

Lecture 12 Part 1

CH5230: System Identification

Response-based Description 5

Okay. Very good morning.

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Response-based Descriptions

Finite Impulse Response (FIR) Models

Consequently, we can modify the convolution equation (3) as

$$y[k] = \sum_{n=0}^M g[n]u[k-n] \quad M \in \mathcal{Z}^+ \quad (10)$$

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Let's continue our discussion with the finite impulse response models. So if you recall the primary motivation for looking at FIR models is, when you look at the convolution equation from an identification viewpoint there are infinite number of coefficients that have to be estimated. And we said one of the strategies is to truncate this up to a certain number of terms. Now an implicit assumption behind the truncation is that, the system is stable, okay? You should not be under the impression that system identification always is always concerned with stable systems. No, there is provision to handle unstable systems as well. But as far as the FIR model assumption is concerned, it is implicitly assumed that the system is stable when you assume that the impulse response coefficients are negligible beyond a certain point in time, right? That is the prime assumption. And as I said yesterday, this is one of the ways of handling stable infinite impulse response systems. What is the other way that we talked about? Parameterization, where we will map this infinite number of unknowns to a finite number of nodes. We will pursue that option a bit later, maybe tomorrow.

Okay, so let's move on and then now write the FIR model. Although we write an equality here, it should be remembered that it becomes an approximation, when the actual system is IIR. So you should get used to this terminology IIR, FIR and so on. And one of the things that you should remember philosophically is whether you're looking at FIR model or the infinitely long convolution equation. If you look at the equation that is a convolution equation, philosophically what it is saying as far as input-output modeling is concerned is the response of the system is purely because of the inputs that are acting on the system. Right. That is one way of looking at a system. If you look at, for example, if you extend this to human behavior, then you say, "Well, I am," I have responded this way because this is, these are the things that happened in my life, they acted on me. Otherwise I'm a very innocent person, I wouldn't have actually behaved this way, right? That is one way of explaining the behavior of a system. It is not the only way. Question is if it is a correct way. Question is if it is a suitable way of explaining every system.

Correctness as far as these questions are concerned technically they are correct, because we have assumed linearity and time invariance. That there is no debate about that. Whether this philosophy suited for all class of processes is the question. What do you think? The philosophy of expressing output or attributing changes in the response of a system solely to the changes in the inputs that occurred in the past, present, or for non-causal systems future. Is this philosophy suited to all class of LTI systems? We will talk only about LTI systems. What do you think? In other words is this model, is it sufficient to just work with this model? Let's not worry about identification viewpoint at this moment. Does it actually suit all kinds of situations? Let me

give you a hint. Suppose, I want to compute the response of a system and I give you the following information. I tell you that these are the inputs that are acting on the system. It's an LTI system. I tell you what is the input sequence and I also give you the initial condition of the system. Would you be able to use this convolution equation to calculate the response? Why there is so much silence?

I clearly spelt out what I will give you. I give you the input sequence, I give you the impulse response coefficients, and I also give you the initial conditions. Will you be able to use a convolution equation to calculate the response? Yes or no? It can be a yes or a no. Yes. How do you incorporate the initial condition? You want to change parties? At least we have a valid reason to change. What do you think now? Is there a provision of incorporating initial conditions? There is no provision, correct? So from that standpoint this convolution equation although technically it can describe all linear time invariant systems, almost all, within the class itself it may not be suited to calculating the response or for systems whose initial conditions are known. And input of course is always known, but on the other hand one can always argue now. You should have actually argued, if you are-- if you have thought or-- if you would have thought a bit deeper, you should have ask, how come the system is at a non-zero initial condition? Something must have cause the system to move it away from its own initial equilibrium position that is steady state.

Some input must have acted on it. A system can be at non-zero initial state for two reasons. One, some input acted on it pulled it away from the steady state or maybe indigenously something is driving it. We will rule out the second option. If you look at the first possibility, some inputs in the past must have force the system to move away from its equilibrium, so that it is at a non-zero initial condition. So you should say that theoretically if you can, you can always map the non-zero initial conditions to some other past inputs that acted on the system. And if you can give me those past inputs, then again, convolution equation can be used to calculate the response. But practically speaking, we may not know the inputs that must have acted on the system to pull it away from its equilibrium. Generally, we would know the non-zero initial condition. Correct? What caused the system to move away from its equilibrium, we may not know. But what I know for sure is whether the system is at its steady state or not. That fact I can observe. So from that viewpoint, the convolution equation comes in for criticism saying, it has its own limitations, there is no way of incorporating initial conditions.

