CH5230: System Identification

Part 1 – Journey into Identification

(Case Studies) 11

We'll continue with our discussion and hopefully complete the case study today. What we are doing is we are identifying a model for the liquid level system from, of course, input output data. We looked at the estimation of impulse response model yesterday. That's where we concluded the discussion and we'll take it up from there. What we learned is that this impulse response estimates, these are called response-based descriptions. This is not the only way of characterizing an LTI system. We begin with this because as I said yesterday, the convolution equation is the central model that describes a linear time invariant system. There are other models as well. So do not be under the impression that this is the only model but the good thing about this description is that and a few other response-based descriptions is that they do not make any assumption on the delay or the order. They only rest on the assumptions of linearity and time invariance. And as we discussed yesterday, the impulse response estimates give some valuable information, namely the delay, right? The input, output delay and what we observed is that this system has a delay of one sample. And how do we infer that? By looking at the impulse response plot.

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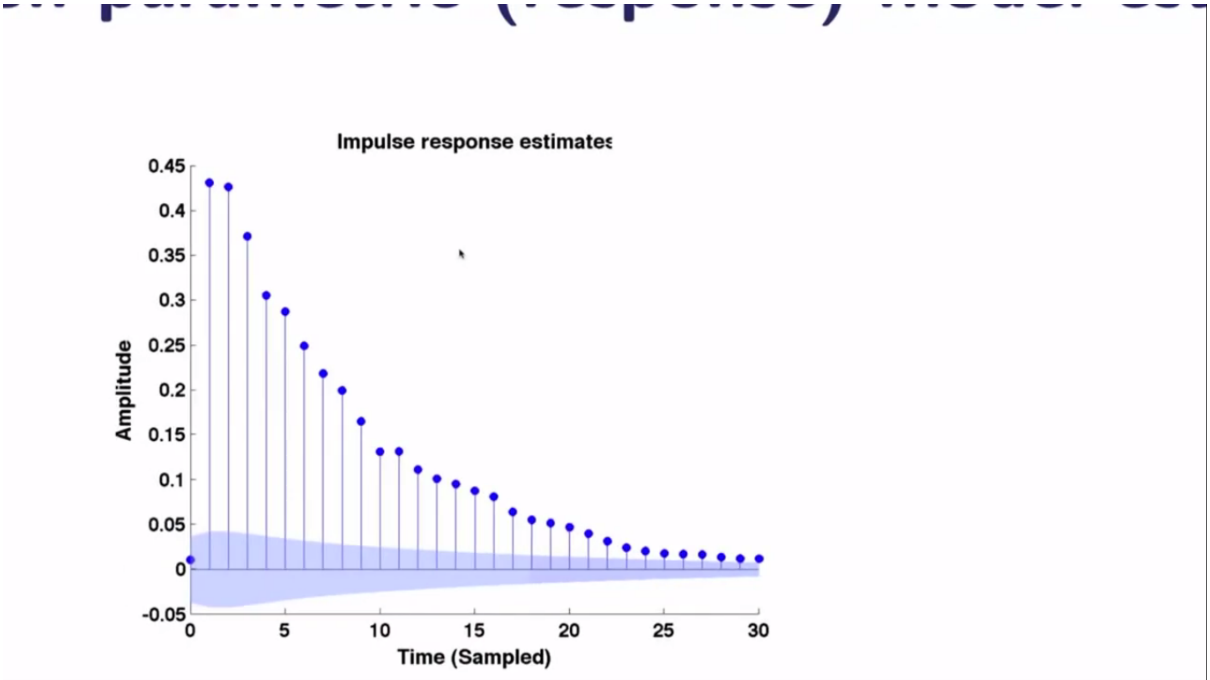

You can see that the first significant impulse response coefficient is at lag 1 or at a Time 1, right? Now, remember that this impulse response estimate has been estimated not directly obtained from an experiment. There is a big difference between these two. We have not excited the system with an impulse. In fact, we have excited the system with a pseudorandom binary sequence, PRBS. And someone asked a question after the end of the class yesterday, is PRBS the only way or are there other inputs that we could have used? Yes, we could have used other kinds of inputs. For example, we could have used a filtered white noise or Gaussian white noise or even a multi sign, there are other inputs that are possible. But there is a reason why we have used PRBS and we'll discuss those reasons later on.

