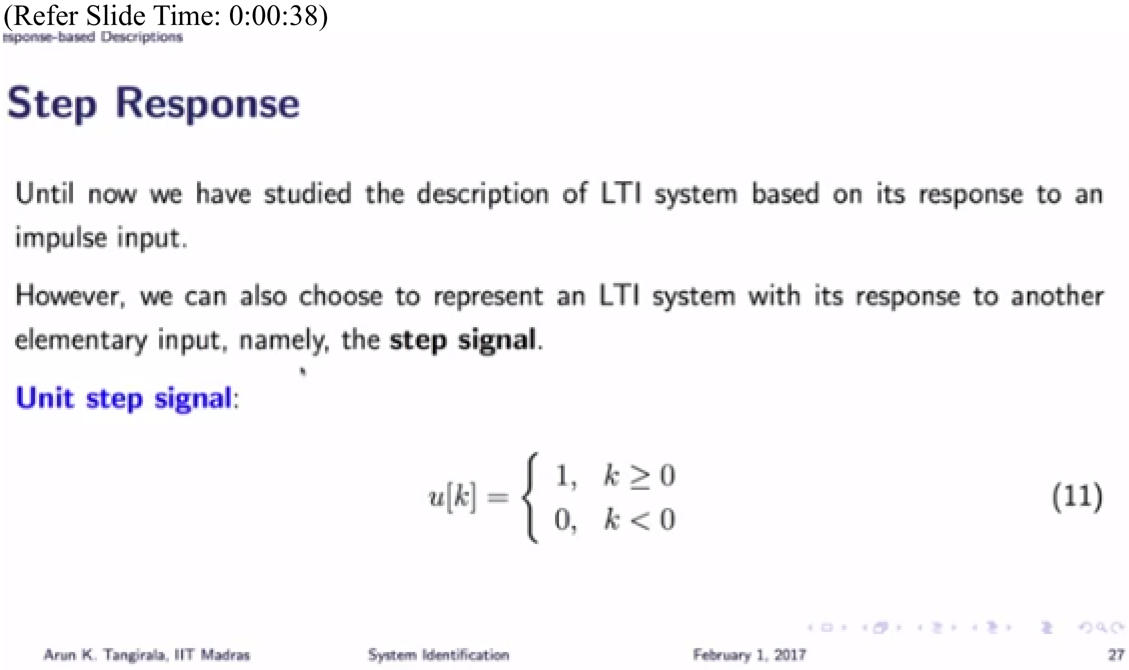
# CH5230: System Identification

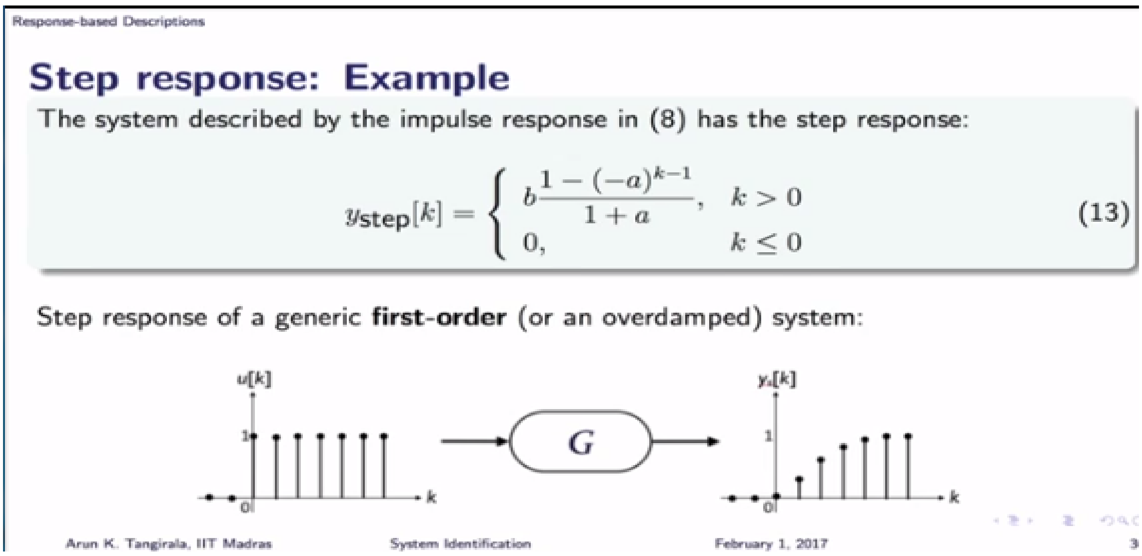
## Response-based Description 6

So, why do we consider this elementary signal step? Impulse was kind of obvious because, you know, it is like the founding stone of your convolution equation. The reason for turning to step responses to step signals, or step type inputs is because a lot of changes that you see in the inputs, too many

processes are stepped like signals. And chemical engineers particularly love step response. You will very rarely see chemical engineers talking about impulse response, whereas electrical engineers, mechanical and sometimes aerospace may talk about impulse responses.
(Refer Slide Time: 0:00:38)




