CH5230: System Identification

Response-based Description 7

Now it is time to move to the third elementary response that we consider for an LTI system which is so called frequency response description. The much feared by not just students but also instructors.

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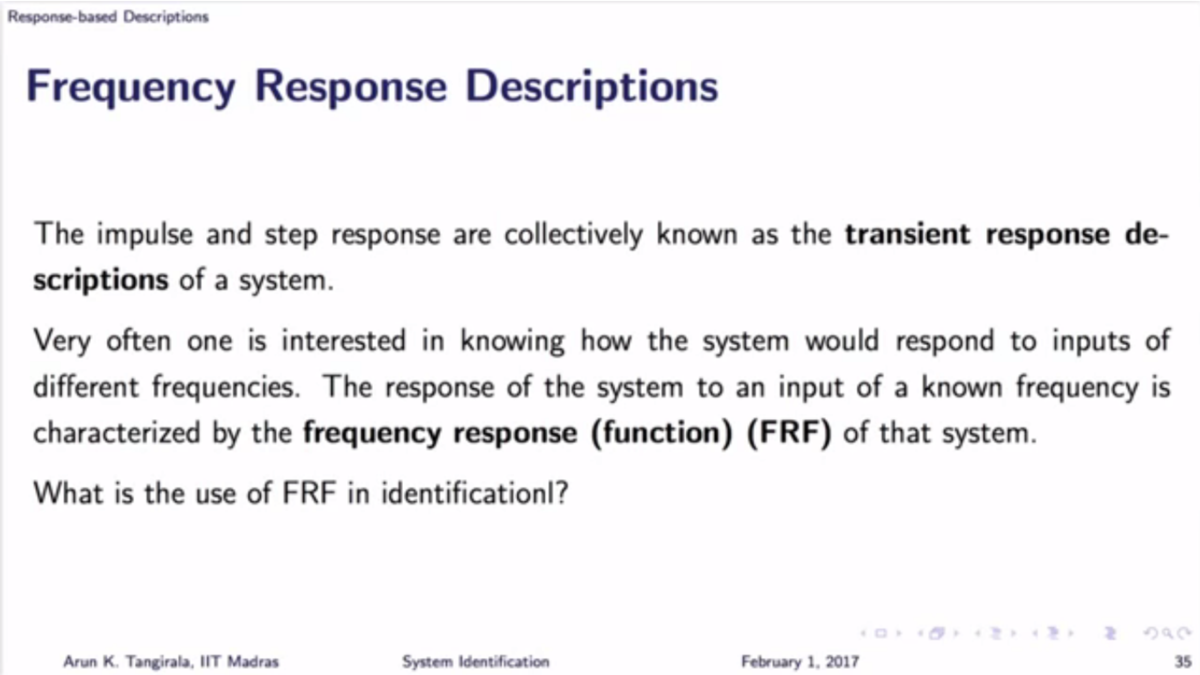

Seriously there are, in the last 15 years of whatever little I have seen across the globe many faculty, many instructors who teach process control. This is across the globe not just within our country. When it comes to talking of frequency responses, they'll say, okay, today is not such a good day. They look at the panchangam and say, well, no, no, I don't think it's a good day for discussing frequency response, let's postpone it a bit. It's not needed after all. Or just scratched the surface say, hello to it and run away. Like, you light a Lakshmi atom bomb during Diwali.

As if it's going to explode. And then what happens is, it gets pushed towards the end and towards the end you say, well, I don't want to rush through and teach you, because it's a very subtle,they're subtle concepts involved there. Already you have learned a lot. It's okay. I have already taught you the spelling of frequency response function from there on you can actually learn. And that so it never gets taught fully. I'm not saying everybody does that but this is a typical generic attitude of instructors when it comes to teaching frequency domain methods. And it's a big pity because as far as LTI systems is concerned, the frequency domain offers such great advantages of analyzing and identifying systems that people seldom underestimate. Okay. So I am absolutely comfortable and I love teaching frequency domain methods. That doesn't mean that I'm going to trouble you with that but hopefully, I'll give you a lot of clarity on it.

And there are many others. There are a few others as well in the process control sphere, who love frequency domain descriptions. So what is this frequency domain description? Question number one. And what is it to use in identification? Question number two. Frequency response is very simple, you don't have to make it look exotic. We have talked about the response of a system to impulse to step. And now we are looking at sinusoids, oscillator signals. You may ask, is this how the intercourse is going to run and you'll going to teach me, response system to impulse, step, sinusoids and then maybe tomorrow chirps and so on. Is this a never ending story? Why can't you stop? Because convolution equation tells me that if I know the impulse response I know everything. Why are you going on talking about responses to different signals? So one question we have answered, frequency response description has got to do with how the system responds to oscillate, sinusoidal not any oscillator signals, sinusoidal type signals.

