Process Control - Design, Analysis and Assessment
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State Space Solution

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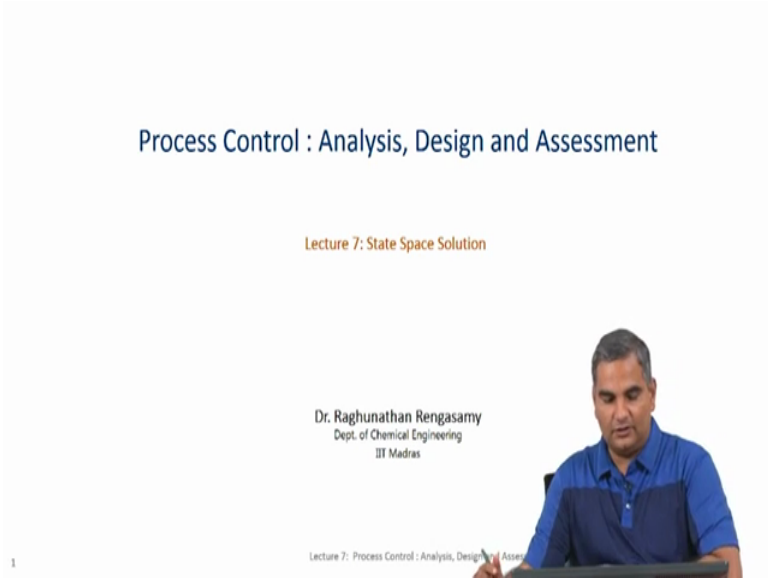

We will continue with the 7th lecture in this course on process control. Till now I have been talking about some fundamental concepts in control and in the last class we talked about modelling processes and we talked about state space modelling and so on. So we will continue with that and then in this lecture we will look at how we can solve this state space equation to get a solution that is usable for control, but before I do that there are certain terms that we keep seeing again and again in control and I want to explain what these mean and give you an idea of you know how we use these terms in control.

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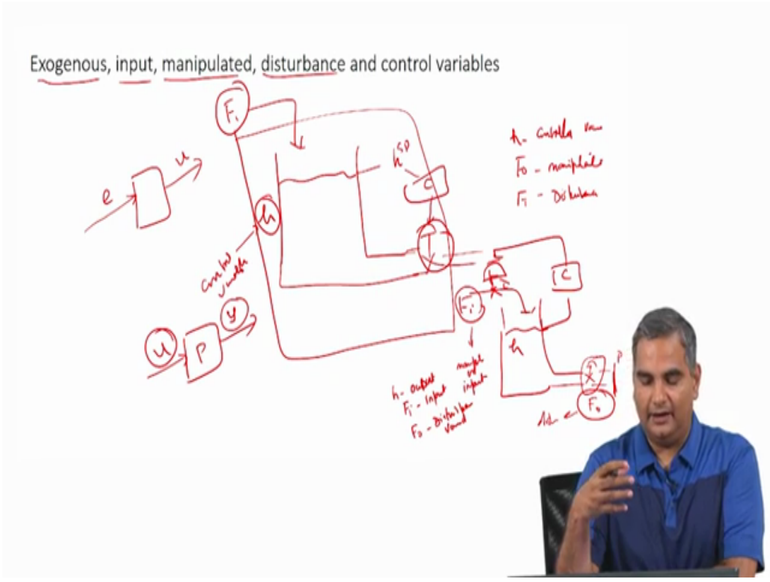

So the terms that we keep coming across are exogenous variable, input variable, manipulated variable, disturbance variable and the control variable. So I just want to kind of explain to you what these are and I will also how you how you know depending on how you design your process an exogenous variable in some case could be an input variable or a disturbance could be made into manipulated variable if you add some instrumentation and so on.

So to do this we will take the same very very simple tank example that we had been talking about and in all of these examples I had valve here this valve I had not to show that this is an input variable but to make sure that when we write the flow equation at the output to introduce the resistance term I want to say there is a valve which introduces that resistance, so that is the reason why we have this here.

But typically if you do see this example we have something that flows in, we have something that flows out and we have the height. In all the lectures till now this control variable is reasonably clear which is the variable that we want to maintain at certain value and in the tank example till now we have been talking about maintaining the height at its set point. So I am going to call the height as the control variable in this case.