But theoretically you should remember that these non-zero initial conditions may be due to some past inputs and if those, if that I information can be given I can always use the convolution equation. So this is something that you should keep in mind, because when we move onto difference equations, we realized one of the many advantages of working with a difference question is, it allows you to incorporate non-zero initial conditions, right? If you think of a difference equation first order, what is it? It's a recursive equation. Right? If you give me the initial condition I can easily plug in to the difference equation. But then there is some kind of a disadvantage to working with difference questions, because it has already assumed the system to be of some order and some delay so on, whereasthe convolution equation does not assume anything of those sorts. The reason why I keep pointing out these subtle points of a model is that you shouldn't, so that, is that you should know when a particular model description is suited, for a given situation. Technically, all the models that we are discussing as I have said earlier also are equivalent, theoretically.

One does not contain more information than the other. But practicality is something that we have to keep in mind. Okay. So remember that FIR or IIR models and then convolution equation models simply attribute the response to only inputs, input changes. There is no past output coming in. There is no--it is not a karmic model, you can think of it that way. What is a karmic model? The karmic model would be saying that whatever the system is responding at this instant is partly a result of how it what response it gave at the previous instance, right? So that means the past is catching up with the present, but that is not how it is, it's saying everything is due to the inputs. Okay. So let's return now to the identification viewpoint. If I choose to fit an FIR model for a given system, typically this is the starting point in system identification. When nothing is given to me about the system and I want to work with an LTI model, then it is best to start with an FIR model. There are other reasons too, but we'll talk about those a bit later. It's a very simple model.

The only decision that the user has to make is the number of FIR coefficients that are required, which is m , in our case. And it varies from system to system. If the system decays slowly then the value of M is going to be large. Likewise, for fast systems with fast dynamics M is going to be low. Fast or slow depends on the sampling interrupt also, you should remember that. But in general relatively speaking slower systems will

have larger values of M . How do I determine this value of M in practice? Well, you just guess, let's say 20 or 30, estimate the coefficients and if you find that the last coefficient is still significant then increases the value of M . So in the routines available in the system identification tool box, there is a default value. You can change that, if you look at the impulse [12:14] that is the routine that does it. Of course impulse just assumes it works for a broader set of situations. Here we are not considering noise at all. It works with noisy data and so on. But you can try this as a simple exercise, simulate an FIR model. And just estimate the coefficients using impulses [12:38]. How do you simulate, I'll show you a bit later, anyway.

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Finite Impulse Response (FIR) Models

Consequently, we can modify the convolution equation (3) as

$$y[k] = \sum_{n=0}^M g[n]u[k-n] \quad M \in \mathcal{Z}^+ \quad (10)$$

- ▶ Since the output is described by a finite set of IR coefficients, the model in (10) is known as a **finite impulse response (FIR)** model
- ▶ The value of M varies with the system. The user specifies this number during an identification exercise.

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So that's the only decision that one has to make as far as FIR model is concerned. Of course the question is how do I estimate g ? As I said the user-- from the user side, the user has to only specify M . Once M is specified the estimation algorithm takes over and estimates the coefficients for you. In general we have-- we will have data that is noisy. So again in that-- from that viewpoint also this equality becomes an approximation. That means even though the true system is FIR I will not be able to fit an exact model because of the presence of noise. Remember, then y becomes the not the true response, but the measured response. So, I will try to seek the best FIR approximation for noisy data, for noise-free data, if the system is FIR then I will just solve an exact problem. If the system is IIR and under noise-free condition again I'll seek the best a FIR approximation.

So more or less as far as reality is concerned whether the underlying system is IIR or FIR, I would be constructing the best FIR approximation. Now typical algorithms that are used for estimating these impulse response coefficients are least square algorithms, which we shall learn later on. As I said we focus on theory. So, so much about the impulse response, now we move on to the second response that is to the elementary signals step. Remember that the impulse response gives us a lot of information, but maybe not all that I desired. As I said one of the valuable pieces of information that I want about a system is gain. And it's hard to read off that gain value from impulse response coefficients. Time constant, that is also a bit difficult to read off from the impulse response coefficients. So we turn to another kind of response, which allows us to get these pieces of information in an easy manner. So it is only for the sake of convenience that we turn to another response. But otherwise theoretically, I do not have to turn to any other response. Why? Because the convolution question tells me, if I know the impulse response I know everything about the LTI system.