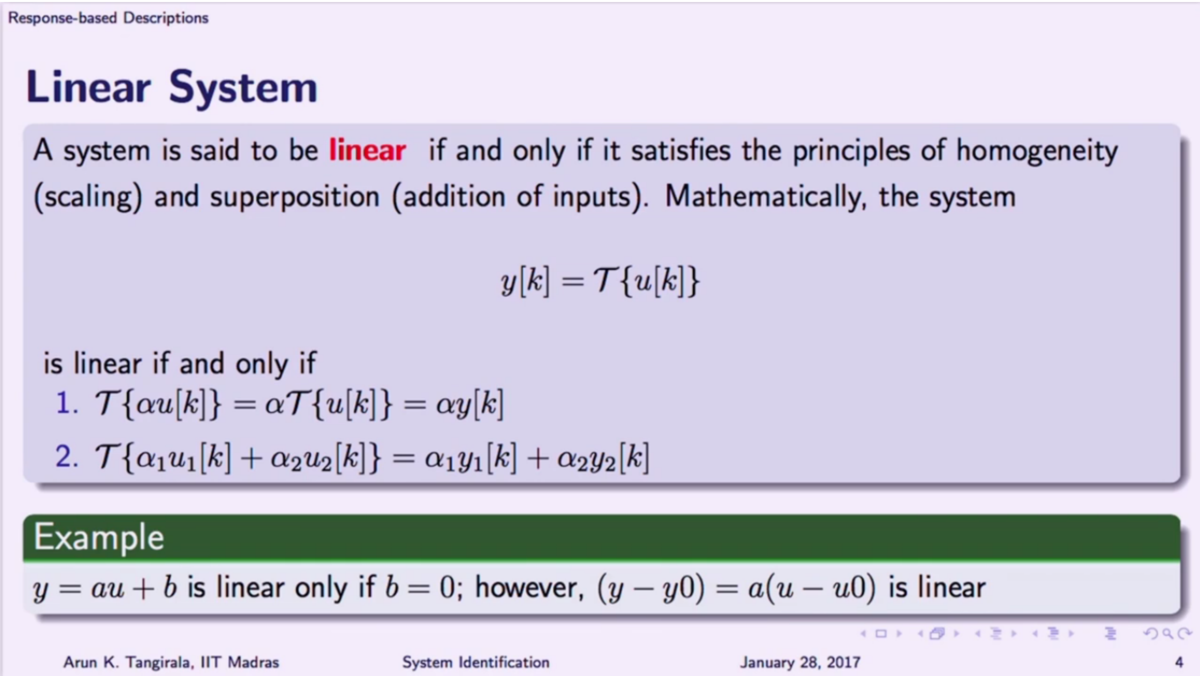
CH5230: System Identification

Response-based Description 2

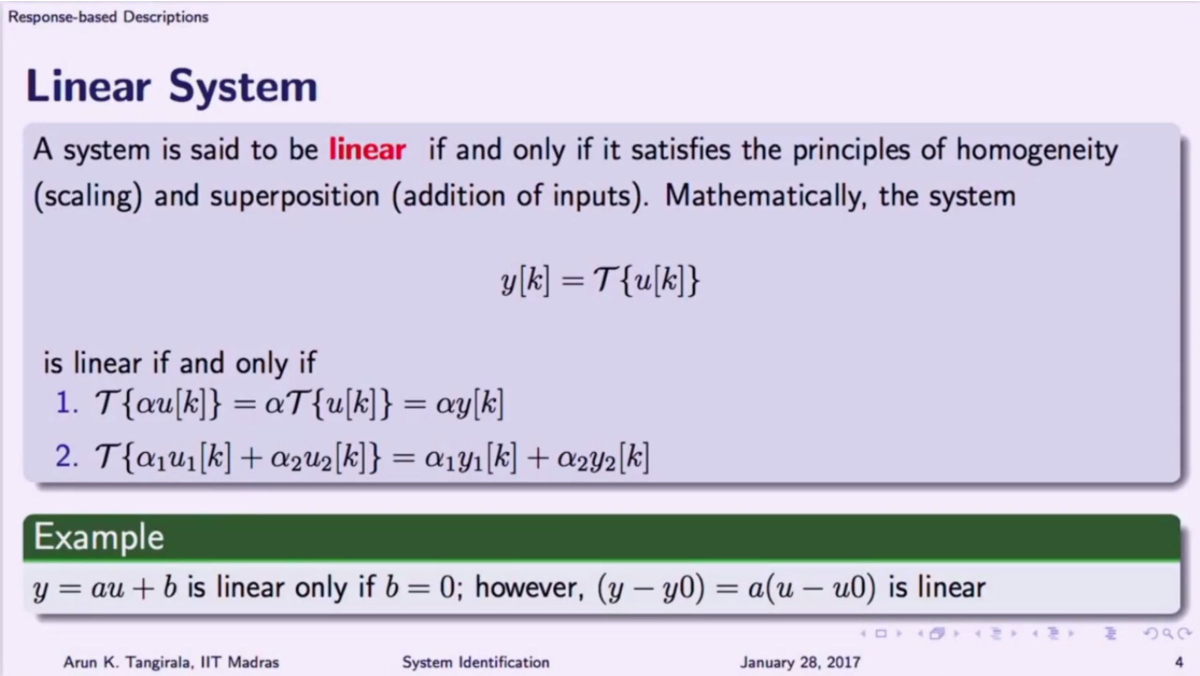
All right. Let's begin now inaugurate the theory with the definition of a linear system. Remember we are going to confine ourselves to LTA linear time invariant systems and therefore it's appropriate that we understand the definitions of linearity and time invariance in a theoretically clear manner.

