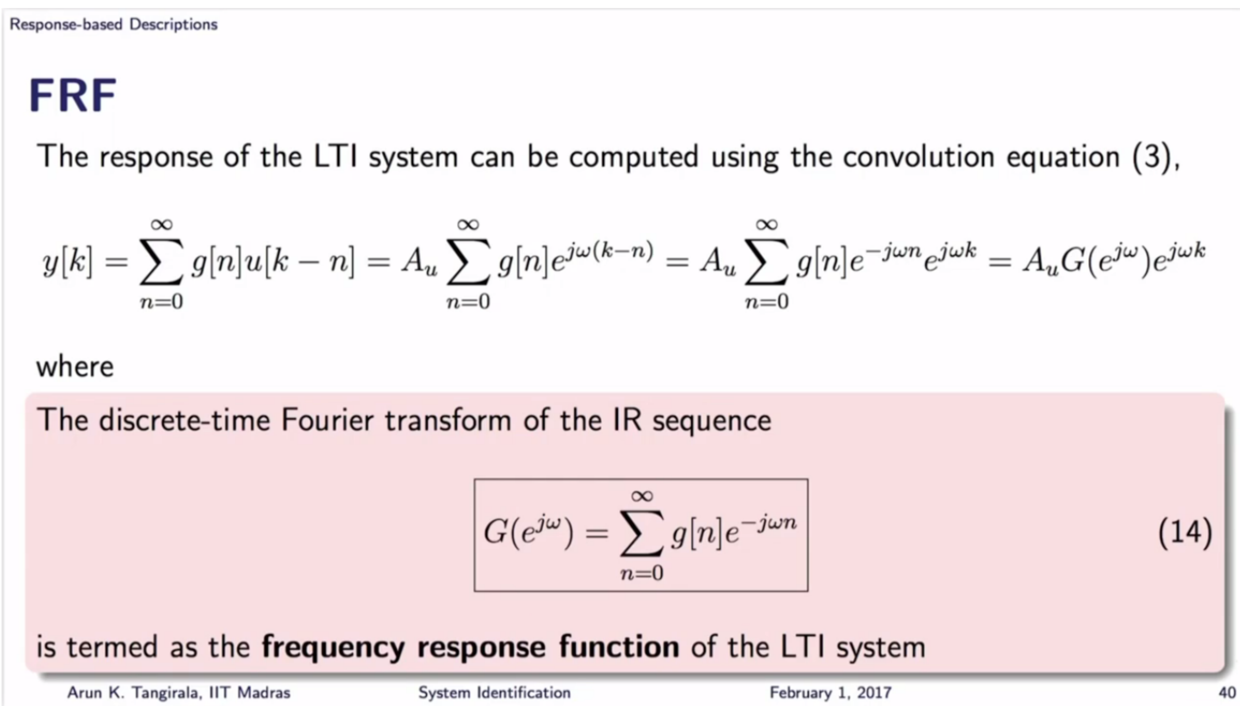

Correct. So x k is absolutely convergent. Right? That is the necessary and sufficient condition for x of omega to exist. Of course without saying k run from minus infinity to infinity. So in these theoretical analysis typically we assume that we have the infinitely long signal with us, only when it comes to practice we asked the question whether we have the infinitely long signal. What do you think? In practice, will I have the infinitely long signal with me? Suppose I want to use this in practice. I want to compute DTFT of a signal in practice.

One of the first hurdles that I would face is this summation that assumes the availability of an infinite long signal no longer holds. And we will talk about that very soon. As a result how the Frequency Response Function itself is modified to handle practical situations. But let's talk about theory for a while.

This DTFT or DTFT of any sequence exists if and only if, this absolute convergence criterion is satisfied weaker requirement is that the signal is convergence in the two norm sense or you can say the signal is a finite energy. This is a weaker requirement.

I will not dwell on what is meant by weaker requirement here. Typically I talk about this in detail in a time see this analysis course, so you can look up the videos as the course right now is presently running live. You can go to the NPTEL website at an appropriate time this concept should come up. At a later stage in this course, I'll just briefly run you through the frequency domain stuff that are required for computing spectral densities and so on.

At that time I may briefly mention what is this weaker requirement. Anyway so let's focus on the necessary and sufficient condition. When this condition is now applied to LTI systems, so you see on the screen the definition of FRF. Correct?