So from the input output data, we have estimated impulse response coefficients. We shall realize later on that this is a much better way of estimating the inputs of response coefficients rather than directly exciting the system with an impulse. I could have, I mean as a crude estimate of the impulse response, I could have just excited the system with an impulse and then obtained a response. That is impulse response by definition. But that estimate or that measurement that you obtain from the experiment may not be as good as the estimate that you have obtained here. And what we mean by that will be clearer later on but at least you should understand what we mean by good is, this will have lower errors. All right. So the first significant impulse response estimate is at lag1. And notice the word significant, when we are talking of estimates, we use the word significant. When we are talking of theoretical ones, we do not use that word. You can show that when a system has a delay of d-samples, an LTI system. I will not keep repeating LTI because throughout the course most of the times we'll talk about LTI, occasionally we may talk of either a non-linear or a time varying system but by and large it's going to be LTI. So you should prefix the LTI by yourself unless I have made it -- made a statement otherwise.

So for any system with a delay of these samples you can show that the impulse response is going to be zero for that many instances. Naturally so, if there is a delay of d-samples between the input and output, and I inject an impulse at the input, it's going to take d-samples long after which I'll see the response, right? But that is theory, where identically the first d-impulse response coefficients are going to be zero but this is estimate. So this is practice. We will learn both theory and practice in this course. And in fact that's why even my book is titled, Theory and Practice. We are just looking at a case study, then we'll follow it up with a detailed discussion of the theory starting next week and then towards midway in the course we'll start looking at the practical aspects of identification.

So when it comes to practice, we are going to deal with estimates and therefore the term significant is very important. We are now relying on statistics to tell whether an estimate that we have obtained should be considered as insignificant or significant. All right. So the first significant impulse response coefficient as I figured out from this band that I have.

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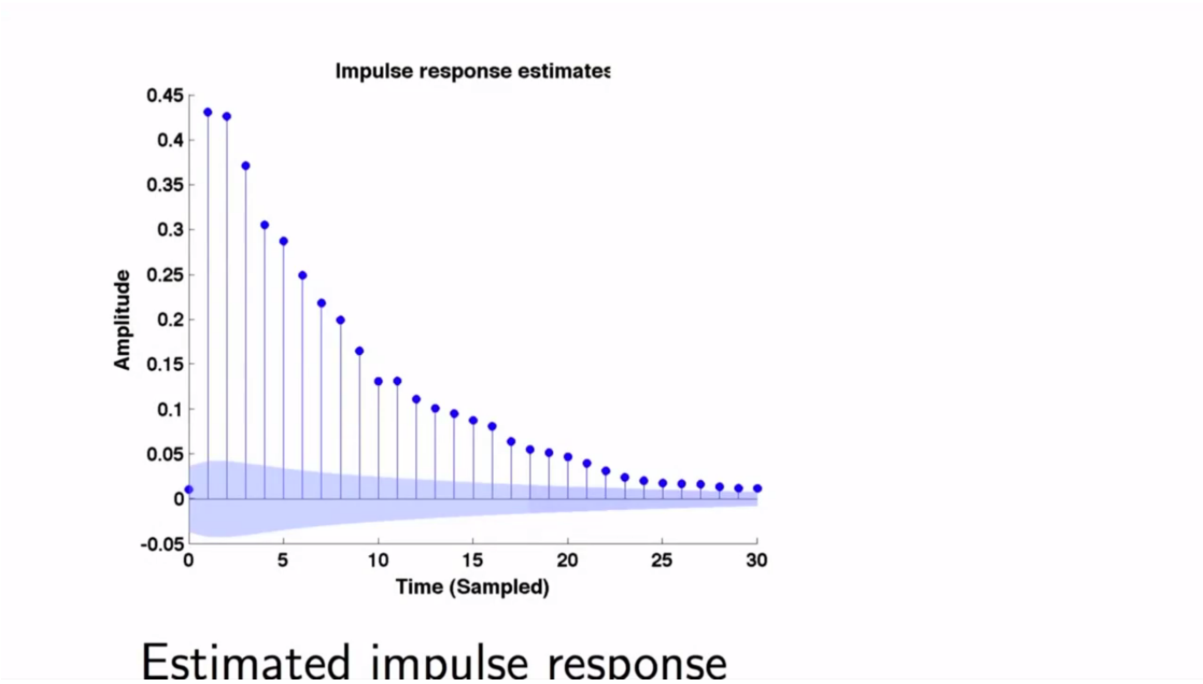