(Refer Slide Time: 00:36)



We know a system is set to be linear if and only if it satisfies the principle of superposition. Now, when we talk of linearity you should understand that there are two quantities or two variables that we are talking of. Right. Here it is u and y. Tomorrow it could be something else. But essentially we're talking of a mapping. And in any mapping there is a domain, there is an image. Okay. You can say u is a domain and y is the image here. Tomorrow or sometime later on we may talk of linearity or something else with respect to something. For example in the liquid level case study, we talked of linearity of predictors, prediction equation. There we were worried, whether y hat that is the expression that function called y hat is linear function of parameters. So it is not always that linearity means I look for y verses u. There's nothing like that. So whenever someone asks you whether something is linear you have to seek information and be clear in your mind, which mapping you are referring to. So here we are saying there is a system, although I write t there but of course we are looking at G, T is some, simply some transformation that u undergoes, by the way I have already, hopefully explained the notation to you. That's why I do not have a separate slide for notation but by now you should understand that we'll use scalars at least in the slides you'll see scales for and, as a function of k would mean a discrete time signal, boldfaced lower case would mean vectors. And bold faced uppercase would imply matrices. Okay. And for random variables typically we use uppercase variables, regular faced. Okay. So, this transformation that maps u to y is said to be linear if it satisfies the principle of superposition, right. In other words, if you know, Alpha u gives you y or u1 gives y1 and u2 gives y2, then Alpha1 u1 plus Alpha2 u2 should give you Alpha1 y1plus Alpha2 u2.

(Refer Slide Time: 03:07)