Why are we now considering this class? When it came to step response, I gave you the arguments as to why step response should be studied separately. Why it deserves a separate attention. Now we will ask, why frequency response deserves a separate attention, when I might as well calculate its response using the convolution equation, whenever required. Well, the frequent, one of the most important uses of frequency response descriptions is, it tells me, what is the filtering nature of a system.

What kind of inputs does it filter? What kind of input does it attenuate? Or amplify or completely block them, which we called as rejection and so on. Why is this useful? If this were, this theory were not be there we wouldn't be able to use any of the communication devices using this or even you're, you know, military communication devices, our telephones, our remotes that we use, everywhere this concept of filtering is there, right in the devices that you use from dawn to dusk.

When I'm tuning into a radio station, FM102.3, that's a 102.3 is a frequency, right? What is it? Megahertz? Gigahertz? Megahertz. That's a frequency. What is happening, what does that device do? The device is capable of receiving, it is continues to receive so many signals, but the moment a tune to turn the knob or press the button or 2 whatever. Set the station to 102.3, the circuit filtering nature changes. And only allows the 102.3, frequency to come in.

Well, not so precisely, but more or less precisely, because I don't know if you still have these olden radios where you would have a mechanical knob that you would turn. And you won't exactly reach 102.3 straight away. You have to go slowly to that station. As you are approaching that station you start receiving. And then as you're leaving also you do this. So there's a small bandwidth that on 102.3, right? But that's pretty small. There are no other stations within that vicinity. Sometimes there can be. Okay.

So the circuit inside this device is acting like a filter and as you are changing certain resisters and capacitance and all those devices that constitute a circuit, those values are changed. So that over all this circuit inside acts like a filter and only allows 102.3 plus delta, plus or minus delta to be received. And this is the case for all communication devices. So you may ask, okay. That's fine. Then why are you in chemical engineering? Why don't you go to electrical engineering and teach them? From other engineering disciplines viewpoint also this is extremely important, because every system experiences some kind of disturbances. Or, of course, there are also inputs.It is quite useful to know what disturbances this system can reject?

To which ones it will respond significantly and to which ones it is, you know, it is immune to? It doesn't care. It's okay. High frequency disturbances, I don't have to worry about. Suppose, I take the liquid level system and let us say, the value is very jittery. So it induces high frequency fluctuations. If I want to know whether these jitteryness of the jitter in the valve is going to affect my level reading. How would I know? One way of knowing it is by turning to the frequency response function, because you can model this jittery nature as a high frequency signal. Imagine it being made up ofsinusoids of high frequencies and then ask the question, how will the system respond if I were to excite the system with high frequency sinusoids, If my theory tells me don't worry it'll attenuate it, I can relax. I only have to worry about replacing the valve but otherwise there's no other worry. Correct? Now we'll talk about identification.What is the use of frequency response functions and identification? We have already spoken about it earlier in input design, when we were discussing the case studies.

We know very well now at least after the case study that the kind of frequencies that I pump into the system that I inject into the system plays an enormous role in identifiability. One as I said, a single frequency allows me to estimate a two parameter model. More frequency content in the system, more number of parameters better is a resolvability. And the other second aspect is what to do with the bandwidth? I should not excite the system with such frequencies to which it won't respond. Then I only get noise. So and there's a third reason, the third reason why we want to look at frequency response function is, we know that all models are approximations.

Which means there's going to be a difference between the truth and what I believe. Suppose, that means there's going to be an error. Suppose, I want to share this error in frequency domain, which means that I want better fits in a certain regime for whatever application that I am looking at. It doesn't have to be always within the bandwidth. There is some frequency of interest for me. Range of interest and I want that error or the bias to be low in that frequency region. I don't worry about other frequencies. This is called Bias shaping in identification. I want to shape the bias. Then once again I need to know the frequency response characteristics. So you see there are three, at least three top reasons why we want to look at frequency response function and identification. So let's know understand. I've already talked about it. We'll, of course, I use the term pre-filter, but that's also related to the approximation error.