So, how does this sound look like, we have seen, right? Step looks like, I'll show the schematic first and then go back to the previous slide. So, a discrete time unit step looks like this. The signal is shown on the left. And what you see on the right is a qualitative sketch of first-order systems response to a step input. Okay? And the first-order system is given here and the response of the first-order system is given at the top in equation 13. What do we mean by first-order system? Here first-order system with a unit delay, right?
(Refer Slide Time: 0:01:35)

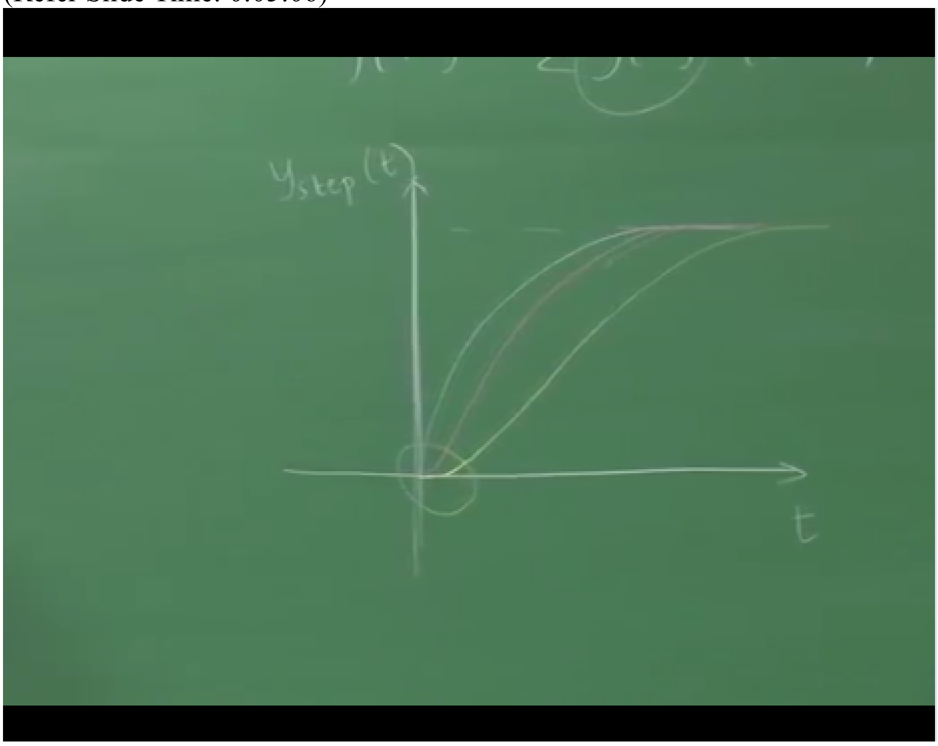


What kind of system are we looking at, the same system that we have talked about earlier. The same first order with the unit delay this is something that will keep using from time to time. So as you can see from the qualitative sketch of the step response, we can also spot the delay, right? So the step responses also capable of giving the delay.