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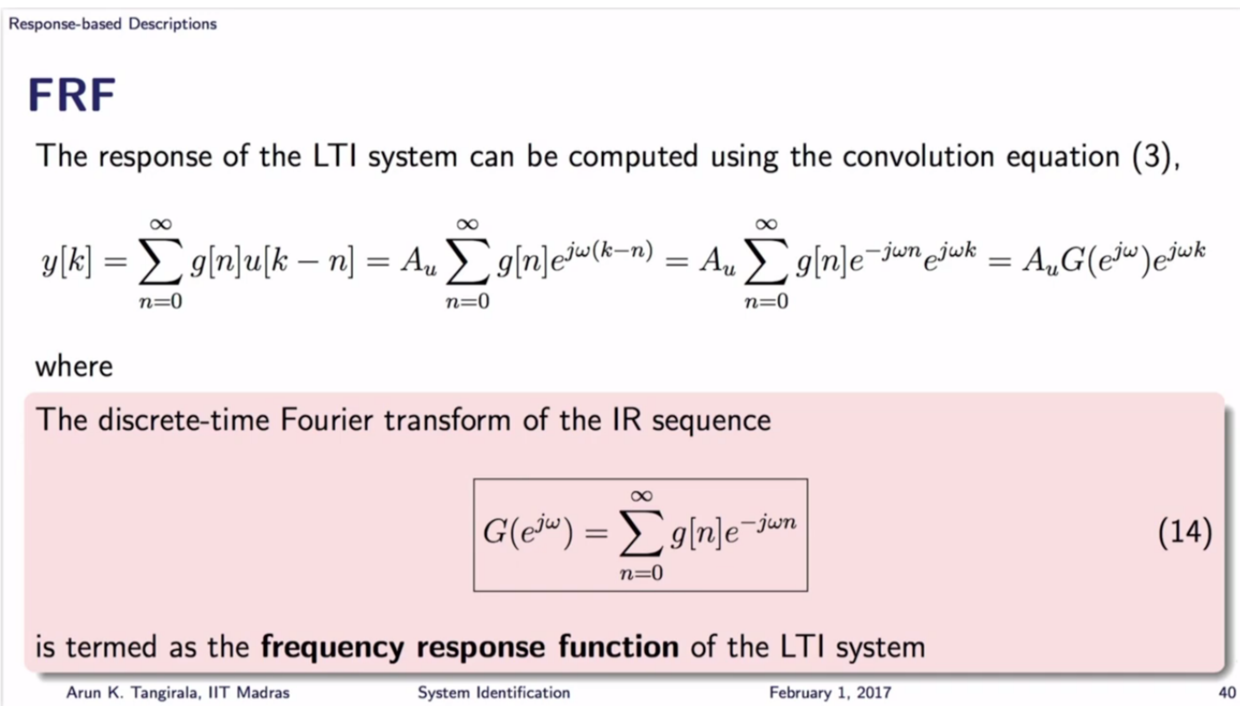

What is FRF now? It is a discrete-time Fourier transform of the Impulse Response Sequence. That's pretty clear. Correct? Now, how do you take the result that I have written on the board and apply it to the expression or the definition that we have on the screen and come to a conclusion as to what class of systems is the effort of defined for?

Okay. So first condition, we straightaway see is that the Impulsive Responses, it should be absolutely converging for the FRF to exist. We will ignore the weaker part requirement for now. So a strict requirement is that Impulsive Response should be absolutely convergent. What does it mean?

What does it tell you about the system? What is the requirement? What kind of steam? No. We discuss two different types of stability. So the system should be, [07:52 inaudible] both stable. Does it make sense? Does this requirement make sense that the system, leave aside the [07:59 inaudible] part. The conclusion that we've kind of arrived at is for the FRF to exist. The system should be stable.

Does this make sense? Does this equipment make sense? See always, there's a mathematical requirement. We are now translating that mathematical requirement of physical requirement. Some physical property of the system. Right? Mathematical equipment we could stop and by saying Impulsive Response should be absolutely convergent. Let's move on,

But it's important to interpret mathematics in the context of the physics. And we have done that, now I am asking you whether this physical requirement that the system should be stable, does it make sense? For the Frequency Response Function to exist. Why there is so much silence? Think about it. We have talked about it yesterday also. What is FRF giving you. Oh, sorry? Correct. It's giving us the-- Let us not use the word steady state as it is strictly. There is no steady state after the transients have died down. Right.?