This is called the significance band. We will learn in this course how to construct the significance band at a later stage, when we learn estimation theory. And by the way, the MATLAB routines and the scripts for the entire case study right from step 1 are all available on my website. You can actually go and download. They're also of course available in the book but you can download. And I strongly recommend that you type them out rather than just simply running them of course. In the first run, you can run them to see if the codes are fine. But then you should type them out by yourself and it's, it will be a good practice for you, so that you'll know what the routines are, make mistakes, correct them and so on. So that's a good reinforced learning for you. And I strongly encourage that you do that. Okay. So the routine has computed the input the significance bands and also the estimates. And from this I infer that the system has a delay of one sample. And that it is a stable system as we said yesterday. Right? And any other inference that we can draw from the impulse response coefficients? Can you say what is the order of the system by looking at the impulse response?

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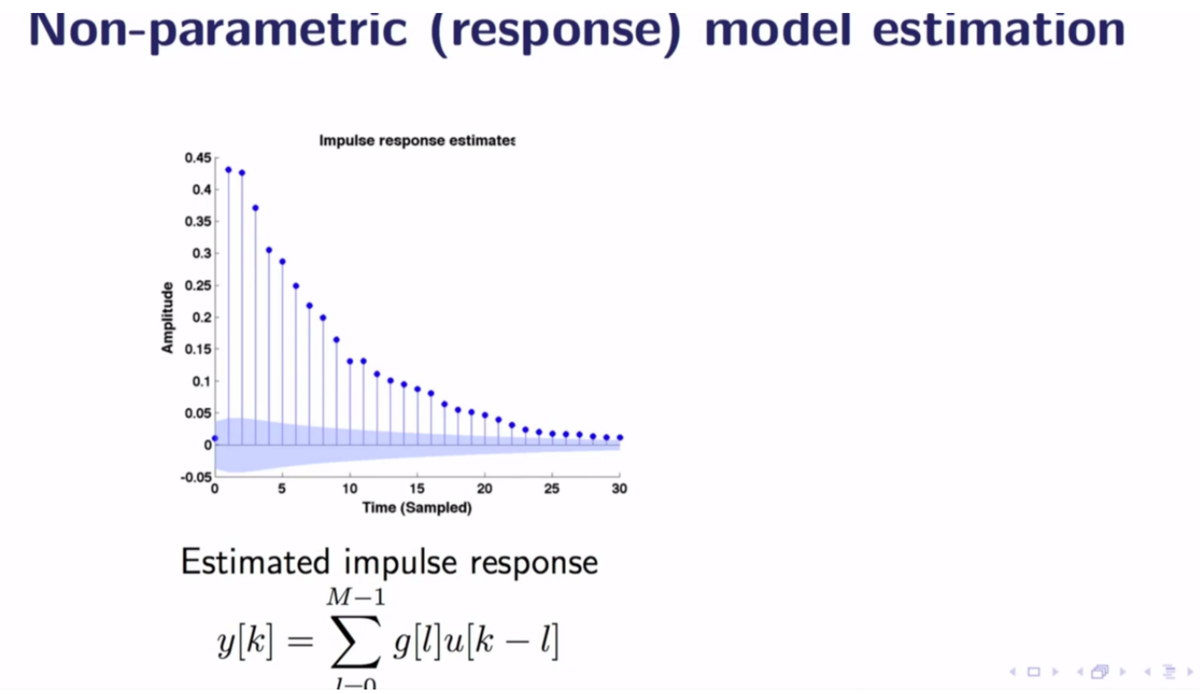

Is there a guess? I mean, can you make a guess that it could be first order, second order and so on? Or it's not so easy. What do you think? Is it easy to infer the order from the impulse response?