The first instant at which you see a significant or step response because we are talking of noise free case, nonzero step response is the delay. And gain is also easily readable. We know how to calculate gain. Gain, when we say gain, typically it's called DC gain. Because today we will talk about frequency response, I'm going to use this term DC gain which will fully understand once we talk about frequency response. So, the steady state gain is what we call as gain and how is it defined? It is a change in the output to the change in the input at steady state, which means there is an implicit assumption that the system is stable. Right? We can't talk of gain for unstable systems. And that can be also easily read off from the step response plot and I've already shown you in the liquid level case study how to read off the time constant.

If the system is a first-order, if the system second order, depending on whether the kind of damping that you have, if it is under-damped, then you would see a step response having some oscillatory characteristics, which eventually die down, because it is stable. And for an over-damped system, the oscillations are completely suppressed, and the response looks a bit more sluggish than the first-order. And as you increase the order of the system, the step response looks sluggish and sluggish and more sluggish. Now, at this point, I should also mention something that you will frequently run into. When I-- if I were to draw the step response, I'm going to just draw the continuous time step response. What you see on the slide is a discrete time one.

I'm going to draw a continuous time step response because it makes the discussion a bit easy. So the x-axis is now continuous time t, the first-order system, continuous time system without delay, let's assume that there is no delay in the system. This is how a typical first-order step response would look like. And a second-order, over-damped would look like this. Let's assume all of these systems have the same gain. I'm not drawing it very well, but you should understand what I'm trying to show here that you'll find some sluggishness here. And suppose I look at a fifth-order system, it may go this way. Well, it can reach quicker that's okay, don't worry about the settling time part. So I would like you to pay attention to the initial portions of the response.
(Refer Slide Time: 0:05:06)



Okay, because there's a board. I can't zoom in further. But look at the initial portions and compare the responses of the first-order, second-order and some kind of a fifth-order, fourth or fifth order. What do you notice there? For the fifth-order system, there is a lot of sluggishness. So you can think of, let's say, a rat, as a first-order. Maybe Rat is even a fractional order, I don't even know. Right? It just moves so quickly, the inertia is so smart.

But it has some inertia. Compare this with the Bull, the controversial Bull, okay? Or if you don't want controversy, let's go to Elephant, as of now, elephants are not in any kind of controversy. So you can't believe that humans were the ones who are in controversy, now animals are also being dragged into controversies. Okay. Anyway, so now if you compare the initial responses here, you notice that in the initial portion the first-order and the fifth-order let's compare these two, it's very sluggish, takes time.

So you look at a Mahavat actually asking Elephant to rise, doesn't stand instantly, like this slaying down take that some time, [speaking in Hindi] it will happen. You know, where you hear this kind of responses, right? [speaking in Hindi] happens don't worry. Now, whereas the Rat, you just, you don't even have to say anything, it looks at it moves. Because it has such a low inertia. So, this portion of sluggishness, suppose I didn't tell you that this is the response of a higher order system, it is I just tell you, it's a response of an LTI system.