Then you should see an oscillatory response. Correct? So the system if it is stable only then I can think of its transients dying down and then only I can speak of the amplitude of the sinusoidal waveform item that I get at output and so on. If the system is unstable, I have to wait forever for sinusoidal waveform to appear in the output. Because the system characteristics never [09:49 inaudible] Therefore, it makes complete sense that Frequency Response Function s are meaningful, A meaningfully and interpretable only for stable systems.

Because only when the system is stable, I can ask the question, what is the amplitude? How is amplitude of the output related to the amplitude of the input at after the transients have [10:13 inaudible]. What is a phase difference between the input and output? So these questions become meaningful only when the transients have died down. And that can happen only when the system is stable. So you should you should remember this, a very important aspect that FRFs are defined and meaningful only for stable systems. All right? So don't forget that.

Okay. So there is yet another definition of Frequency Response Function that we will come across shortly. But one of the things that I told you yesterday is that this Frequency Response Function is useful and that is its primary role in describing the filtering characteristics of an LTI system. And yesterday when we derived this expression that you see on the top here, it was pretty clear to us that the ratio of the amplitude of the output to the input is given by the magnitude of the FRF. And that is what I am showing you here.

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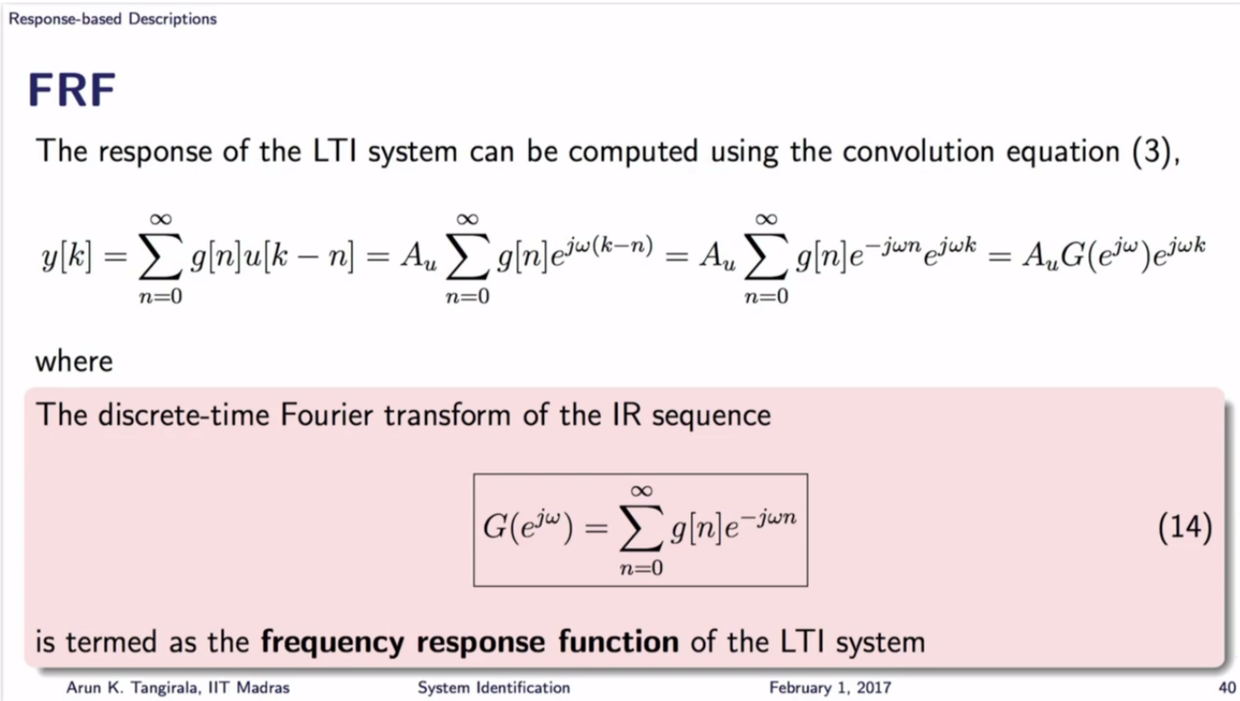