Now the definition of exogenous variable is a variable that is outside your system definition. So in this case for example if you think of your tank as your system, then this F i is a variable that is from outside the physical system which is tank so this variable kind of vary on its own based on what comes before this process and really we have no control over this so this is right now you might want to think of this as an exogenous variable or for example if it is going to vary on its own we also call it a disturbance variable.

So in general exogenous variables are variables that are outside the battery limits of the system however, exogenous variables effect how the system works. So for example is this inlet flow rate changes then correspondingly outlet flow rate will change and so on so we talked about this. And these exogenous variables are variables also sometimes called as disturbance variables, these are variables which again have an effect on the process but we are not able to control them to any degree. So we have to simply live with those and then still make sure the controller works when we have this disturbance.

Now the input and manipulated variable in control terminology atleast as far as this course is concerned we are going to kind of use it interchangeably. So the reason why we call it input or manipulated variable is, so if I take a controller which we will see later we said that we are going to get an error and we are going to find the value for the manipulated variable. So in other words the error which is a difference between the output and the set point is going to be used to set the value for this u or in English you might say the error is used to manipulate this variable so that the ultimate variable of interest which is hide in this case is controlled properly.

So this is the reason why we call the variable that is an output of a controller as a manipulated variable, this is also generally called as an input variable because if you think of this as a process then u is an input of the process and that is what dictates what y is. So if you think about this from a process view point then u is the input to the process and y is the output from the process that is the reason why this is also called input variable.

So when we look at it from the controller view point we manipulate this variable so that the ultimate control variable takes its value and if you take it from a process view point this input to the process gives you an output y. So that is the reason why sometimes we use this input and manipulated variable interchangeably. Now if I showed you this picture and then I ask you now define each of these variables it becomes little I would not say complicated you have to be very careful about how you define these things.

For example is this were a control valve and if it is at fixed position then basically it gives me a resistance, but now if I were to design a control structure in such a way that this height is compared with some set point and that information goes to a controller which then manipulates this valve opening if I have a control structure like this then in this case what would happen is the h is the control variable, F not becomes the manipulated variable and why does this become the manipulated variable this becomes a manipulated variable because whenever I change this control valve position then the flow rate is being manipulated.

So in other words we are controlling the height of this tank by manipulating the outlet flow rate, okay. In which case now F i which is inlet flow rate it is uncontrolled this is now the disturbance variable, okay so this is one configuration that you could possibly have. Now if I take the same tank and I could still put this control here, valve here but I am going to show you something slightly different then let us say I have this height of the liquid in the tank and I have F i.

Now for example if I were to put in a control valve here and then use this measurement compare it with the set point and then use a controller to go and manipulate this valve. Now what I am doing is that the height or the control variable of the tank is being controlled by manipulating this valve which means I am manipulating the inlet flow rate into this. So something which was basically a disturbance variable in the last example becomes the manipulated variable or an input variable.

In this case because this is at a fixed position here there is no manipulation of this however depending on what is the pressure at this side of the system which is downstream even for a fixed position this F not can keep changing because the flow rate across the valve depends not only on the resistance but also the delta P across the valve, so if the downstream pressure keeps changing that will make this F not change now in this case F not will become a disturbance variable.

So the definition of manipulated variable and the disturbance variable depends on what your control architecture is, in this case I have h as the output, F i as the input or manipulated variable and F not as the disturbance variable. So whenever you look at examples in the future you just want to be careful about where the manipulation is and what are things that I cannot manipulate or I choose not to manipulate and what are the things I am controlling?

Once you have a good idea of this then you will be able to say exactly what is an input or a manipulated variable, what is a disturbance variable and what is a control variable and the key point that I want to make here is how you define, which variable is what depends on what kind of instrumentation you have and what kind of control architecture that you are thinking about.

So in this case if I am going to manipulate the outlet flow rate with this control valve to control the height then F i becomes disturbance, in this case if I manipulate the inlet flow rate with height then this becomes a manipulated variable and F not becomes a disturbance variable. So this is something that I thought, I should introduce at this point so that there is some clarity in terms of this definition of variables in control.