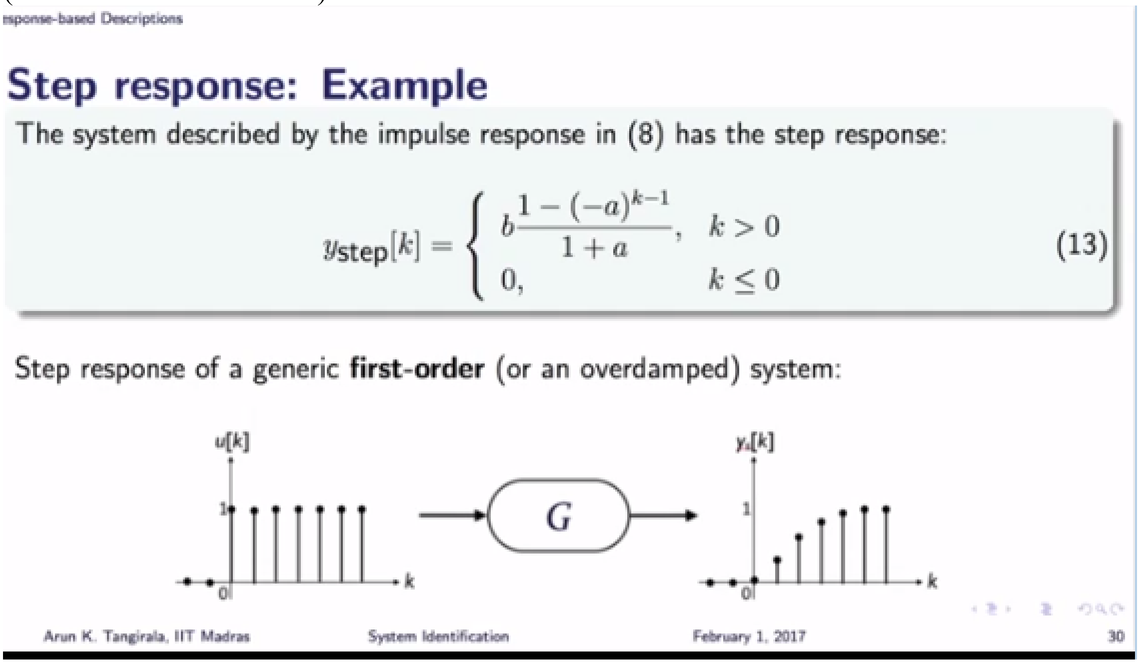
Then there is every possibility that you may think that there is a delay in the system, this is called the apparent delay, okay? There is a true delay and then there is an apparent delay. So, this portion here manifests as apparent delay. The observer would not know whether such a delay isn't is a true delay, maybe due to transportation lag or whatever measure and so on or heavy inertia, that is sluggishness do too heavy inertia manifesting as delay. It may be very hard for the user to distinguish without probing further. This is the basis on which you will see in a lot of control literature particularly Process Control literature, what are known as first-order plus time delay approximations, second order plus time delay approximations, and so on.

Once the sluggishness is crossed, it looks more or less like a first order, more or less, I won't say exactly. So, you can or if you don't like first order, second order, so, you can approximate reasonably well the response of a higher order system as a first-order or a second-order system with delay, but that delay is apparent. And if you turn to the literature on time delay estimation system identification, only very few articles talk about this distinction, but of course, you know, those are good ones. And you, whether they discuss the difference between true and apparent delay or not, you should be aware of whether this estimation algorithm is getting you is referring to the true delay or the apparent delay. And from the observer's viewpoint, it may not be possible to resolve sometimes. But you can come up with some methods which can distinguish between true and apparent delays.

Visual inspection may not give you much resolution here, resolvability maybe poor. So, you should keep this in mind, a step response can mislead you in terms of the actual physical nature of the process, but as far as a model is concerned, lower order plus time delay approximation is reasonably well justified. Okay? So this is something to keep in mind, but of course for discrete time systems operating with a Zo, zero order [09:42 inaudible] discretized systems, I won't say discrete time system, so the discretized system which is something that will, we'll talk about later. With a zero order [09:50 inaudible] we know the zero order [09:51 inaudible] introduces a delay. Why it does, we'll see later. So you should expect to see a minimum of one delay in all discretized or sample data systems with a zero order [10:04 inaudible] at the input side. Any questions?

