

**NPTEL**  
**NPTEL ONLINE COURSE**  
**CH5230: SYSTEM IDENTIFICATION**  
**STATE-SPACE/SUBSPACE IDENTIFICATION**  
**PART 8**  
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Alright, so welcome to the you know concluding topic of this course, all along we have this being actually the most important is reserved for the last, because this is the first step in identification, suppose with design inputs but we learnt it towards the end, that's always the case we reserved the good things for the end, so we are going to talk the input design for identification very briefly, I should tell you at the outside that input design is still an evolving topic, sub-branch of CCID, there are number of open ended problem such remain to be solved, simply because mathematically it can be quite challenging, and it's an iterative process, there is mix of statistics and systems theory involved in designing inputs and so on, some good results are available for linear systems, some preliminary results are available for nonlinear systems, we will only discuss the linear systems case as with the rest of the course.

Outline the basic ideas learn what is PRBS in particular learn the concepts of persistent excitation and informative experiments in a formal way, so to begin with let's recall this example and it's be fitting now that we recall this example at the end of the course, this is the example that I had used to highlight the role of input in identifiability,  
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Input Design References

## Role of input in identifiability

Recall Example

A process

$$y[k] = b_1 u[k-1] + b_2 u[k-2] + b_3 u[k-3]$$

is excited with  $u[k] = \sin(\omega_0 k)$ . Then, the response to this input can be re-written as

$$y[k] = \left( b_1 + \frac{b_2}{2 \cos \omega_0} \right) u[k-1] + \left( b_3 + \frac{b_2}{2 \cos \omega_0} \right) u[k-3]$$

From the input-output data, thus only two of the three parameters can be estimated **uniquely**.

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and we remember from this example that if I use a sign wave for this kind of process here, if I use sinusoidal input of a single frequency I can only identify two of the three parameters and you can look at it in two, in many ways but the best way to look at it is from a linear algebra view point, we know that there are three unknowns, and if you set up 3 equations the regressor matrix  $U(k-1)$ ,  $U(k-2)$  and  $U(k-3)$  and likewise here few march ahead in time, the first column is essentially this, and you can fill the rest of the entries, this matrix here which plays, the rank of which plays a critical role in obtaining unique estimates of  $B_1$ ,  $B_2$ ,  $B_3$ , this matrix is going to be rank deficient when you use this kind of a sinusoidal input of single frequency.

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Input Design References

## Role of input in identifiability

Recall Example

A process

$$y[k] = b_1 u[k-1] + b_2 u[k-2] + b_3 u[k-3]$$

is excited with  $u[k] = \sin(\omega_0 k)$ . Then, the response to this input can be re-written as

$$y[k] = \left( b_1 + \frac{b_2}{2 \cos \omega_0} \right) u[k-1] + \left( b_3 + \frac{b_2}{2 \cos \omega_0} \right) u[k-3]$$

From the input-output data, thus only two of the three parameters can be estimated **uniquely**.

$$\begin{bmatrix} u[k-1] & u[k-2] & u[k-3] \\ u[k] & & \\ u[k+1] & & \end{bmatrix}$$

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On the other hand it becomes full rank, the moment you include two frequencies, and you can also recall that we had set with one frequency you can identify two parameters, basically you can go back to your systems theory bode A plot and let's say at a single frequency I can identify the magnitude and the, that is the amplitude ratio and the phase, and if the system is a first order I can identify the gain and time constant uniquely, but beyond that I cannot, so I have two point data with the single frequency, therefore two parameters can be estimated, that is another way of looking at it, but the linear algebra way of looking at it helps later on understand the concept of persistent excitation, anyway so the idea is now to design input such that this matrix is full rank, of course we will not always necessarily look at FIR models for a general LTI systems what is the idea is what we are going to discuss.