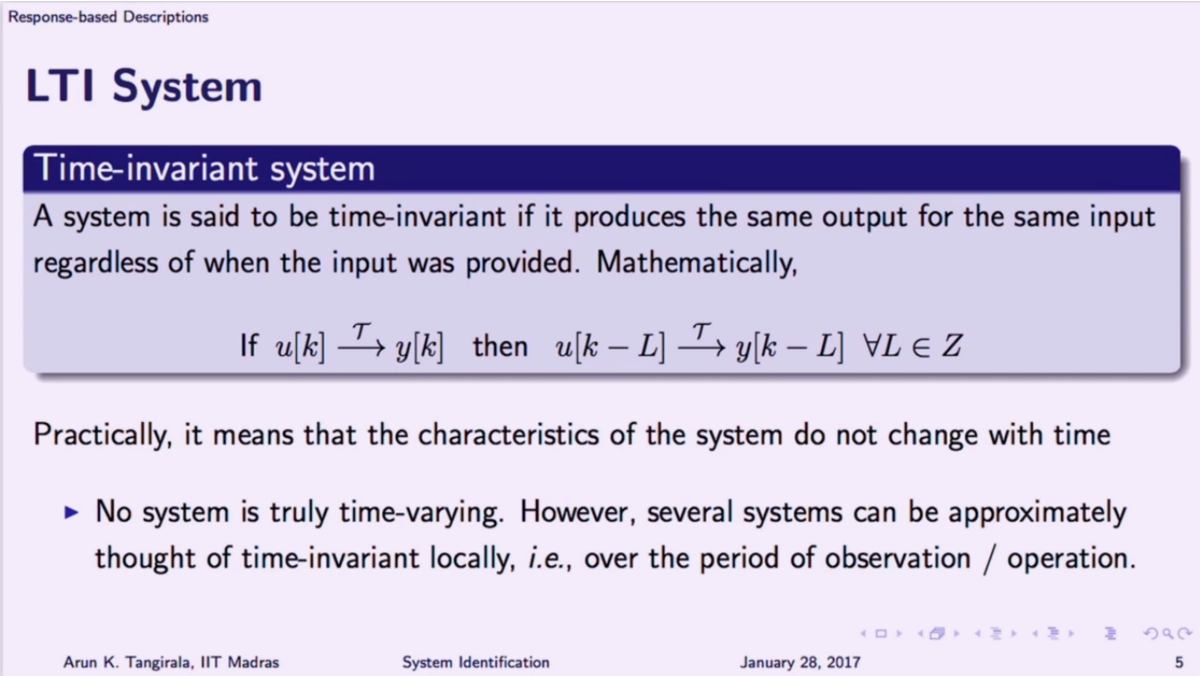


In this respect, if y k is Alpha u k plus beta, this is not linear. Because it simply does not satisfy the principle of homogeneity. First of all. Which is a part of the superposition also. We say that these are affine models. If I find models. And, of course, when you have a situation like this very it's very easy to shore for this system that y k minus y(0) not is Alpha times u k minus u(0) not, where y not(0) and u(0) not are some reference points that satisfy this transformation.

So in this y k minus y(0) not is a deviation variable. u k minus u(0)not is another deviation of the input. In terms of deviation variables the relation is linear. This is a very simple linearization. Where there is no approximation involved here. It's pretty straightforward. Okay. Any questions? Typically when students are asked if something is linear our pre 11th and 12th standard coaching, whatever the approach is to see for the powers. Right. If u has a power of 1.5 or 1.2 anything from 1. And then therefore many people conclude that this is linear. But don't do that. Always keep this superposition principle in mind.

The second property that we're looking at is timing variance. Right.

(Refer Slide Time: 05:05)



What is the notion of time invariance, if an input generates an output response y then if the same input is given hundred years later or you know 764 days later, the response should not change. The response should be the same, regardless of the time at which I give the input, the same input should give me same output. That's it. Both these are clearly idealizations. There is no doubt about that. You cannot think of a system that exactly satisfies this two requirements. Am I right, it is not possible. Can you find one system which is linear which is time invariant. It's impossible. Right. Even when I say it is linear or a large range of operation, time invariant is something that's very, very difficult. We see this philosophical statements, right. Anything that is constant is change. Change is constant. I mean all these things, which means. Yeah. We here this, what this means is that things keep changing with time. Right. That is only constant thing.

What this means is that the response of the system is bound to change by your [6:35 inaudible]. It's not going to be exactly identical, right. You know, you come and perform an experiment today. And then maybe a few hours later itself you come and do the experiment, you will find some difference. The question is now,, whether I should treat the system as time invariant. What do you think?

Should I treat the system as time invariant. I find some manure differences in the numbers, in the response as sensed by my instrument. What do you say? Should I consider this time invariant. Is it okay? Or you say no, no, I learned in the class it has to be identical to the 150th decimal. No way I treat this as time invariant. What do we do? It's not possible to get such identical responses. What do we do?