Before we move on to the next response, it is instructive to know the relation between, in fact not only instructive, it's useful even for identification, to know the relation between step and impulse response. See, if you look at the step input here at the bottom, you'll notice that this step is made up of impulses. And we know that there is a relation between step and impulse, right? A step, like we wrote yesterday in the previous lecture, any arbitrary input as a sum of scaled and shifted impulses, here also, I can think of the step as, made up of several unit impulses just shifted in time and added you know, superposed on each other. Since there is a relation between the inputs themselves that is a step and the impulse and given that the system is linear and time invariant, we should expect some relation between the step response and impulse response.

(Refer Slide Time: 0:11:10)




And what is that relation? This is the relation. How do you derive this relation? Really easy. Go back to the convolution equation here. And all you have to do is, replace the input there, with a step-- unit step, by default it's assumed we're looking at unit step. So here you have, why step is a step response? Once you plug in for step in the input, you get the step response being the sum of impulse response coefficients up to that instant, which means it's an aggregation of impulse responses up to that instant.

(Refer Slide Time: 0:11:53)

## Step and Impulse response

A step signal is a simple superposition of shifted impulses. Given the linearity nature of an LTI system, the response should also be expected to be a superposition of impulse responses.

**Step and Impulse Response**

The step response can be expressed in terms of the impulse response using (3)

$$y_{\text{step}} = \sum_{n=0}^{\infty} g[n]u[k-n] = \sum_{n=0}^{k} g[n] \tag{12}$$

Fair enough, because if you look at the step signal, it is also an aggregation of impulses up to that point. So the response will also be an aggregation of the impulse responses up to that instant. And this is used in the system identification toolbox to estimate step response. Previously, I remember in the previous versions of the [12:17 inaudible], Toolbox, and MATLAB, the step responses were estimated directly. What we mean by directly is, you can re-write now this conversion equation in terms of step response coefficients.

(Refer Time Slide: 0:12:17)

Response-based Descriptions

## Step and Impulse response

A step signal is a simple superposition of shifted impulses. Given the linearity nature of an LTI system, the response should also be expected to be a superposition of impulse responses.

**Step and Impulse Response**

The step response can be expressed in terms of the impulse response using (3)

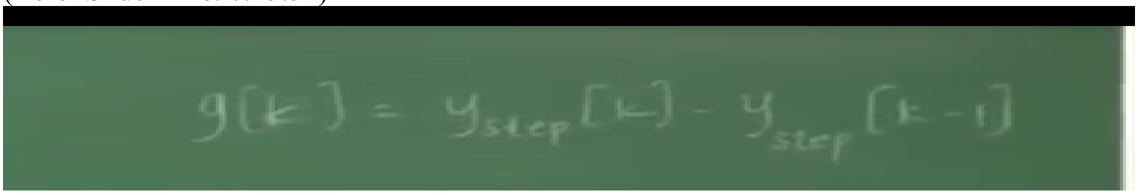
$$y_{\text{step}} = \sum_{n=0}^{\infty} g[n]u[k-n] = \sum_{n=0}^{k} g[n] \tag{12}$$

That's a simple exercise that you should take home with you. Use the relation that is given here, which allows you to calculate step response from impulse response. How do you calculate impulse response on step response? If I give you step response, how would you calculate impulse response?

STUDENT 6: [12:59 inaudible] impulsive difference.

ARUN: Correct. Simply differentiae know. The impulse response at any time k is the difference between the step response at k and step response at k minus 1. Very simple. So you should not put up a blank faces for these kind of simple questions. It is simply a g[k] is y step at k minus y step at a k minus 1.

(Refer Slide Time: 0:13:31)




Right, that's the difference between two successive step responses. You can use this relation and plug it here, plug it into the convolution equation. We write the convolution equation directly in terms of the step response coefficients, and estimated step response coefficients. You will have to take care of some border effects because they're going to be some negative time instance and so on. All those are practical things to worry about. But theoretically, you can rewrite the convolution equation in terms of the step response coefficients and estimated step response coefficients directly. That used to be the case with the identification toolbox.