And a phase difference is given by the argument of the FRF. And predominantly in filtering applications the quantity of interest is this amplitude ratio. Then we turned to the face. What does amplitude ratio tell me? Amplitude ratio tells me whether an input of a certain frequencies attenuated or amplified, if the amplitude ratio at that frequency. Now although you don't see that clearly in the left hand side here but on the right hand side it's pretty obvious that AR, amplitude ratio is a function of omega at any given omega if AR is greater than one, then we say amplification occurs. And likewise if it is less than 1, the signal is attenuated.

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Response-based Descriptions

Amplitude ratio and Phase shift

We can make an interesting observation by writing the complex response as

$$y[k] = A_u|G(e^{j\omega})|e^{(j\omega k + \angle G(j\omega))}$$

Thus, the output of an LTI system is also a sine (cosine) wave of the same frequency, but with modified amplitude and phase.

The **amplitude ratio** and **phase shift** are given by

$$\frac{A_y}{A_u} = |G(e^{j\omega})|; \qquad \phi = \phi_y - \phi_u = \angle G(e^{j\omega})$$

Both the amplitude ratio and phase shift are functions of the input frequency ω.

Arun K. Tangirala, IIT Madras System Identification February 1, 2017 41

And sometimes this amplitude ratio can be exactly zero. What does that mean? The input is completely blocked. That's called blocking. It will just not let that frequency go through. You won't get any response. Many times when I ask questions I don't get any response from the students. So you're completely blocked it, for whatever reason, reasons known to you. Okay. However at least for Linear Time Invariant systems, typically you don't run into situations where the system blocks a range of frequencies. That means amplitude ratio cannot be zero over a continuum of frequencies.

At specific frequencies it may block, the amplitude ratio may be zero. But to really block out a band of frequencies is very rare. You don't see. All right. Typically what you would see is the significant attenuation or the range of frequencies or significant amplification over a band of frequencies. But when it comes to zero values for the amplitude ratio it can only occur typically only at some finite set of frequencies. Otherwise you can't design such filters easily. So fine, so the amplitude ratio is the primary factor that describes the filtering nature of the system, whether I call the system as a low pass filter or a high pass filter or a band pass filter or a band reject filter and so on. All of this, these names are acquired by the nature of the amplitude ration.

What does the phase sift? Tell me? Although we don't focus so much on phase shift with respect to filtering, for us the fears would be useful in estimating delays. How is it possible? So suppose I take a very simple system. Okay? Pure delay system. For a pure delay system, how do we construct Frequency Response Function? Let's see if you can construct the FRF for a pure delay system.

The purpose of this discussion is to give you the basic idea behind using phase for delay estimation. How do you start with? When in doubt always go to the definition. In fact, whether we are in doubt or not, good starting point is always a definition. I would like you to tell me what is a Frequency Response Function for a pure delay system? How do you construct it? Where do you start?

I hope you are thinking and not blocking. No, this is that time. Seriously, this is the time that you should train your brains to think and if you're in doubt you should just use this opportunity to the fullest extent and ask questions. If you are quiet here then you are going to be quiet during the quiz also. But I know quiz has to be written quietly but I am referring to a different kind of quietness.

I'm not scaring. I'm just telling you that typically my quizzes will test you on concepts more than your memory. So get your concepts clear and reinforced. Why? No, where do start? You go back to the definition of FRF. Right? How is FRF defined? I want to know what is a FRF, right?

What is G of e to the j omega for such a system? So where do you start from? Look at the definition. What does the definition demanding? How is FRF defined? Oh, really. We just saw in the previous slide. You all agreed with me, we have kind of chanted it many times. I can wait. I just want to know what is the definition of FRF? Because I would like you to think, see the major difficulty that I see for the students to do well in exams is lack of appropriate thinking. Thinking in the right direction, going in a systematic manner, all of this I see missing heavily.

Generally if you follow a systematic thinking with some alertness you will do very well. So work backwards. How is FRF defined? Why this is some much [17:35 inaudible] Yeah. So that's it. It is that G of e to the j omega, definition is simply DTFT of the Impulsive Response. Okay. Let's say the system has a delay of D sampling units. Is everyone okay with that?