When we discuss that it will eventually lead us to the concepts of one informative experiments and persistent excitation,  
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Input Design References

## Role of input in identifiability

Recall Example

A process

$$y[k] = b_1 u[k - 1] + b_2 u[k - 2] + b_3 u[k - 3]$$

is excited with  $u[k] = \sin(\omega_0 k)$ . Then, the response to this input can be re-written as

$$y[k] = \left( b_1 + \frac{b_2}{2 \cos \omega_0} \right) u[k - 1] + \left( b_3 + \frac{b_2}{2 \cos \omega_0} \right) u[k - 3]$$

From the input-output data, thus only two of the three parameters can be estimated **uniquely**.

On the other hand, if the input contained two frequencies we could estimate the three parameters uniquely. This leads to concepts of

- ▶ **Informative experiments:** Data should be informative w.r.t the parameters
- ▶ **Persistent excitation:** Inputs should contain as many frequencies as possible.

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informative experiment at least qualitatively as a name suggest, we want to perform an experiment such that the data is informative with respect to the parameters that I want to estimate, and straightaway you should recall the concept of fisher information here, so which means you should expect in the input design literature, fisher information to be appearing everywhere, right, it does a number of hits there, but we will not be pursuing that route, we will not be designing input using the optimization route here and so on, we'll only study some elementary results, nevertheless I encourage you to go and read the literature on, sorry, the fisher's information, okay.

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Input Design References

## Role of input in identifiability

Recall Example

A process

$$y[k] = b_1 u[k-1] + b_2 u[k-2] + b_3 u[k-3]$$

is excited with  $u[k] = \sin(\omega_0 k)$ . Then, the response to this input can be re-written as

$$y[k] = \left( b_1 + \frac{b_2}{2 \cos \omega_0} \right) u[k-1] + \left( b_3 + \frac{b_2}{2 \cos \omega_0} \right) u[k-3]$$

From the input-output data, thus only two of the three parameters can be estimated **uniquely**.

On the other hand, if the input contained two frequencies we could estimate the three parameters uniquely. This leads to concepts of

- ▶ **Informative experiments:** Data should be informative w.r.t the parameters (Fisher's informative)
- ▶ **Persistent excitation:** Inputs should contain as many frequencies as possible.

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And then comes a concept of persistent excitation, again as a name suggest what this means is persistent is not in time necessarily, okay, you can think of it that way, but what this means essentially that the input should contain as many frequencies as possible, again as evident from this example.

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Input Design References

## Informative experiments

Basics

- ▶ To obtain informative data sets it is important to answer: (i) what to measure? (ii) what to manipulate and (iii) how to manipulate?
- ▶ Equally important it is to answer what is meant by "informative"? An experiment is "informative" if it allows us to distinguish between two different models
- ▶ The experimental conditions should generally resemble those under which the model is used.

The information content in an experimental data is strongly tied to the input excitation. There are typically two aspects of concern: (i) the frequency content of the input and (ii) the SNR.

We shall study this aspect somewhat formally.

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Now in order to generate informative data sets it is important in general to answer what to measure, when if I have access to the experiment, and if I have the freedom to decide which variable should be sensed so that I get to estimate the parameters with great efficiency, then I have to ask this questions what to measure, what to manipulate and how to manipulate, we will not focus on what to measure, this is not what is of interest to us, so we will not focus on that,