Now when we did this modelling exercise in the last class I showed F i as an input variable. So if I did that then the intension is really to actually put in a valve there and then control the flow rate of F i with a valve which I did not show in the last example but whenever you say something is an input variable then the intension is going to manipulate that and then control variable.

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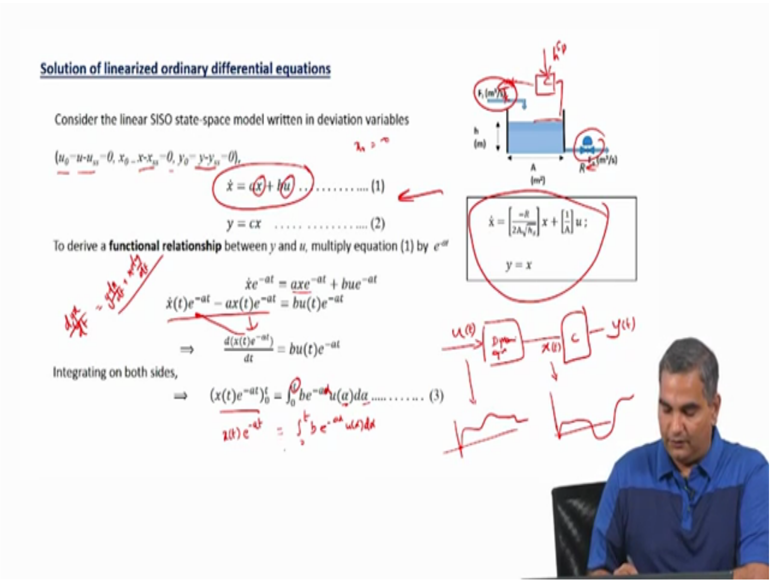

Now let us go back to the state space form that we have and again though I put in a valve here this is to introduce the notion of R and not really to finish the control diagram. So now if I have a tank like this and let us say I am actually going to manipulate this with a control valve, right so I have this as my F i, then we derive this equation which is of this form, right. So basically what I am saying is I am going to so this is a structure that I am going to use ultimately, I do not have it right now but this is the idea I am going to manipulate this and that is the reason why I am calling this as the input variable, okay.

So that is from a control perspective but we are still not closing the loop, so we are still looking at a open loop model and then we are trying to understand how to solve these equations. So if you look at this we showed that this could be written in the deviation variable form which basically means the u, I defined as u minus u steady state. So at a initial value it is 0 and x as x minus x steady state so the initial value is 0 and y as y minus y steady state so the initial value is 0.

And if you look at this this is of this form which we already talked about x dot equal ax plus bu, y equal to cx. So basically what this is telling me is that when I look at this equation if you want to think about this in plane language then it says the rate of change of the state is a function of the current value of the state itself and also this input which I can manipulate to take any value I want.

So when you think about this if you start this system at some initial state let us say x not in this case it is going to be 0, then if there is no change in u there is not going to be any x dot, if u stays to be 0 x dot is 0 that means I am at steady state. So my process starts moving away from steady state if I make a change to my u in which case x will change because there is a rate of x and the minute x changes then this term also comes into picture and then you will have the rate of change of x as a function of both, its current value, and the value of the input.

Now this is a dynamic equation, whereas the second equation is a static equation which just says will there is an output which is some constant times x. So when we say we want to solve these equations then what we are looking for is if okay so I can put a block diagram here, so let us say I have this, I have a u so when I want to say I want to solve for this let me write this y.

So what we want to do is if I give you a time profile for the input then by solving this dynamic equation which is x dot equal to ax plus bu, I want to get a profile for the state as a function of time and once I have the profile for the state as a function of time then this block is simple multiplication by constant which will give me y of t.

So if you want to think about this pictorially basically we always start with time 0 where u is 0, then let us say I give some profile for u whatever that profile might be, then corresponding to this dynamic equation I am asking this question as to what the profile of the state will be so that will also start at 0 and may be this there something and the profile of y is simply a constant multiplied by this so that is (())(13:26).

So how do I solve this equation to do this? So a simple approach to solve this equation is by using what is called an integrating factor, so I can take the first equation which is x dot equal to ax plus bu and multiply by e power minus at on both sides so that will give me x dot e power minus at equal ax e power minus at plus bu e power minus at, then what I do is I take this to the other side and I get x dot e power minus at minus ax e power minus at is equal to bu e power minus at.