See at the risk of sometimes not being able to cover or whatever teach the topics that I intend in the class, I do go slow. But you can't actually make it even slower. You have to be quick. So what do I do from this. So, yeah. So now your task, what is a Impulsive Response of pure delay system. Right? In fact, you can write the equation for a pure delay system. The input output relation, what is it? y k equals. Okay. Very good.

I can even have a factor here, maybe some factor here alpha, doesn't matter. Still that doesn't make it. It's not pure delay, some amplification. That's okay. But having an alpha there won't hurt. Okay. Then I'm giving away, I have given away already 25% of the answer. In fact 50% I would so. Good. So the Impulsive Response would be. So this implies G k. What is it? Alpha times.

How do you get impulse response? [19:37 impulse] It is as simple as that, alpha times what? Are you not answering because you don't follow or you answering it's too obvious. This guy is just asking too trivial questions. It has to be one of this. What is the answer? That's it. So why didn't you answer. Do I am going to specifically is it.

There is a tradition during Navaratri festivals. I don't know, I guess you must be, now understanding what I'm hinting at. That we have dolls at homes, it's purely a woman, you know generally Navaratri is what women celebrate. And it's a celebration of also and respect for women during that time. So women invite other women to come to their homes and see the dolls and so on.

And with that they take a small box of kumkum. And say, "Please come to our house." I don't know if you want me to follow such a tradition here. Please tell me the answer. Just tell me the answer, that's it. I don't have to specifically ask you, fine. So what happens to G of e to the j omega? Log in that into that expression. What do you get for? Correct. True. So now I having realized that, what is answer? You're right.

Correct. So you'll get here alpha times, don't forget the alpha e to the minus j omega D. Correct. That's all the answer is. It took so long because I had to probe you. I have to prod you and so on. But that's all to it. Now what can you say about amplitude ratio and phase. What is the amplitude ratio for pure delay? How much is it? Alpha. Very good. And what about phase difference? That's right. this is also a function of omega. This is AR of omega minus, lets write us minus D omega.

Good. That's all it is. Now can you explain how we can estimate delay given the phase? Yeah. So just plot the face or just omega. And the slope of the phase will give me the delay. This is a completely different way of estimating the delay. If I give you Impulsive Response, how I going to estimate the delay, look at the first nonzero value, the lag at which you get the first nonzero value. Whereas, if I give you the phase you will would look at the slope, although we have derived this for nice free conditions, under noisy conditions it's not going to be exactly we have to do a curve fitting there.

But this is the basic idea behind using phase for estimating delays. It may not be in this simple form but the core idea is always this, that the delay appears explicitly as a parameter. Do you see the big difference between using Impulsive Response coefficients and phase in estimating delay? What is a prime difference that you notice? What is the unknown parameter that we are estimating D, delay? When it comes to impulse response coefficients this parameter that I am estimating delay is an implicit form. Do you see delay appearing explicitly in the impulse response coefficient? It isn't.

So it's implicitly, I'm searching for this delay, impulse response is an implicit function of it. Whether it makes a difference to estimation? Yes it does. Because, my search algorithm has to be different. The decision variable is not appearing explicitly. Whereas you look at the face the delay appears explicitly as a decision variable. My optimization becomes a lot easier and simpler whenever the decision variable is explicit rather than implicit. Implicit you have to go and extract, it becomes kind of qualitatively I'm saying this. Whereas when a decision variable appears explicitly it becomes a lot easier. I'm not saying it's going to be trivial but relatively it's going to be easier to obtain the optimal parameter estimate. So that is one of the prime advantages of using frequency domain methods to estimate delays.

Okay? There are other methods, there are other advantages which will not talk about, but hopefully now you understand how the phase is useful in estimating delays. As for a system identification is concerned. When you look at amplitude ratio, it tells me what kind of filtering nature the system has. Again which is quite useful in system identification. Any questions? This is a typical way that you, in which you derive the Frequency Response Function.

Now I will go through some examples, at least I'll show you one demonstration in MATLAB of the FRF.