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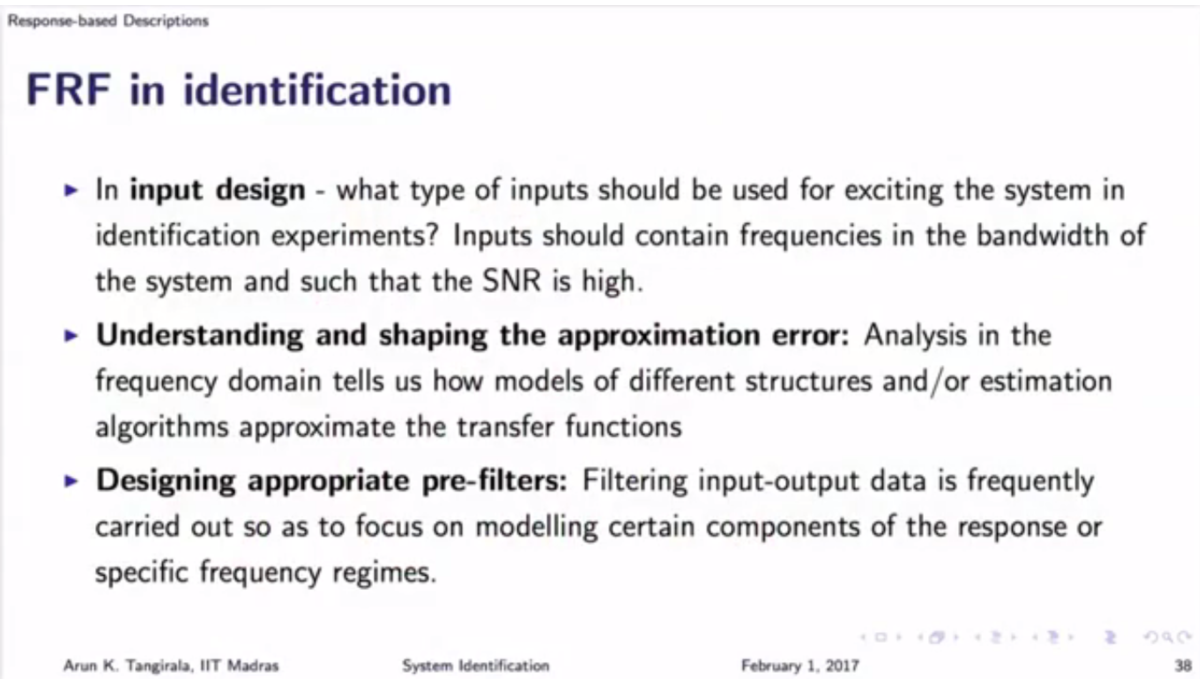

Let's know understand the technical definition of a frequency response function. As I said, the frequency response function as we call it. Why is it called function? Because as we will see shortly. It's a function of the frequency that I inject.

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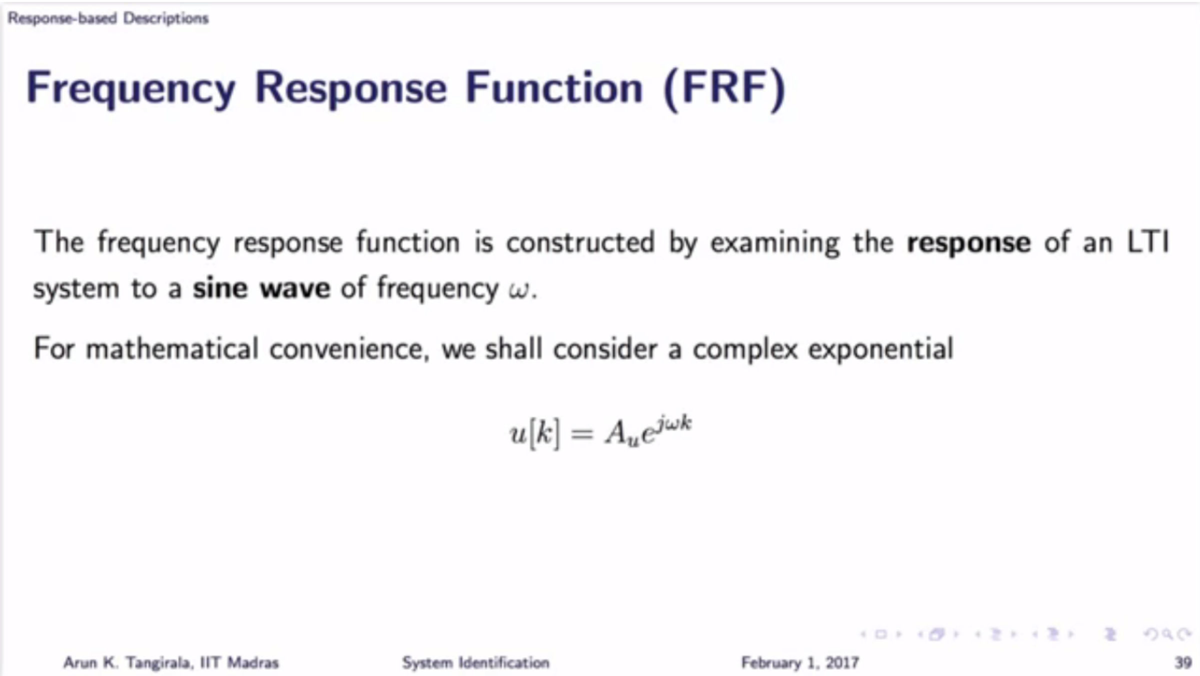