See, impulse response analysis is widely prevalent among electrical engineers. They use the impulse response a lot because they also -- these are the ones that are used extensively in filter design. On the other hand, chemical engineers rely on what is known as a step response, which basically gives you some other set of insights on, for example, the order of the system. Theoretically, the same information is contained in both impulsive response and step response, right? Why is that? Because for an LTI system, the convolution equation says, if I know the impulse response of the system, I know the response of the system to any other input. So recall the convolution equation, yk is sigma g n u k minus n. Of course, the summation runs from minus infinity to infinity or whatever is appropriate. But this equation tells me for an LTI system, if I know [7:56 inaudible], the impulse response coefficients, I can in principle compute the response of the system to any other input. Which means, if I give a step input, how is the step input defined? U k, input, let's consider one at k greater than or equal to zero and zero at negative times. This is called a unit step. Exactly the step that we gave earlier.

Now you compare what you have on the right plot there is a step response. And let me zoom that for you.

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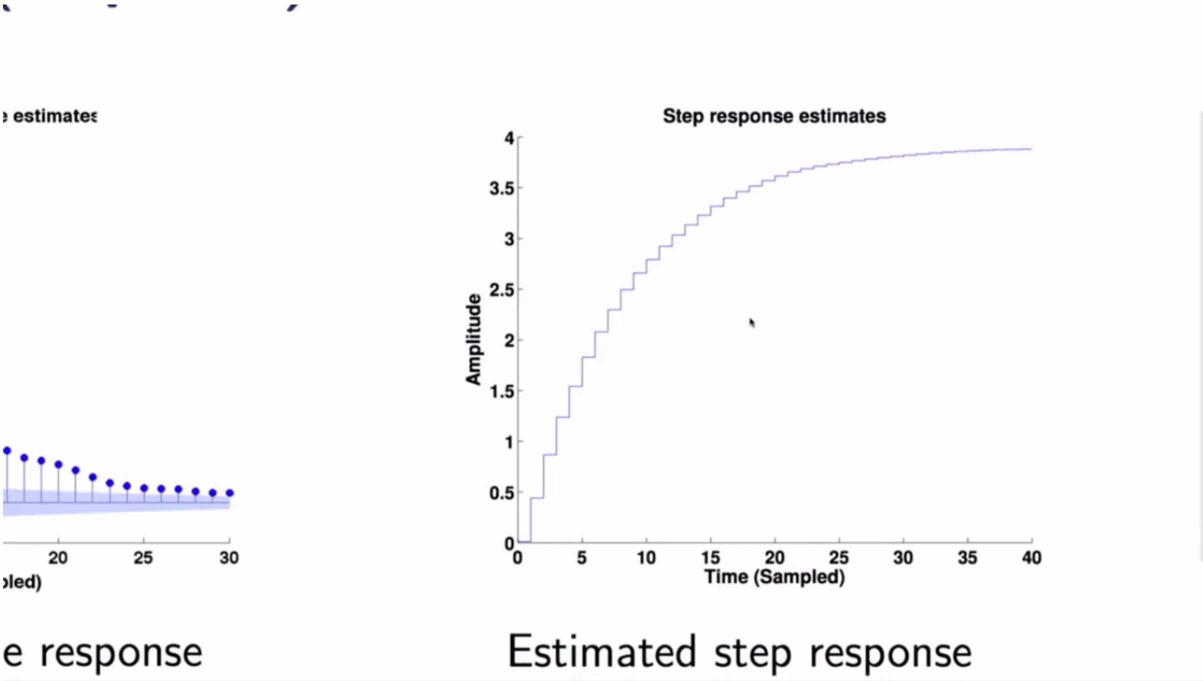

So here you have, this is estimated step response. Earlier we obtained a step response, how did we obtain that? By directly injecting a step input, right? So there is a difference between these two. What is the difference that you notice by the way? This is the estimated step response and earlier I showed you.