But nowadays, if you ask first response, if I'm correct, what it does is, it estimates impulse response coefficients and [14:19] relation. Whether this is the correct way of doing it or not for noisy data, we will raise this question later on. Because estimation theory says that if you want to estimate a parameter directly in general, it is recommended that you estimated directly. Don't try to estimate some other coefficient, unknown and then use some relation and estimated, that may not be a good practice. We'll worry about that a bit later.

But you should know these facts about step responses. So to summarize, step response, we turn to step response because it gives me information in a very convenient manner on the gain, delay, any dynamics time not only time constants but if it's an under-damped summarize time and overshoot and so on. And you can see apparent delays and true delays but you will not be able to distinguish you may have to have a good algorithm that distinguishes between these two. One of the things that you should understand, generally I see students confused between delay and time constant.
(Refer Slide Time: 0:15:36)

## Step and Impulse response

A step signal is a simple superposition of shifted impulses. Given the linearity nature of an LTI system, the response should also be expected to be a superposition of impulse responses.

**Step and Impulse Response**

The step response can be expressed in terms of the impulse response using (3)

$$y_{\text{step}} = \sum_{n=0}^{\infty} g[n]u[k-n] = \sum_{n=0}^{k} g[n] \quad (12)$$



There is a lot of confusion prevailing in the mind of students; on what is delay and what is dynamics, or what is settling time? So if you look at the colloquial usage of delay, suppose I file an application, let's say passport application, and it takes maybe two weeks, these days you've just given two weeks. We say there is a delay of two weeks. No, that is not delay that is your processing time. The delay is--let us say you handed over your application, you went to clerk and say here is my application, and the and the staff there, takes half a minute to respond to you, even look at you. That is delay. Okay, it can vary. But that is what is delay. Delay is the time taken for the system to respond for the first time after you have applied the input. Once the system starts to respond the role of delays done. Okay. So do not mix up the entire settling time with delays. Delay is completely different from dynamics. I can have systems at large delay and no dynamics. You remember, one of the examples that I wrote in the first class, y(k) is simply you came in as the u(k) minus d. u(k) minus 2 or whatever. I said, it's a static system with a delay. No dynamics. After the delay it instantly reaches a steady state. You can have such a system, you can have systems with no delay and only dynamics, and you can have systems with both.

Generally people talk of delay dominant systems and dynamic dominant systems and so on. What we mean by delay dominant systems is, if you look at the overall time, from the time you apply the input, to the time for the system to be steady state, if the delay is predominantly contributing to the overall time, which is delay plus settling time, then we say it's a delay dominant system. Maybe the techniques required for controlling, for identifying such systems, sometimes different from general systems were delay is not so dominating. But please, do observe this distinction between delay and dynamics. As far as systems theory is concerned, there is a clear distinction, which means also that I can estimate delay without knowing the dynamics. Haven't we done that for the liquid level case study? When we looked at impulse response coefficients, without knowing the ordeal system, I was able to infer the delay by searching for the lag at which the first significant impulse response question comes about. So, when you turn to method for estimating delays in system identification, you will find a host of methods.

Some of these methods try to estimate delay and dynamic simultaneously, and again, if those methods are estimating apparent delay, then they try to optimize, for example, in system identification tool works, there is a command called delay ESD, delay ESD, and you supply the model, it says you also have to specify what dynamics you are assuming, whether it's the first-order, second order, then what it does is, it tries to optimize the time constants and [19:18 inaudible] simultaneously. That's not such a great idea to do. It's best typically, to estimate time delays without having to estimate the dynamics. It is possible-- very much possible to do that and frequency domain methods are very well suited for that. That is a frequency response function is very well suited. Now that we have uttered the name frequency response function, it's time to move on. Okay, we'll skip we have already completed step response versus impulse response.