Unlike, impulse and step, where it's only a function of time t, there's nothing more. Here it's a function of the frequency that I inject, right? Hoping that the system will respond differently to different frequencies. It is a response to a sinusoidal input. Although we say sinusoidal input for mathematical convenience, we'll consider a complex sinusoidal. Instead of consulting sine omega k or cosine omega k, we'll consider e to the j omega k. Do we really excite systems with complex sinusoids? May not be, but for mathematical convenience the results won't change at all. For mathematical convenience, it is useful to consider a complex sinusoid, because then the main result that is usually discussed in frequency response analysis comes out that easily. So let us assume that the input is a0 e to the j omega k. Okay.

I didn't intend the rhyming there. But now I want to calculate the response. How do I calculate the response? Here I have, y. I go back to the convolution equation that is still the mother of all equations there. So I plug in the input here. Very simple, these all very simple algebra there's not much there.What is of interest to us is this expression that you see here. So assume that the input has an amplitude a0 and that the frequencies omega or omega 0, whichever you like. Let's keep it to omega, because that's what we're using in the equation. What do you notice there in the final result the right most expression that you see? I plugged in. I injected a0, e to the j omega k, right? What did I get back? It's like saying I mean, I invested so much time what did I get back? So it's like this here. I injected a0, e to the j omega k and I get back a0, g of e to the j omega. What is g of e to the j omega? It is defined here in equation 14 and we need to talk about that. Times e to the j omega k. Now a subtle point that you may not have noticed in this derivation is that we have assumed. We have derived this expression for large times, although it may not be so obvious in this derivation, but it is implicitly assumed that I am looking at some kind of a steady state behaviour. There is no steady state here, because the input itself is oscillatory. We can't use technically the term steady state, but let us say that we have reached a time where the systems characteristics have died on.

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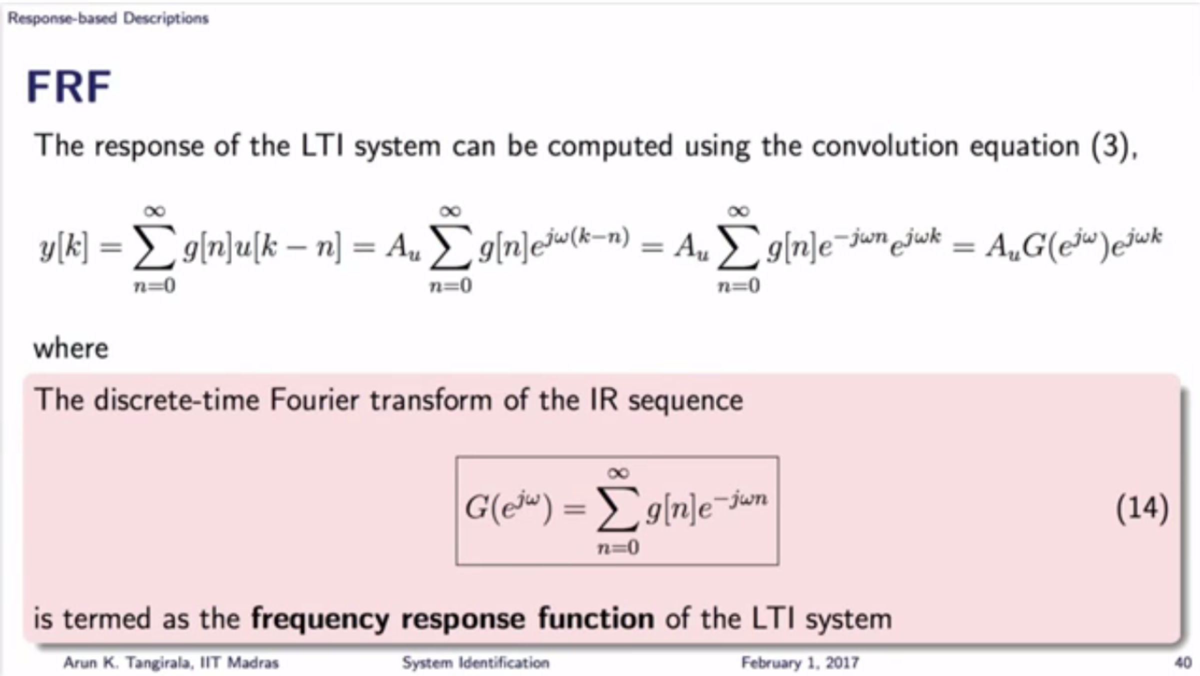