Yes. This one, doesn't appear to be corrupted by noise. I mean, I've not done any photo-shop gimmicks or anything like that. It is a consequence of estimating the step response from another set of data. This data that I've used to estimate a step response has been obtained from a different experiment. And in this different experiment, I've used a PRBS. So this should throw light on the difference between obtaining a response directly and indirectly. Which is better in your opinion? Don't hesitate. See you're here to answer, make mistakes and maybe you're right. Which is better in your opinion? What do you think?

[9:50 inaudible].

Is better. Why? Why do you feel that way?

It depends on what, what is the use of the data, I mean, how we are going --

No. When we asked which response is better, what yardstick do you have in mind?

That is what you have to think. When we say which estimate, which set response is better. What do you think?

If there is no variations, less variations--

Yeah. So which do thing is better. This estimated one or the one that we obtained directly?

Estimated one.

Estimated one. Why? Because it's less noisy, as he said. That's correct. It's a good way of thinking. It's a correct way of thinking. We want as smooth estimates as possible, that's number one. And as less noisy, right. Of course, one has to quantify that is obtained here and there and so on. But by and large it is true that this step response estimate obtained this manner is much better. Just like I said earlier for impulsive responses estimate. Few minutes ago, I said you could obtain impulse response by directly injecting an impulse. And from the experiment you could fetch the impulsive response or you could estimate the impulse response like the one that we have done. So what you should do is a similar diagram that I have used. By the way, this is not an experiment that I performed. So now, one secret I'm revealing. It is a simulation that I have performed on this liquid level case study where I have simulated the nonlinear ODE, right. I could perform an experiment but this was much easier for academic purposes. It's easier because I have so many things under control and it helps me illustrate a few concepts.

So you should download the Simulink model, which is also available on the website. And of course, you know, depending on which version you are using, the Simulink version was prepared under 2014B,MATLAB 2014B, I think now you have 2016 A or B or 17 perhaps. I do not know the latest release. And the plots may look a bit different, I should caution you. Some of these plots, because the toolboxes are evolving, some of these plots that you have seen until now and that you will see later on, they may look different from the ones that you will see when you run the latest version of MATLAB, but estimates won't look so different. They will look different from what they are numerically because when you simulate, you will be simulating a different reflection of the noise, right? But by and large, the nature of the results will be the same. There is no doubt about that. So you should do this for the, with the Simulink blog diagram, go ahead and inject an impulse. There is the blog diagram has a provision. Towards the end of the case study maybe I'll show you that and inject the impulse input and generate the response, estimate the impulse responses and compare. And then get a feel of what's happening.

So the beauty of this approach is, I can estimate many other responses in a much more so-called efficient manner. Efficient meaning low error. Okay. So that's one aspect. Now let's come back to the other aspect that is deriving some information about the process characteristics from these response-based descriptions. The impulse response estimates did not give us much direct information on what could be the order of the process, whether you could, I mean if you are comfortable with impulse response but generally its lot more easier to read off things from step response. For example gain, which is a very important piece of information, not only in processes but also for students. They ask, how much do I gain by solving this assignment? That's how the weights are perceived, 10 percent for six assignments, 10 by 6, 1.67. Is that all I gain by solving that assignment? Unfortunately, those calculations are wrong. Okay. There are only as far as marks are concerned. But in terms of learning, there is no number that you can attach. Anyway, so in processes the gain is an important -- one of the most important piece of vital statistics that is required in all applications, be it control, monitoring, design, everywhere. So the step response directly gives me an idea of the gain.