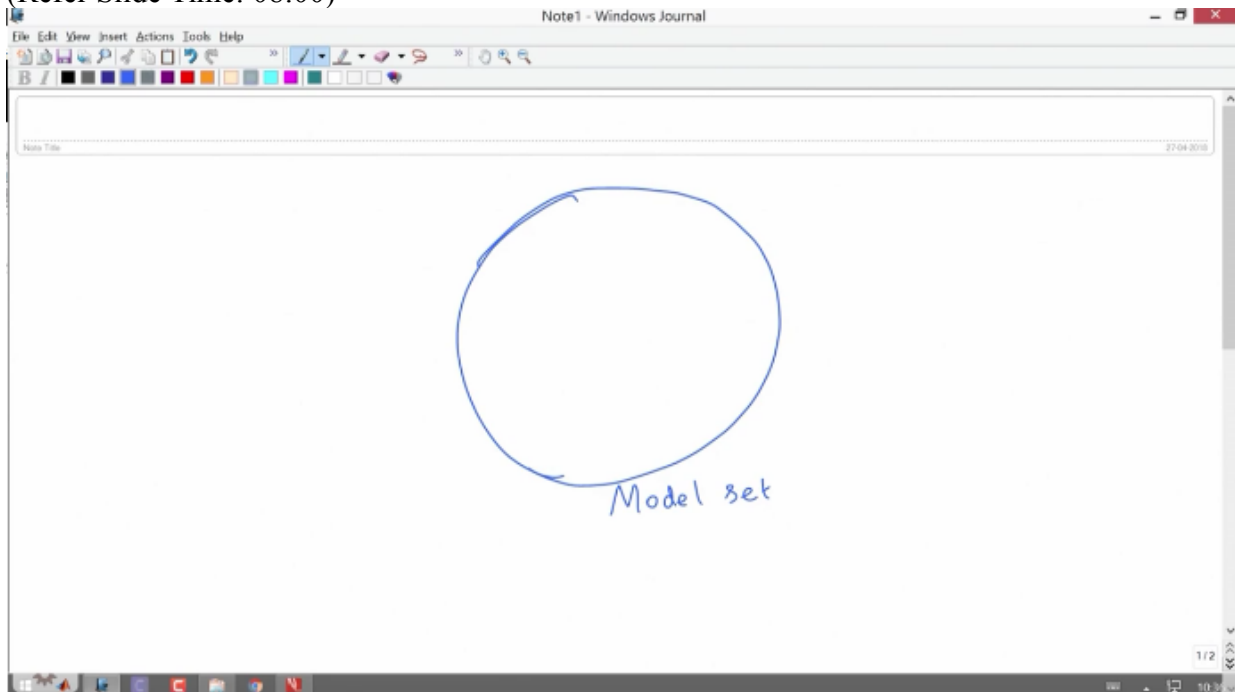
we will not also ask what to manipulate, let us assume that we know already which input to manipulate, we will only focus on the discussion on how to manipulate, which means some related to input design.

And we also have to ask what is meant by informative, and we will learn later on what is meant by informative experiment as essentially that if it is informative, if it allows us to distinguish between two different models, so go back to the interview analogy and ask if I have conducted the interview in an informative way, okay.

So what does the success of an interview rely on? On many factors but one key factor is the kind of questions that I have asked in the interview that means the kind of inputs I have used, and I should ask the questions in such a way that it allows me to distinguish between the good or the so called top candidate and not so top candidates, so that I can pick the candidate most suitable candidate for the position that has been advertise without any ambiguity, if I ask for example and it could be an interview, it could be an exam, suppose I want to figure out who is that star performer in a class, and I ask please answer what is sine square theta or that is probably going to be difficult, sine square theta + cos square theta at the B tech level you can claim memory loss and not answer it,  $1 + 2$ , what is  $1+2$ ?