And you can quite easily recognize that this can be written like this so here we are using this result which says differential of let us say two functions y, x with respect to t is y times dx by dt plus x times dy by dt, okay. So if you use this if you do this so the first term will be x dot e power minus at, you will get this, the second term you have to differentiate this when you differentiate this it is minus ae power minus at which you have and then xt so this is the form that you have so this equal to this.

Now since you have this as d by dt you can take it to the other side and integrate this so when we integrate d of this we will get this itself and you have to do this as the definite integral so you do this integration from 0 to t. So on the right hand side you will have bu t e power alpha d alpha 0 to t, the reason why I changed this alpha is because I have t here I do not want to keep the running variable also as t because that will lead to all kinds of confusion so I just say (t) e power minus au alpha d alpha 0 to t, on the left hand side I have x t e power minus at 0 to t.

Now at t this will be x t e power minus at, at 0 it will be x 0 which is 0 so that will go out. So now simplifying this you will get this equation which is x t e power minus at minus 0 because x have 0, 0 because we have written things in the deviation variable form is equal to 0 to t b e power minus a alpha u alpha d alpha. Now what you can do is you can take this e power minus at to the other side and you can take it into the integral because this is a definite integral upto t, whereas running variable is alpha so you can definitely take it inside.

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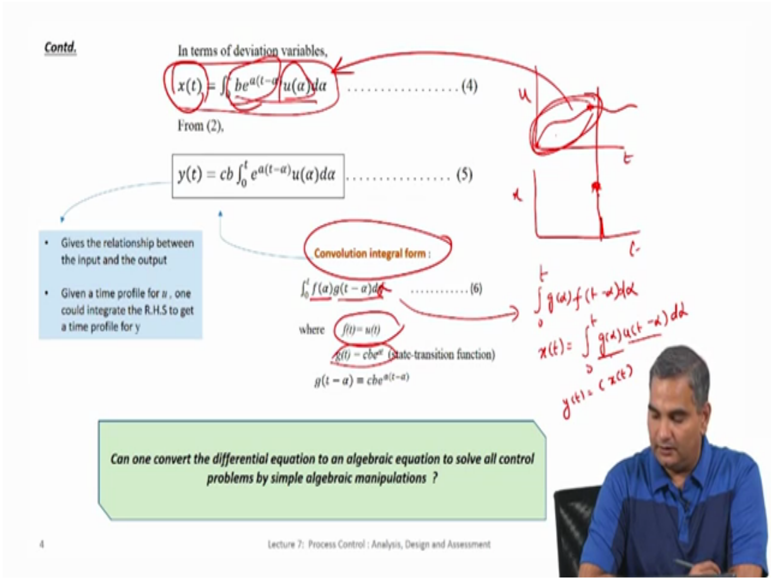

And once you take it inside and simplify this, this is the equation you will get, okay. Now this running variable alpha is also equivalent to t except that since we are using this variable definition t here I do not want to confuse you with using the same thing here as well. So when you look at this equation so basically what it says now is if you want to find what the state variable value will be at x of t then what you need to do is you need to integrate from 0 to that time and inside the integral you have this.

So when you do this integration basically what it says is this u alpha, alpha goes from 0 to t so basically you need to know how the input behave from the beginning initial state all the way upto time t, okay. So if you think about it this way, so let us say this u is like this, if you want to find the value of x at this point then what this says is I need to use how u behaved upto that time t, use that information in this integral and do the integration and you will get a value for x of t.

So in other words what this says is what state value you find at time t is a cumulative effect of all the input values that were there before. So basically 0 to ut every part of this curve has an effect on this prediction. So in put in simple terms the past values of u have an effect on your x of t and how much effect they have is given by this multiplication factor which is kind of weighting the input.

So you would expect an input value here to have a higher effect on this state as supposed to an input value here. In other words if you think about it in general common sense, if you do something today there is going to be an immediate effect tomorrow but if you do something today and think about what will happen 30 days later what happens 30 days later is not really so much dependent on what happen today, so that kind of idea comes here.