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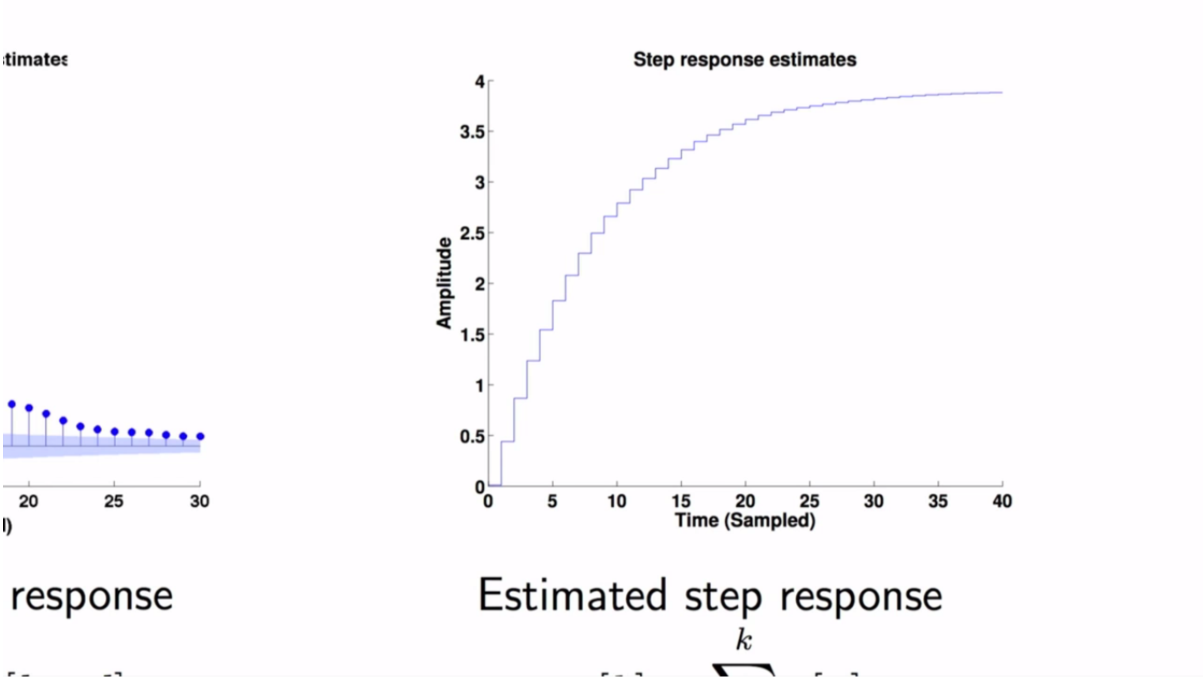

And also it gives me an idea of order. What kind of order can you guess at least to begin with, when you look at this?

First order, I mean it does tally with the dynamics that we know already for the liquid level system. But even if I didn't know, at least a decent guess as first order. What else can I infer from this? Sorry, somebody said something. Correct. So what is roughly the time constant from this plot that you can infer?

Ten.

Ten and 15.

So roughly between 7 and 8 because at 63.2 percent of the final value or the change because this is a step response that has been estimated, it is directly giving you the change, right. The change is nearly [15:16 inaudible] and 63.2 percent of that is about between 2.4 and 2.5, that occurs at around about 8, which is the same that we inferred from the step response as well. In fact, later on when I give you the theoretical model that I have used for simulation by linearizing the non-linear ODE around the operating conditions that I've used. You will find that the time constant is also 8. So our estimates are coming out right. So these are the inferences that we can draw from this impulse response and step response.