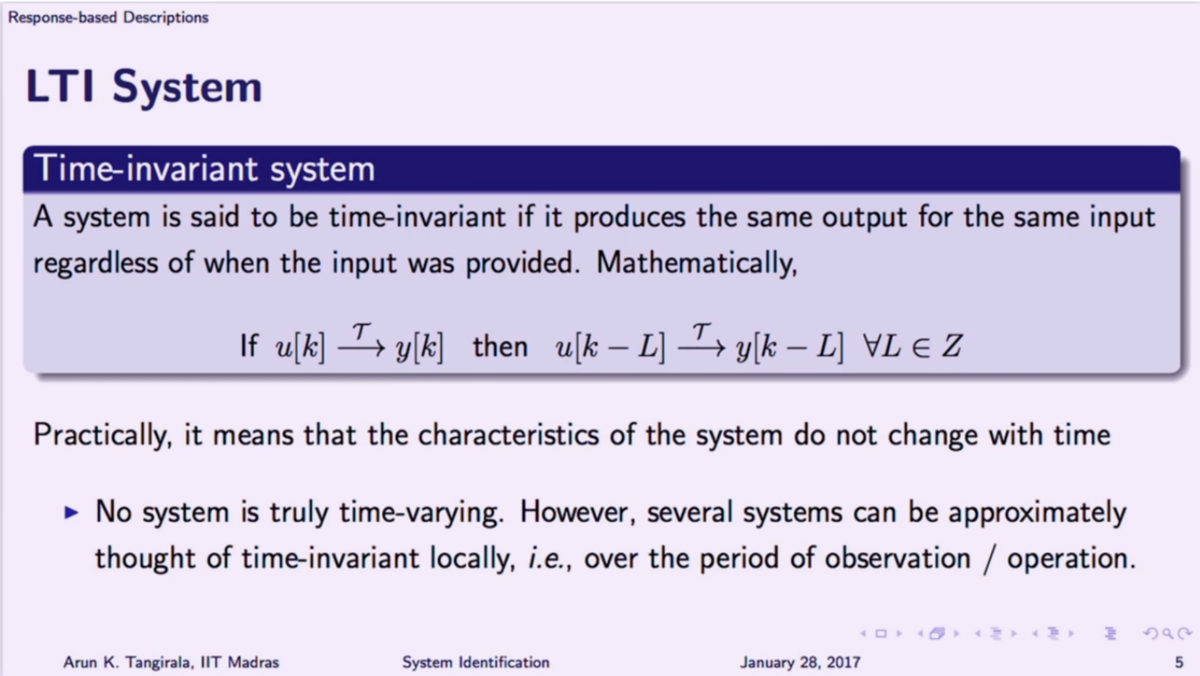
[07:42 inaudible]

What do you mean by the statistical prospect?

Statistical prospect is [07:46 inaudible]

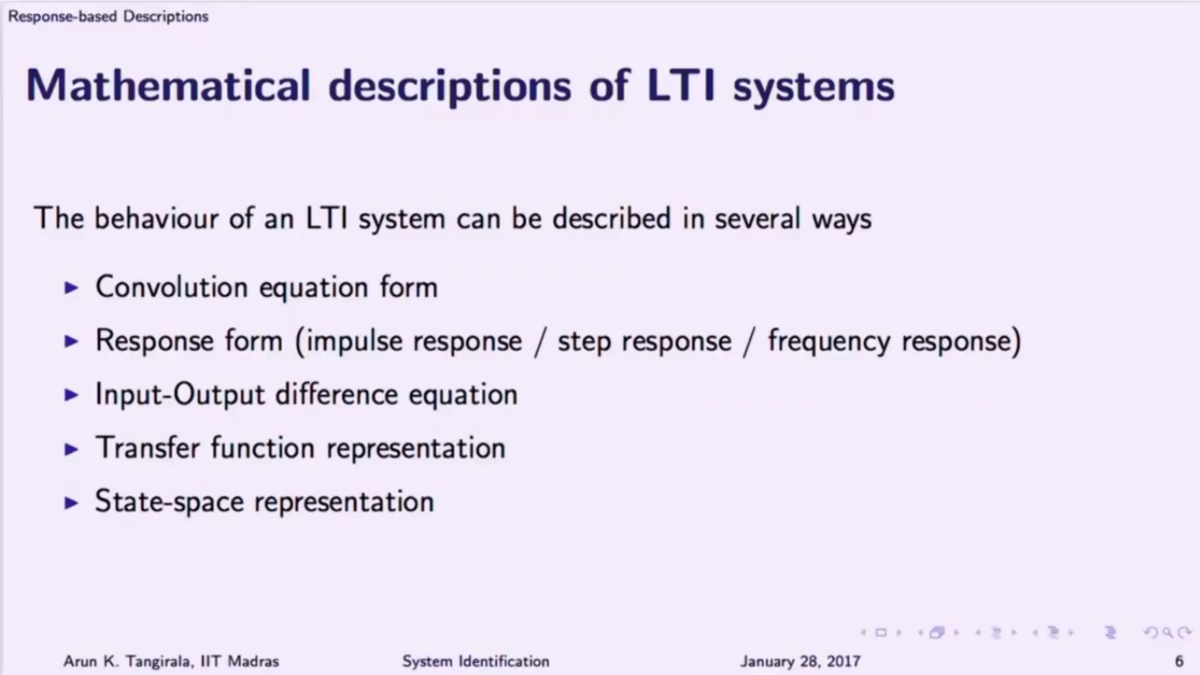
Blame it on the sensor. That's all you're saying. Correct. You say that, yeah maybe the true system is time varying slightly. But it's okay. After all nobody knows the truth. Right. I'm going to assume that the there is a composite system there. So a deterministic system that is time varying but then it also depends on what is the difference whether the time varying nature, itself is stochastic or not, which will allow me to think of the system being made up of as time invariant plus some stochastic signal or if there is a deterministic nature to the time varying things. Many cases the parameters can vary of the system can vary in a deterministic manner. Then we say that it is linear time vary but I know that the nature of the time varying characteristics of the parameters and so on. But anyway let me come quickly to the bottom line. The bottom line is no system is truly linear. No system is truly time invariant.