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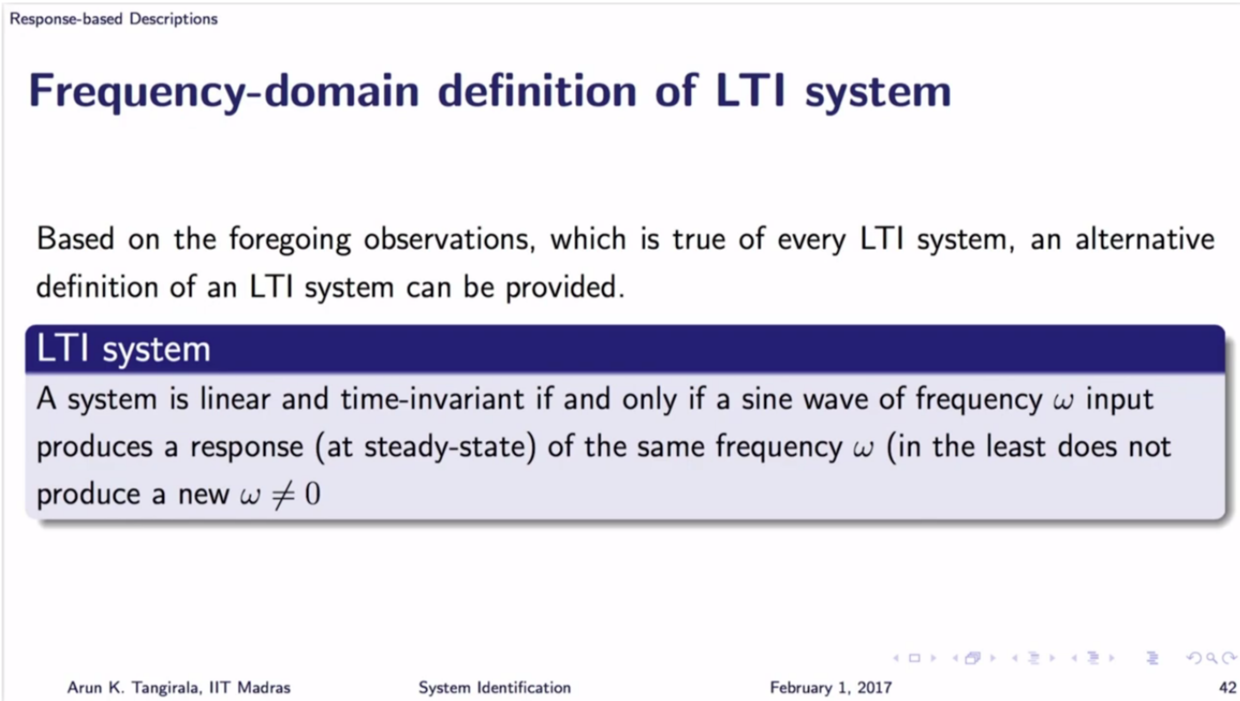

And show you how do you obtain the bode lot, so-called bode plot. By the way, let me actually draw the bode plot here. A bode plot is a plot of AR versus omega. It consists of two plots. One, the amplitude ratio and the other is the phase. Both versus omega and typically you draw on a log, log scale for amplitude ratio and a semi log scale for phase. Let us not assume a log scale here for easy drawing here.

How does the face look like for a pure delay at omega 0. So let's see here is the 0 for phase. Just fall off linearly. And how would amplitude ratio look like? Sorry. [26:20 inaudible] Correct. So it will be alpha, a flat line. So as you can see what has happened beautifully with the help of FRF is what we mentioned yesterday. We said Delay is not necessarily to be clubbed with dynamics. Of course, phase contains dynamic contributions also.

But one thing that we have managed to achieve is to come up with a measure that is only a function of delay. There are also going to be dynamics in this, because you have considered a pure delay, your phase has only contribution from delay. But one thing that you should observe is at least we have decoupled the gain from delay. Whatever be alpha, phase doesn't change. Right? Phase is independent of it. The amplitude ratio is a function of the gain alpha in this example, but in a general dynamic for the general dynamics system it will be a function of both gain and time constant. And phase would only be a function of delay and time constant. We have not managed to completely decouple delay from dynamics, but at least some decoupling has occurred. And that is again one of the advantages of working with Frequency Response Functions. We'll talk about, the bode plot again more shortly.