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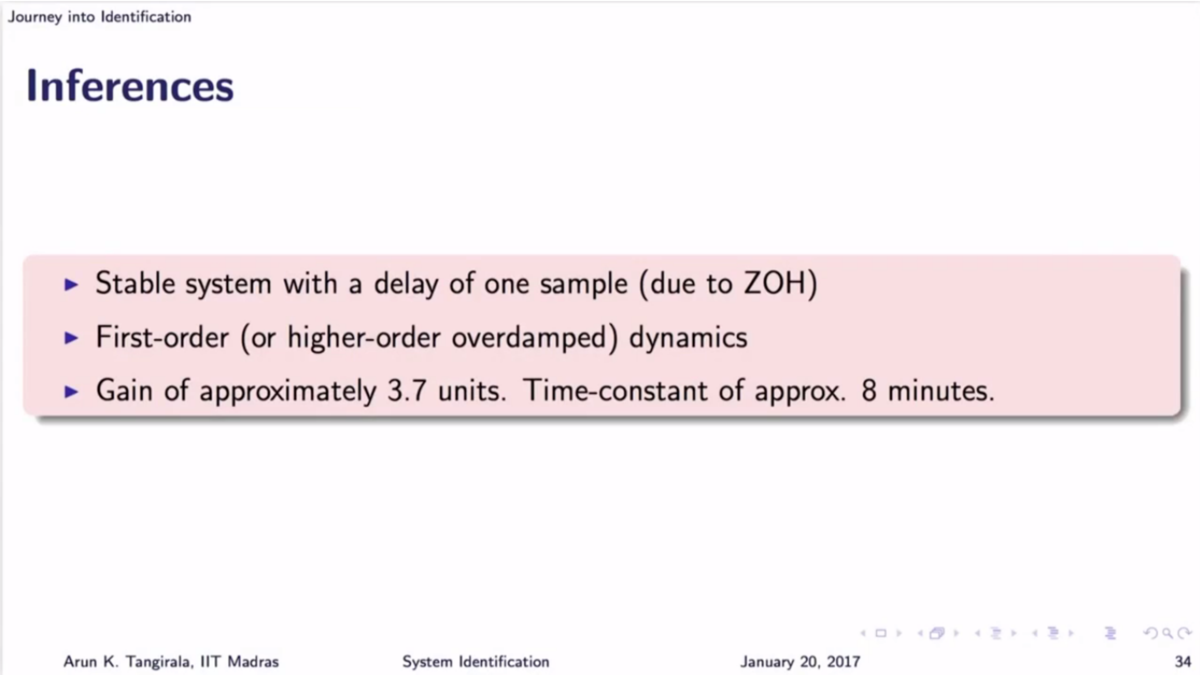

First that it's a stable system with a delay of 1 sample. Now this delay of 1 sample, I had asked a question yesterday. The true system, if you take the physical system, it doesn't have any delay, right? So if you look at the true system, there is no delay per se in a liquid level system, at the rates that you are observing, you can't expect a delay between the change in the flow and the change in the liquid level. So where is this delay coming from. Remember that this is not the only system that is participating in your experiment. We have said that earlier, right?

So let us say Gc the continuous time process which is a liquid level system is the one that is eventually being excited but if you recall the discussion that we had earlier, we said there are other elements participating in the experiment. What are they? There is an actuator, all right, good. Let's call this as A and then. So there is a D to A converter. Let's call this as actuator. And then on this side you have a sampler which includes both a quantizer and a converter and so on, which will eventually get me yk. Of course, I have not drawn the noise part here but leave -- keep that aside that for now. Now, what's happening is, here this is the input that I have designed. In fact, in an actual experiment you wouldn't design changes in flow, you would actually design -- What would you design? You would design changes in the value movement, that is how much the valve should open and close. The flow rate is then determined by that. So in fact, in practice, in reality, you would not build a model between [18:00 inaudible] level. What is a model that you would end up building? Between valve opening and level changes, that I should tell you. But right now we will not worry. And will therefore not worry about the actuator part. Okay. So we will keep that out of the picture for now and focus on this D to A device.

Now this D to A is typically what is known as a hold device which we shall learn later on. And typically we use what is known as a zero order hold, abbreviated as ZOH. The role of this ZOH is to construct an approximate u of t from uk. Right. Because the process needs a continuous time signal. And how does it approximate, it approximates using a piece wise continuous -- piece wise constant interpolation. What we mean by that is, the signal here is going to look like this. I mean just as a sketch. This is how the discrete time input is going to look like, although in the plot you may not have seen that. Strictly speaking, the discrete time input is only defined at specific instance in time, right? Whereas u of t here would look continuous and the output of a ZOH corresponding to this would look like this. This is how it would look like. And what the ZOH has done is, it has assumed a signal to be constant between two sampling incidents. Now, there is a sampler as well on the other side and both this ZOH and sampler are in sync, in the sense they both have the same interval, sampling observation [19:49 inaudible].