(Refer Slide Time: 08:58)



We assume that over this scales of operation the system is LTI. That means, I don't find a significant deviation from these identities for the scale of operation. And if there is a deviation from this. Yes, it may show up in a modelling error. And this, the deviations from this could be deterministic, could be stochastic and so on. Generally, we hope that it is stochastic so that I can lump them into V. But if I find a significant deviation from this and that has a deterministic nature to it I may have to incorporate that in the model. So the best thing is to start off with this LTI and see if the model works. If it works, very good. That means if it approximates a system very well, very good. If the approximation is poor then I may have to discard this framework. Okay. So that is a basic thing. In fact, many linear time varying systems are modelled as locally time invariant. I may have a system that is time varying over months. But on the scale of a week it is time invariant relatively. So I'll have a model that keeps changing every week. So you need a recursive algorithm that we'll talk about later on. Likewise, many non-linear systems can be modelled as locally linear models. So if I have a curve, if I have a relationship like this between y and let us say any, some function like this, I can actually have locally linear models here depending on the extent of non-linearity. This is also common practice. In order to be able to do that I need to do this, I need to know how to deal with LTI systems. That is why we are starting with LTI systems. So that whatever you learned in this course you can apply it to identifying either linear time varying or non-linear time invariant or non-linear time varying systems or some other multi scale, multi rate, complex, non-linear, whatever system that will give you a fancy paper. Okay. I'll only list the descriptions of LTI systems and then we'll actually go through the convolution equation and so on, starting Tuesday.

(Refer Slide Time: 11:35)



So there are different ways in which you can describe a linear time invariant System. One is the convolution question form, which we have seen earlier at. The other is so-called response form which actually stems from the convolution equation. We have already spoken about it, impulse, step and frequency, the trinity. And then you can go to difference equation forms. You've seen that as well, in a sense we have encountered these equations but we'll go through the theory now. And then you have transfer function representations which also we have spoken about briefly and state based representations. We have looked at all of this in the context of the liquid level case study. The first two, which is a convolution in question and the response from, they belong to non-parametric family because you do not have to make any assumptions. Whereas the remaining three that I listed here, again I should say at least the remaining two here, input output different equation form and transfer function representations they belong to the parameter family. And in the states based representations again you have a non-parametric and you have a pragmatic family. But look at this, there are so many different mathematical ways in which a linear time invariant system can be described. And let me tell you no modern contains are describes less or more better than another one.

In the sense that all contain the same information. If I give you one model you will be able to derive all the other forms. So theoretically they're all equivalent. Then why should I actually study all these three different this five different forms or you know, six maybe? Why? Any idea?

[13:39 inaudible]

Good. Is of guessing the model structure. Anything else?