Now when you look at this, this is also called the convolution integral form where the output y is a function of some weighting matrix and this g of t minus alpha. So in this case actually you can look at this and then say well this is u of t and if the other term is g of t minus alpha then you can say g of t is cbe power at. Now you can also show very easily that this is also equal to 0 to t very simple to show this, this can also be written as g of alpha f of t minus alpha d, okay so this is something that actually you can easily show.

So that basically means that I can write the output or the state as a convolution between the input and (another matrix) another function and that function is typically called as the impulse function which we will see later, but for now I just want you to remember that what we are basically saying is x of t is going to be equal to 0 to t some convolution function so you could use this g of alpha u of t minus alpha d alpha.

So it is a convolution between u and some other function and this is called the impulse response function and you can get x t as this convolution integral and once you get this y t is basically c of x of t which is easy to calculate. So this is an important result because we are going to come back to this and we will use this convolution form when we look at Laplace transforms later.

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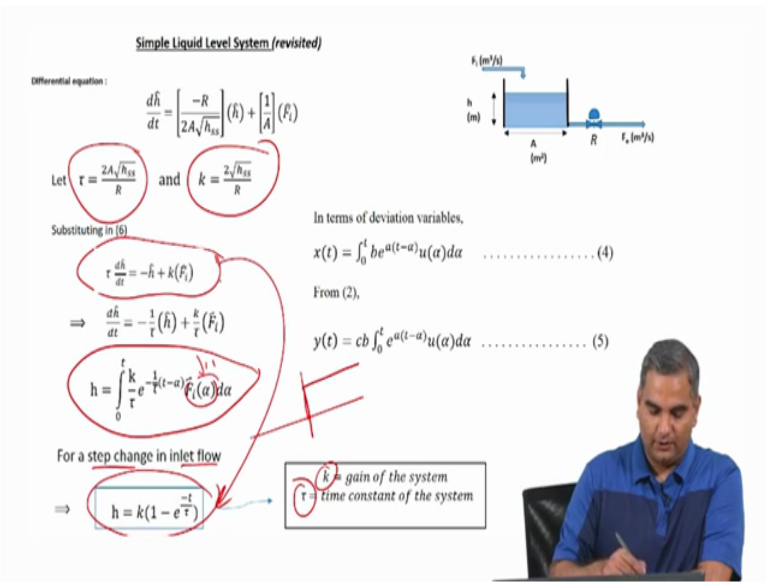

So if you take the liquid level example again I have this form here dh hat dt so this is just to show that these are in the deviation variable form. Now if you make these definitions for tau and k you can write this in this form the reason why I am writing it in this form you will realize once we do the transfer function models later because these terms k and tau are used in transfer function model definitions.

So once you have this basically move the tau to the other side and you get this equation and if you solve this equation using the convolution form you will get this, you can go back to the previous slide get the convolution equation form and put the correct terms there you will get this equation like this. Now what we can see is a following, supposing I wanted to predict how the height will behave as a function of changes in my inlet flow rate, so I have to say how my inlet flow rate changes.

So if I assume that my inlet flow rate changes in this way, so at time t equal to 0 suddenly let us say it becomes 1 and stays as 1. So this is what is called as a step change in inlet flow. So in which case what we are saying is at time equal to 0 it becomes 1, so it is always one after that so this I can replace by 1, if I want to predict how the height will change as a response to a step change if you do that integration then you will get this as a solution this I will leave it to you as an exercise just substitute 1 for F i alpha and then integrate this and see what happens.

In this equation we call this k as the gain of the system and tau as the time constant of the system, okay. So if you look at this then basically what it says is if you were to write your state space model in this form tau x dot is equal to x plus k u then the solution can be directly written down as this which is k times 1 minus e power minus t by tau, so this is something that you want to remember this is the response to change in the inlet flow rate as a step change, okay.

So remember this form and the result here and when we go to Laplace transforms later we will start with this form and I will show you what is the transfer function equivalent of this, then you will see that the solution to the transfer function equivalent will have to be this because we have already derived this result starting from this form to this equation.

So with this I will stop this lecture where we have talked about how to solve a state space equation and how to get the convolution form of the integral and how we can solve that for atleast one type of input which is the step change in the input and I showed you what the result is. So we will pick up from here in the next class and discuss more interesting aspects of control in detail, thank you.