When you put together this device, this system called a sample data system we can show that eventually we end up with a delay, 1 sample delay, between the discrete time input and the discrete time output, although there is no delay within the process and we will prove that later on. So this one delay contribution is coming from zero order hold. And that's always going to be the case.

So, the source of this one delay is ZOH, in other words. It is not coming from the process. What this means is additionally the process had any delays either due to inherent delays in the process, there are processes that can have inherent delays or maybe due to transportation lags. For example, you may inject a change somewhere upstream because there is no provision of injecting a change right at the processing input, that transportation delay is also going to contribute and so on. So this one delay is regardless of all the other delays. If there are other delays, they will also be reflected. And accordingly the impulse response estimate will show you that. Remember that.

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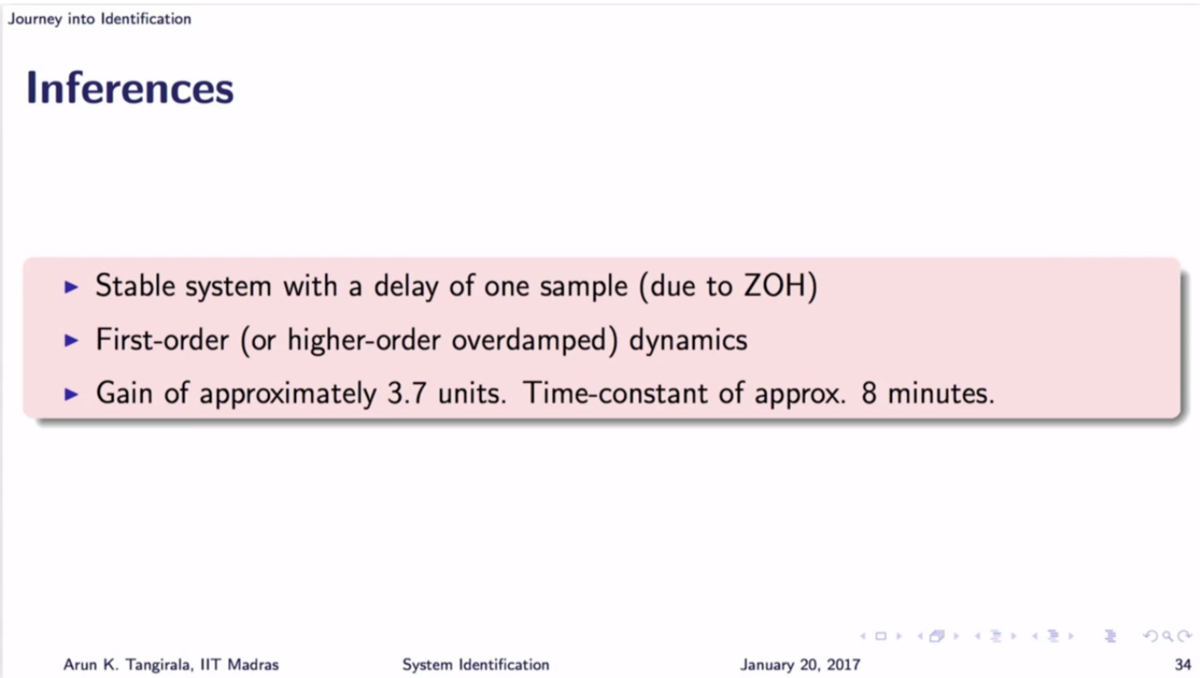

And then, what is other inference that we have drawn? We have said that it, well, it could be higher order but by and large it appears to be faster order. For example, it could be second order over [21:14 inaudible], we do not know. And the gain is approximately 3.7 units, between 3.7 and 4, we have just read it off from the step response. We have not made a rigorous calculation of it. And the time constant is approximately, you could argue maybe between 7 and 8.

So with these inferences now we are set to build another model that we are eventually interested in. Now let me motivate that discussion. Already I have spoken about it yesterday. We said ultimately we want to predict, we can also predict using these models.