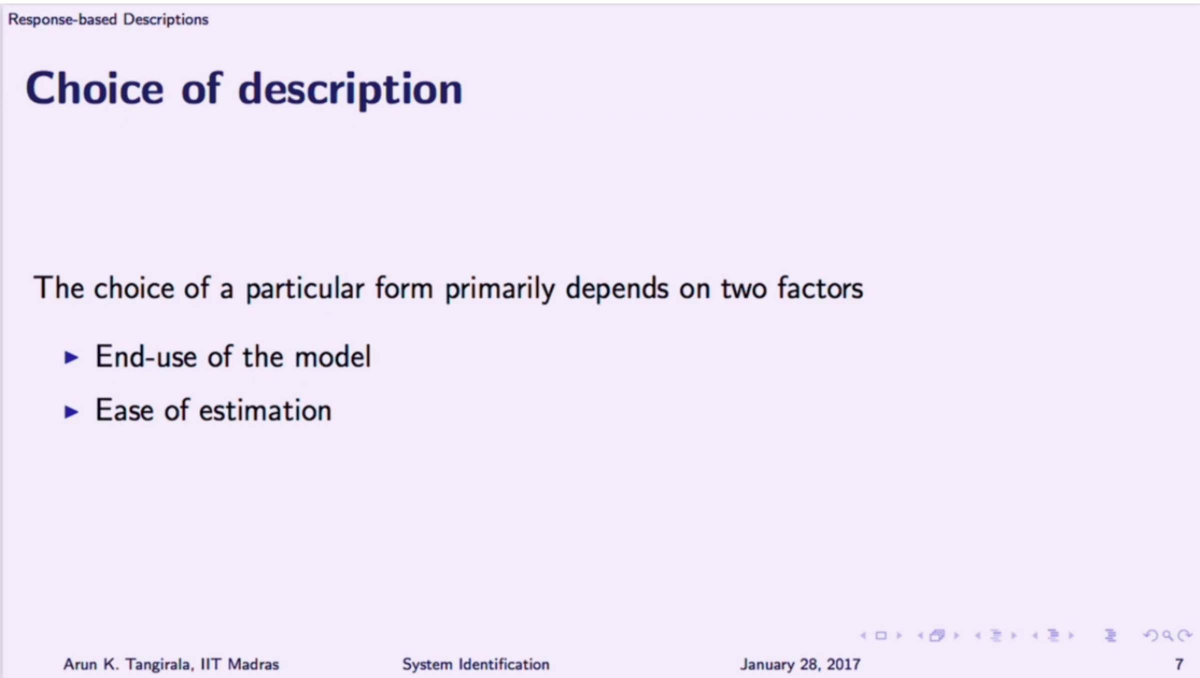
Unique [13:47 inaudible]

Okay. Identifiability may be an issue with state space but the remaining are identifiable. That's not an issue. Yeah that's not a bad point. Any other idea?

Different piece of information from different representations.

Yeah. Theoretically I can convert one from the other. Here we are not worrying about estimation at all. It's all about theory, deterministic, ideal, fantastic world that never existed but still it's fine. So in terms of information I can derive one from the other. From an identification viewpoint the choice of these different models have to be known because one, the ease of estimation may be different. We have already discussed that like a different equation form, it may be easier to estimate fewer parameters, ease of implementation. Typically if you want to implement a model online a recursive form is very amenable. Correct. Which of these models give me a recursive form. You understand what is a recursive form, I know the value at the previous instant, I just had to predict the next one or the next two steps. Which among these model descriptions, straight away give me a recursive form? Difference equation. Right. The difference equation forms are very amenable. So there are different criteria. One is of course from an identification viewpoint, how much information I need a priority to guess the model, ease of guessing, to end use what do I want. And then of course ease of estimation. So there are these difference criteria that typically govern the choice of the model. Regardless of choice we will study all these different forms over the next two lectures or two or three lectures. But you should remember ultimately when you're choosing there are going to be these criteria, End-use, Ease of estimation, Ease of guessing, it includes Ease of guessing also.

(Refer Slide Time: 16:05)



And End-use includes so many other things. Maybe a recursive maybe the DCS, in fact in some cases FIR models, I mean so-called finite impulse response models or a truncated convolution forms are preferred to different equation forms. So one has to look at the application. If there is no application given then there are some standard criteria. You understand. So you always remember this, studying linear systems theory is one part of story, studying it in a context is another thing. What I mean to say is that if you turn to signal processing, textbooks, they would present the linear system theory, they would also talk of convolution forms, impulse response, descriptions frequency response, difference equations and so one. But their perspective, their approach to dealing with this are which one would you choose and so on would be completely different. The criteria would be different compared to what you would encounter in identification. So in the book that I have written and in the lectures that I give, I am presenting the linear system series in the context of system identification. As an example if I take the convolution equation, what is the convolution equation. Output is being represented as a weighted sum of the past, present and future inputs. Correct.

For a filter design person or for filtering, suppose I'm looking at a filtering application. The input is a signal that I want to filter. G is a impulse response of the filter. I'm going to study how changing G is going to affect y. Right. In filtering design there are considerations on phase, what the filter does to the phase of the input. What kind of attenuation or amplification it perform, these are all the things that I'm worried about. Whereas in identification the concern is different. I'm given the output and the input and I wanted to determine G. Right. Then different questions arise. So the same equation that is describing a linear system can appeal to different applications in different ways. And that's the beauty and that is a perspective that you want to develop. And that is why it is important to study this linear system theory even though you're aware of it again in the context of identification. Okay. So we'll meet next Tuesday. Thank you very much.