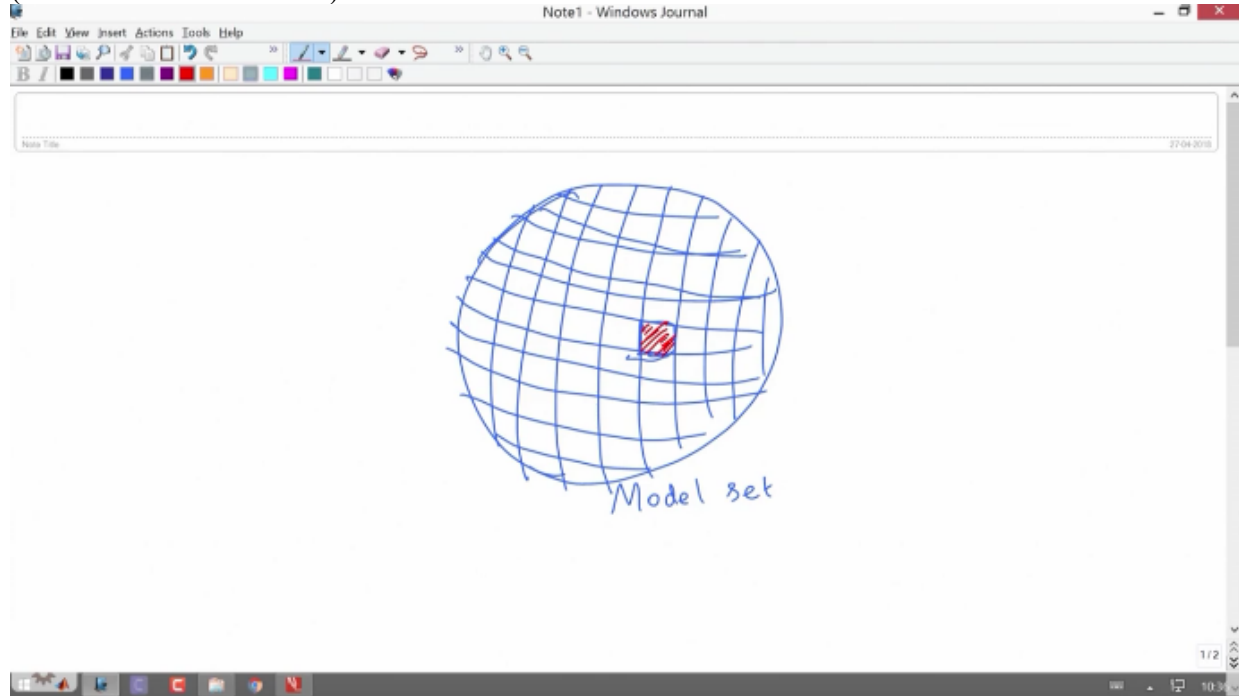
Will it allow me to distinguish between a star performer and mediocre performer, no way, everybody will be able to answer that question, so that is not an informative exam, I have to ask even more questions so that I start separating out the models or the star performer from the mediocre ones, and I pick the one that is most important to me, you recall that we had this diagram, I think I have drawn this long ago, but I know correct to draw now, so remember that we have this model set, so this is our model set that is a choice, the short listed candidates you can say,

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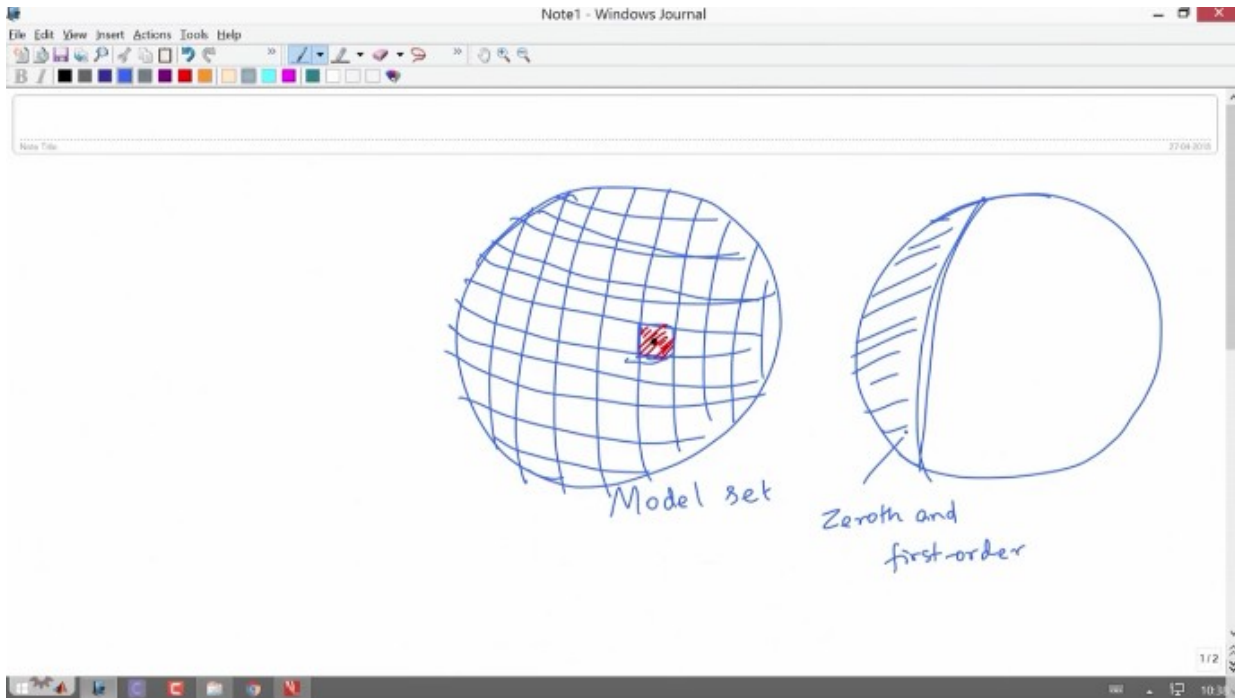
and now I want to pick the most appropriate model for the process based on the given data, but now we are asking data is not given I have to generate data, I have to generate data in such a way that it finely partitions this model set into chunks, okay, so it should, I should ask questions or I should design inputs such that I get a very fine partitioning of this model set, and then it is the duty of the estimation algorithm to go and pick the right chunk, so this is what I want the estimation algorithm to pick.