Remember, the response of any system to an input consists of two parts its own transients and that is why you see ,this kind of sluggishness even in step response. Plus the input side, if you recall a step response, look at the step response, how does it look like? There is this portion and then it reaches a constant value. Compare this with the step input that we give. Need not to be a same magnitude, but this is your u step, step input versus time t. I fed this to an LTI system and it gave me this response depending on the order. So if you look at clearly the LTI system, the response of the LTI system, very interestingly, the input, the shape of the input eventually appears in the output. Even for impulse response. If I were to give an impulse how would the response look like, the impulse is actually located at a highly localized in time.

But if I look at the response, the response is like a stretched impulse. Eventually, if the system is stable, we are only talking of stable systems for now. Eventually, it will go to 0. Input also goes to 0. You can attribute this portion here due to the dynamics. This is because of the inertia. Filtering people would say this is a distortion of the input. There are several ways of looking at it. Ultimately it's a convolution that is causing this and the convolution is coming about because the system has inertia. If the system had a pure delay, how would the response look like, the step response? Simply, I shifted step. Correct? No dynamics at all. Likewise, if I freed a sinusoid to a pure delay system, no response until delay after that the sinusoid will appear.

But the response of a system that has inertia, LTI system that has inertia to impulse, step, sinusoid, whatever it may be is such that initial portions will contain the system's transients, once those have died down the input shape appears. That is the hallmark of an LTI system only. No other system will give you that. So we say here, if you look at the g, we have not talked about g of e to the j omega. We'll talk about it tomorrow. But g of e to the j omega is some complex function of omega. Okay, it's a complex number.What do you see here? I gave a complex input, a complex sine wave of frequency omega and amplitude a0.

And what I got back in return at large times is, again a complex sign wave of what amplitude? You cannot say, a0 times g of a to the j omega, because amplitudes are always real value. a0 mod of g of e to the j omega. So the amplitude of the input is being modified and that why maybe we're saying mod, but mod of g of e to the j omega is the one that is responsible for either amplification or attenuation. But is the frequency of the output an input the same? It's the same. But there is also phase shift. If you write g of e to the j omega in polar form, there is a mod times e to the j argument of g of e to the j omega. That argument of g of e to the j omega contributes to a phase shift.

So to summarize, the response of an LTI system to sinusoidal input at large times is the same sinusoid that is of same frequency, but with modified amplitude and phase. How do I know, how much a modification has occurred? I simply need to look at g of e to the j omega for that system, which I can calculate. If I give you impulse response the mathematical equation there tells me, I can calculate g of e to the j omega. That tells me, whether this frequency has been amplified or attenuated and how much phase shift has occurred. The most important characteristic for us, although phase is important is the magnitude of g of e to the j omega, because that tells me the filtering nature.

If that is going to be very small for some omegas, then we say, those frequencies are being attenuated and likewise, for amplification. Since g of e to the j omega contains all the information that you need to know of how a system responds to sinusoidal inputs. It is called a frequency response function. It's a function of the omega. Quite different from the impulse and step response which were only functions in time t. Here it's a function of omega. It consolidates all the information that you required to know of how this LTI system responds to sinusoidal inputs. Why frequency? Because sinusoidal inputs are characterize by frequency. Why function? Because it's a function of the omega. That's all.

So whenever we think of frequency response function, we should think of g of e to the j omega. Okay, because it denotes the magnitude of which denotes the amplitude ratio, the phase of which tells me, the phase shift. We'll close the discussion today by saying, this frequency response function is nothing but the discrete time Fourier transform. When you look at equation 14, it is actually the discrete time Fourier transform of the impulsive response sequence. DTFT as it is called. Not DFT, don't confuse it a DFT. If I know impulsive response therefore I can calculate FRF. Likewise, since Fourier transforms are unique, if I give you FRF, you should be able to recover the impulse response. So once again, showing that I can technically recover one response from the other, I can calculate. Okay.We will talk about this more in detail tomorrow.