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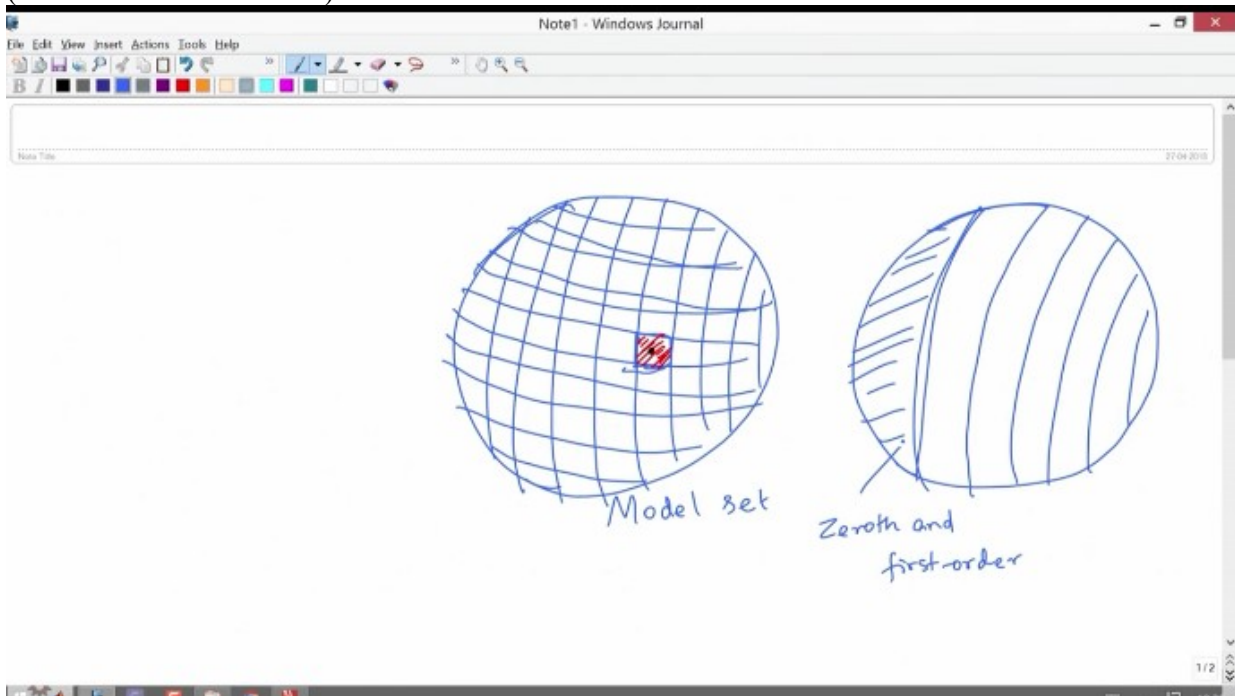
We know that we can never estimate, we can never pick a particle, there is no way we can do that, that is like saying and hopefully that the truth is contained in this, okay, that means whatever pi, chunk I'm picking up it should be containing the truth, so the truth should be somewhere here, so clearly it depends on the input design, and it depends on the estimation algorithm, but the first step is input, because that is what is going to generate the data, if the data has not partitioned this properly and it has missed out getting you the truth or it is not giving you enough resolvability, let me not say missed out the truth, but it is not giving you enough resolvability, then what happens is that you will not be able to distinguish between two models, so if I use a single frequency I do not know if a first order model is correct or a second order model is correct, I can only say whether a zeroth order and the first order, which one is more appropriate? So going back to our model set the way we want to do it is with a single frequency I can, I will be able to partition this into two parts, this is a single frequency cut, you can say this is zeroth order and the other ones are first order and so on, I mean this set here contains zeroth and first order, and the rest of the models, right.

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With another frequency it is like another cut of this pi, so I will be able to now distinguish between the suitability of a first order and perhaps, so another frequency I can estimate two more parameters, so let us say there is no delay, so we can go up to third order, so I will obtain another partition, so I can clearly say whether a zeroth order or a first order or a second order or a third order is suited, but I cannot say beyond that.

Now you extend this argument, you extend this discussion you can see that with each frequency you are making a cut, and you are making fine cut,  
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so if you want a very fine cut, right you can also say that it is being cut this way, don't worry with every frequency, with every frequency you are fine partitioning the model set and the more the frequencies that you have, the more fine partitioning that you have and that is essentially the idea of persistent excitation. The persistent exciting input means that the input contains as many frequencies as possible which means that you have managed to really generate information that will allow you to later resolve, see I have made the partitioning, but then it is the ability, my ability I have to choose a fork to pick that one, if I choose a big fork then it will pick many pieces together, and that fork is your estimation algorithm, hopefully you like the analogy and hopefully it is not made you hungry, okay.

So the information content is definitely depended on the frequency contained of the input and then there is also the SNR, because that maybe dust, there may be unwanted things, instead of saying thus there may be unwanted things,  
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Input Design References

## Informative experiments

### Basics

- ▶ To obtain informative data sets it is important to answer: (i) what to measure? (ii) what to manipulate and (iii) how to manipulate?
- ▶ Equally important it is to answer what is meant by "informative"? {An experiment is "informative" if it allows us to distinguish between two different models}
- ▶ The experimental conditions should generally resemble those under which the model is used.

The information content in an experimental data is strongly tied to the input excitation. There are typically two aspects of concern: (i) the frequency content of the input and (ii) the SNR.

We shall study this aspect somewhat formally.

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so if you look at children the only one to pick the cherry there, sometimes they don't want anything else around surrounding it, so you will have to get the cherry there and also you have to use the instrument to get that, right.

And also when you are generating experimental data, a guideline, a natural and the commonsense guideline is to use experimental conditions that resemble those under which the model is used, so you should not ask a questions in an interview that are completely unrelated to the job that you are hiring this candidate for, right, suppose you are advertised for a nurse and you ask about police, question, the candidate is of course shocked, but even otherwise the nurse is not going to police around, you have to ask questions that resemble the conditions under which the nurse is going to operate, okay, don't ask what happens when somebody robs and what would you do, okay.

So remember that,



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Input Design References

## Formal approach

The information in data should be such that we should be able to

1. Uniquely estimate the parameters of a given model.
2. Distinguish between (the suitability or predictability of) two models at a given frequency.

In order to state these requirements formally, we need to first define what is meant by equality of models.

Two models can be compared primarily using the **prediction error yardstick**.

so the information and data formally should be such that you should be able to uniquely estimate the parameters of a given model, there are two things first you should be able to uniquely estimate the parameters and two you should be able to distinguish between the suitability of two models, both, I should be able to assess each candidate uniquely, and I should also have the resolvability, both are tight to each other, in some sense they are tight to each other, but they are not necessarily the same, for example with a single frequency---.....

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